

Mathematics 10th Unit 5

Sets and Functions

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Set

(K.B + U.B)

A well define collection of distinct objects is called Set. A set is represented by capital English alphabets.

For example: A= Set of integers,

$B = \{1, 2, 3, \dots, 10\}$ etc.

Presentation of Sets

(K.B + U.B)

A set is presented by 3 ways

- (i) Tabular form
- (ii) Descriptive Form
- (iii) Set builder notation

Some Important Sets

(K.B + U.B)

N = The set of natural numbers = $\{1, 2, 3, \dots\}$

W = The set of whole numbers = $\{0, 1, 2, 3, \dots\}$

Z = The set of all integers = $\{0, \pm 1, \pm 2, \pm 3, \dots\}$

E = The set of all even integers

= $\{0, \pm 2, \pm 4, \pm 6, \dots\}$

O = The set of all odd integers

= $\{\pm 1, \pm 3, \pm 5, \dots\}$

P = The set of prime numbers = $\{2, 3, 5, 7, \dots\}$

Q = The set of rational numbers

= $\left\{x/x = \frac{m}{n}, \text{ where } m, n \in Z \wedge n \neq 0\right\}$

$Q' =$ The set of irrational numbers

= $\left\{x/x \neq \frac{m}{n}, \text{ where } m, n \in Z \wedge n \neq 0\right\}$

R = The set of real numbers = $Q \cup Q'$

Empty Set or Null Set

(K.B + U.B)

A set having no element in it is called an empty set. It is represented by ϕ (phi) or $\{\}$.

Singleton Set

(K.B + U.B)

A set having only one element in it is called singleton set.

For example: $\{a\}, \{3\}$ etc.

Subset

(K.B + U.B)

(LHR 2016, D.G.K 2016)

“If A and B are two sets, such that every element of set A is present in set B, then set A is called subset of set B”. It is represented as $A \subseteq B$.

For Example

If $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$

Then $A \subseteq B$.

Note

(K.B + U.B)

- Number of all possible subsets = 2^n
- Number of all possible proper subsets = $2^n - 1$.
- Number of improper subsets = 1

Proper Subset

(K.B + U.B)

“If A and B are two sets, such that every element of set A is present in set B and there is at least one element in set B which is not in set A, then set A is called proper subset of set B”. It is represented as $A \subset B$.

For example:

If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$

Then set $A \subset B$.

Improper Subset

(K.B + U.B)

If A and B are two sets, such that every element of set A is present in set B and there is no more element in set B which is not in set A, then set A is called improper subset of set B.

For example: $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$

Then A is improper subset of set B.

Equal Sets

(K.B + U.B)

If A and B are two sets, such that $A \subseteq B$ and $B \subseteq A$ then $A = B$

For example

If $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$

Then $A = B$

Power Set (K.B + U.B)

A set consisting of all possible subsets of a set A is called power set of set A represented as $P(A)$.

For example:

If $A = \{1, 2\}$ then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Note (K.B + U.B)

- Formula to find number of elements in the power set = 2^n

Disjoint Sets (K.B + U.B)

Two sets having no common elements are called disjoint sets.

i.e. If $A \cap B = \phi$ then A and B are called disjoint sets.

For example

If $A = \{1, 2\}$, $B = \{a, b\}$, then A and B are disjoint sets.

Overlapping Sets (K.B + U.B)

Two sets are called overlapping sets if they have:

- At least one element is common
- Neither set is subset of other.

For example

If $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8, 10\}$

Then A and B are overlapping sets.

Operation on Sets**Union of Two Sets****(BWP 2014) (K.B + U.B)**

The union of two sets A and B written as $A \cup B$ (read as A union B) is a set consisting of all the elements which are either in A or in B or in both.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B \text{ both}\}$$

For example

If $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8, 10\}$

Then $A \cup B = \{1, 2, 3, 4, 5, 8, 10\}$

Intersection of Two Sets**(LHR 2015) (K.B + U.B)**

If A and B are two sets then a set consisting of common elements of set A and B is called A intersection B , represented by $A \cap B$.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

For example:

If $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8, 10\}$

Then $A \cap B = \{2\}$

Difference of Two Sets (K.B + U.B)

If A and B are two sets then their difference written as $A - B$ or $A \setminus B$ is a set consisting of all the elements of A which are not in B .

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

For example:

If $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8, 10\}$

Then $A - B = \{1, 3\}$

Universal Set (K.B + U.B)

A set consists of all the elements of the sets under consideration is called universal set. It is represented by capital English alphabet U or X .

For example:

U = Set of integers, $A = \{1, 2, 3\}$,

$B = \{2, 4, 6, 8, 10\}$, then U is universal set.

Complement of a Set (K.B + U.B)**(GRW 2014, SWL 2015, BWP 2016)**

If U is a universal set and A is its subset then $U - A$ is called complement of A , represented as A' or A^c .

For example:

If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$

$$A' = U - A$$

$$= \{1, 2, 3, 4, 5\} - \{1, 2, 3\}$$

$$= \{4, 5\}$$

Exercise 5.1**Q.1 (A.B)**

Given

$$X = \{1, 4, 7, 9\}$$

$$Y = \{2, 4, 5, 9\}$$

To Find

(i) $X \cup Y$

(ii) $X \cap Y$

(iii) $Y \cup X$

(iv) $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$
 $= \{1, 2, 4, 5, 7, 9\}$

$$(ii) \quad X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\} \\ = \{4, 9\}$$

$$(iii) \quad Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\} \\ = \{1, 2, 4, 5, 7, 9\}$$

$$(iv) \quad Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\} \\ = \{4, 9\}$$

Q.2 (A.B)**Given:**

X = set of prime numbers less than or equal to 17.

Y = set of first 12 natural number

To Find:

$$(i) \quad X \cup Y$$

$$(ii) \quad Y \cup X$$

$$(iii) \quad X \cap Y$$

$$(iv) \quad Y \cap X$$

Solution:**Here**

X = Set of prime numbers less than or equal to 17.

$$= \{2, 3, 5, 7, 11, 13, 17\}$$

Y = Set of first 12 natural numbers

$$= \{1, 2, 3, \dots, 12\}$$

$$(i) \quad X \cup Y = \{2, 3, 5, 7, 11, 13, 17\} \cup \{1, 2, 3, \dots, 12\} \\ = \{1, 2, 3, \dots, 12, 13, 17\}$$

$$(ii) \quad Y \cup X = \{1, 2, 3, \dots, 12\} \cup \{2, 3, 5, 7, 11, 13, 17\} \\ = \{1, 2, 3, \dots, 12, 13, 17\}$$

$$(iii) \quad X \cap Y = \{2, 3, 5, 7, 11, 13, 17\} \cap \{1, 2, 3, \dots, 12\} \\ = \{2, 3, 5, 7, 11\}$$

$$(iv) \quad Y \cap X = \{1, 2, 3, \dots, 12\} \cap \{2, 3, 5, 7, 11, 13, 17\} \\ = \{2, 3, 5, 7, 11\}$$

Q.3 (A.B)**Given**

$$X = \phi, \quad Y = Z^+, \quad T = O^+$$

To Find

$$(i) \quad X \cup Y$$

$$(ii) \quad X \cup T$$

$$(iii) \quad Y \cup T$$

$$(iv) \quad X \cap Y$$

$$(v) \quad X \cap T$$

$$(vi) \quad Y \cap T$$

Solution:

$$(i) \quad X \cup Y = \phi \cup Z^+ \\ = Z^+$$

$$(ii) \quad X \cup T = \phi \cup O^+ \\ = O^+$$

$$(iii) \quad Y \cup T = Z^+ \cup O^+ \\ = Z^+$$

$$(iv) \quad X \cap Y = \phi \cap Z^+ \\ = \phi$$

$$(v) \quad X \cap T = \phi \cap O^+ \\ = \phi$$

$$(vi) \quad Y \cap T = Z^+ \cap O^+ \\ = O^+$$

Q.4 (A.B)

$$U = \{x | x \in N \wedge 3 < x \leq 25\}$$

$$X = \{x | x \text{ is prime} \wedge 8 < x < 25\}$$

$$Y = \{x | x \in W \wedge 4 \leq x \leq 17\}$$

To Find

$$(i) \quad (X \cup Y)' \quad (ii) \quad X' \cap Y'$$

$$(ii) \quad (X \cap Y)' \quad (iv) \quad X' \cup Y'$$

Solution: Here

$$U = \{x / x \in N \wedge 3 < x \leq 25\}$$

$$= \{4, 5, 6, \dots, 25\}$$

$$X = \{x | x \text{ is prime} \wedge 8 < x < 25\}$$

$$= \{11, 13, 17, 19, 23\}$$

$$Y = \{x | x \in W \wedge 4 \leq x \leq 17\}$$

$$= \{4, 5, 6, 7, \dots, 17\}$$

$$(i) \quad X \cup Y = \{11, 13, 17, 19, 23\} \cup \{4, 5, 6, \dots, 17\}$$

$$= \{4, 5, 6, \dots, 17, 19, 23\}$$

$$(X \cup Y)' = U - (X \cup Y)$$

$$= \{4, 5, 6, \dots, 25\} - \{4, 5, 6, \dots, 17, 19, 23\}$$

$$= \{18, 20, 21, 22, 24, 25\}$$

$$(ii) \quad X' = U - X \\ = \{4, 5, 6, \dots, 25\} - \{11, 13, 17, 19, 23\}$$

$$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$Y' = U - Y$$

$$= \{4, 5, 6, \dots, 25\} - \{4, 5, 6, \dots, 17\}$$

$$= \{18, 19, 20, 21, 22, 23, 24, 25\}$$

$$X' \cap Y' = \{4, 5, 6, \dots, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$\cap \{18, 19, 20, \dots, 25\}$$

$$= \{18, 20, 21, 22, 24, 25\}$$

$$(iii) \quad X \cap Y = \{11, 13, 17, 19, 23\} \cap \{4, 5, 6, \dots, 17\}$$

$$= \{11, 13, 17\}$$

$$(X \cap Y)' = U - (X \cap Y)$$

$$= \{4, 5, 6, \dots, 25\} - \{11, 13, 17\}$$

$$= \{4, 5, 6, \dots, 10, 12, 14, 15, 16, 18, 19, 20, \dots, 25\}$$

$$(iv) \quad X' \cup Y'$$

$$= \{4, 5, 6, \dots, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$\cup \{18, 19, 20, 21, 22, 23, 24, 25\}$$

$$= \{4, 5, 6, \dots, 10, 12, 14, 15, 16, 18, 19, 20, \dots, 25\}$$

Q.5 Given (A.B)

$$X = \{2, 4, 6, \dots, 20\} \quad (\text{LHR 2014})$$

$$Y = \{4, 8, 12, \dots, 24\} \quad (\text{FSD 2015})$$

To Find

$$(i) \quad X - Y$$

$$(ii) \quad Y - X$$

Solution:

$$(i) \quad X - Y = \{2, 4, 6, \dots, 20\} - \{4, 8, 12, \dots, 24\}$$

$$= \{2, 6, 10, 14, 18\}$$

$$(ii) \quad Y - X = \{4, 8, 12, \dots, 24\} - \{2, 4, 6, \dots, 20\}$$

$$= \{24\}$$

Q.6 Given (BWP 2014) (A.B)

$$A = N, B = W \quad (\text{LHR 2015})$$

To Find (D.G.K 2014)

$$(i) \quad A - B$$

$$(ii) \quad B - A$$

Solution:

$$(i) \quad A - B = N - W = \{1, 2, 3, \dots\} - \{0, 1, 2, 3, \dots\}$$

$$= \phi$$

$$(ii) \quad B - A = W - N = \{0, 1, 2, 3, \dots\} - \{1, 2, 3, \dots\}$$

$$= \{0\}$$

Properties of Union and Intersection of Sets (K.B)

- **Commutative property of Union**

$$A \cup B = B \cup A$$

- **Commutative property of intersection**

$$A \cap B = B \cap A$$

- **Associative property of union**

$$A \cup (B \cap C) = (A \cup B) \cap C$$

- **Associative property of intersection**

$$A \cap (B \cup C) = (A \cap B) \cup C$$

- **Distributive property of union over intersection**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- **Distributive property of intersection over union**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- **De-Morgan's laws**

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Commutative Property of Union

(K.B + U.B)

For any two sets A and B, prove

that $A \cup B = B \cup A$.

Proof

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in (B \cup A)$

$\Rightarrow (A \cup B) \subseteq (B \cup A) \rightarrow (i)$

Now let $y \in (B \cup A)$

$\Rightarrow y \in B$ or $y \in A$ (by definition of union of sets)

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in (A \cup B)$

$\Rightarrow (B \cup A) \subseteq (A \cup B) \rightarrow (ii)$

From (i) and (ii), we have $A \cup B = B \cup A$.

(by definition of equal sets)

Commutative Property of Intersection

(MTN 2014) (K.B + U.B)

For any two sets A and B, prove

that $A \cap B = B \cap A$

Proof

Let $x \in (A \cap B)$

$\Rightarrow x \in A$ and $x \in B$

(by definition of intersection of sets)

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in (B \cap A)$

$\therefore (A \cap B) \subseteq (B \cap A) \rightarrow (i)$

Now let $y \in (B \cap A)$

$\Rightarrow y \in B$ and $y \in A$

(by definition of intersection of sets)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in (A \cap B)$

Therefore, $(B \cap A) \subseteq (A \cap B) \rightarrow (ii)$

From (i) and (ii), we

have $A \cap B = B \cap A$ (by definition of equal sets)

Associative Property of Union

(K.B + U.B)

For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$

Proof

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C$

(Associative property)

$\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Rightarrow x \in A \text{ or } x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$\Rightarrow (A \cup B) \cup C \subseteq A \cup (B \cup C) \rightarrow (i)$

Now let $y \in A \cup (B \cup C)$

$\Rightarrow y \in A \text{ or } y \in (B \cup C)$

$\Rightarrow y \in A \text{ or } (y \in B \text{ or } y \in C)$

$\Rightarrow y \in (A \text{ or } y \in B) \text{ or } y \in C$

$\Rightarrow y \in (A \cup B) \text{ or } y \in C$

$\Rightarrow y \in (A \cup B) \cup C$

$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C \rightarrow (ii)$

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

Associative Property of Intersection**(K.B + U.B)**

For any three sets A, B and C, prove that

$(A \cap B) \cap C = A \cap (B \cap C)$

Proof

Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C$

$\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \in C)$

$\Rightarrow x \in A \text{ and } x \in (B \cap C)$

$\Rightarrow x \in A \cap (B \cap C)$

$\Rightarrow (A \cap B) \cap C \subseteq A \cap (B \cap C) \rightarrow (i)$

Now let $y \in A \cap (B \cap C)$

$\Rightarrow y \in A \text{ and } y \in (B \cap C)$

$\Rightarrow y \in A \text{ and } (y \in B \text{ and } y \in C)$

$\Rightarrow (y \in A \text{ and } y \in B) \text{ and } y \in C$

$\Rightarrow y \in (A \cap B) \text{ and } y \in C$

$\Rightarrow y \in (A \cap B) \cap C$

$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C \rightarrow (ii)$

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Property of Union over Intersection (K.B + U.B)

For any three sets A, B and C, prove that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof

Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A \text{ or } x \in (B \cap C)$

$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \rightarrow (i)$

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B) \text{ and } y \in (A \cup C)$

$\Rightarrow (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \in C)$

$\Rightarrow y \in A \text{ or } (y \in B \text{ and } y \in C)$

$\Rightarrow y \in A \text{ or } y \in (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \rightarrow (ii)$

From (i) and (ii), we have

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributive Property of Intersection over Union (K.B + U.B)

For any three sets A, B and C, prove that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof

Let $x \in A \cap (B \cup C)$

$$\Rightarrow x \in A \text{ and } x \in (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B)$$

$$\text{or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \rightarrow (i)$$

Now let

$$\Rightarrow y \in (A \cap B) \cup (A \cap C)$$

$$\Rightarrow y \in (A \cap B) \text{ or } y \in (A \cap C)$$

$$\text{or } (y \in A \text{ and } y \in C)$$

$$\Rightarrow (y \in A \text{ and } y \in B)$$

$$\Rightarrow y \in A \text{ and } (y \in B \text{ or } y \in C)$$

$$\Rightarrow y \in A \text{ and } y \in (B \cup C)$$

$$y \in A \cap (B \cup C)$$

$$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \rightarrow (ii)$$

From (i) and (ii), we have

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De-Morgan's Laws (K.B + U.B)

(LHR 2016, MTN 2016, SGD 2015, BWP 2016, D.G.K 2016)

For any two sets A and B belonging from universal set U.

$$(A \cup B)' = A' \cap B' \rightarrow (i)$$

$$(A \cap B)' = A' \cup B' \rightarrow (ii)$$

(i) Proof

Let $x \in (A \cup B)'$

$$\Rightarrow x \notin (A \cup B) \quad (\text{by definition of complement of set})$$

$$\Rightarrow x \notin A \text{ and } x \notin B$$

$$\Rightarrow x \in A' \text{ and } x \in B'$$

$$\Rightarrow x \in A' \cap B' \quad (\text{by definition of intersection of sets})$$

$$\Rightarrow (A \cup B)' \subseteq A' \cap B' \rightarrow (i)$$

Let $y \in A' \cap B'$

$$\Rightarrow y \in A' \text{ and } y \in B'$$

$$\Rightarrow y \notin A \text{ and } y \notin B$$

$$\Rightarrow y \notin (A \cup B) \quad (\text{by definition of union of set})$$

$$y \in (A \cup B)'$$

$$A' \cap B' \subseteq (A \cup B)' \rightarrow (ii)$$

Using (i) and (ii), we have

$$(A \cup B)' = A' \cap B'$$

(ii) Proof

Let $x \in (A \cap B)'$

$$\Rightarrow x \notin A \cap B \quad (\text{by definition of complement of set})$$

$$\Rightarrow x \notin A \text{ or } x \notin B$$

$$\Rightarrow x \in A' \text{ or } x \in B'$$

$$\Rightarrow x \in A' \cup B' \quad (\text{by definition of union of set})$$

$$\Rightarrow (A \cap B)' \subseteq A' \cup B' \rightarrow (i)$$

Let $y \in A' \cup B'$

$$\Rightarrow y \in A' \text{ or } y \in B'$$

$$\Rightarrow y \notin A \text{ or } y \notin B$$

$$\Rightarrow y \notin A \cap B \quad (\text{by definition of intersection of sets})$$

$$\Rightarrow y \in (A \cap B)'$$

$$\Rightarrow A' \cup B' \subseteq (A \cap B)' \rightarrow (ii)$$

From (i) and (ii), we have proved that

$$(A \cap B)' = A' \cup B'$$

Exercise 5.2

Q.1 Given $X = \{1, 3, 5, 7, \dots, 19\}$ **(A.B)**

$$Y = \{0, 2, 4, 6, \dots, 20\}$$

$$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

To Find

(i) $X \cup (Y \cup Z)$

(ii) $(X \cup Y) \cup Z$

(iii) $X \cap (Y \cap Z)$

(iv) $(X \cap Y) \cap Z$

(v) $X \cup (Y \cap Z)$

(vi) $(X \cup Y) \cap (X \cup Z)$

(vii) $X \cap (Y \cup Z)$

(viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

(i) $X \cup (Y \cup Z)$ **(RWP 2015) (A.B)**

$$= \{1, 3, 5, 7, \dots, 19\} \cup$$

$$(\{0, 2, 4, 6, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$$

$$= \{1, 3, 5, \dots, 19\} \cup$$

$$\{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23\}$$

(ii) $(X \cup Y) \cup Z$ **(A.B)**

$$X \cup Y = \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}$$

$$= \{0, 1, 2, 3, \dots, 20\}$$

$$(X \cup Y) \cup Z$$

$$= \{0, 1, 2, 3, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{0, 1, 2, 3, \dots, 20, 23\}$$

$$(iii) \quad X \cap (Y \cap Z) \quad (A.B)$$

$$Y \cap Z = \{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{2\}$$

$$X \cap (Y \cap Z) = \{1, 3, 5, 7, \dots, 19\} \cap \{2\}$$

$$= \{\}$$

$$(iv) \quad (X \cap Y) \cap Z \quad (A.B)$$

$$X \cap Y = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}$$

$$= \{\}$$

$$(X \cap Y) \cap Z = \{\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{\}$$

$$(v) \quad X \cup (Y \cap Z) \quad (A.B)$$

$$Y \cap Z = \{0, 2, 4, 6, \dots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{2\}$$

$$X \cup (Y \cap Z) = \{1, 3, 5, 7, \dots, 19\} \cup \{2\}$$

$$= \{1, 2, 3, 5, 7, \dots, 19\}$$

$$(vi) \quad (X \cup Y) \cap (X \cup Z) \quad (A.B)$$

$$X \cup Y = \{1, 3, 5, \dots, 19\} \cup \{0, 2, 4, 6, \dots, 20\}$$

$$= \{0, 1, 2, 3, \dots, 20\}$$

$$X \cup Z = \{1, 3, 5, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, \dots, 19, 23\}$$

$$(X \cup Y) \cap (X \cup Z) = \{0, 1, 2, 3, \dots, 20\}$$

$$\cap \{1, 2, 3, 5, 7, \dots, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, \dots, 19\}$$

$$(vii) \quad X \cap (Y \cup Z) \quad (A.B)$$

$$Y \cup Z = \{0, 2, 4, 6, \dots, 20\}$$

$$\cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$X \cap (Y \cup Z) = \{1, 3, 5, 7, \dots, 19\} \cap$$

$$\{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

$$(viii) \quad (X \cap Y) \cup (X \cap Z) \quad (A.B)$$

$$X \cap Y = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, \dots, 20\}$$

$$= \{\}$$

$$X \cap Z = \{1, 3, 5, 7, \dots, 19\}$$

$$\cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

$$(X \cap Y) \cup (X \cap Z) = \{\} \cup \{3, 5, 7, 11, 13, 17, 19\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

$$Q.2 \quad \text{Given } A = \{1, 2, 3, 4, 5, 6\}$$

$$(A.B + K.B)$$

$$B = \{2, 4, 6, 8\}$$

$$C = \{1, 4, 8\}$$

To Prove

- (i) $A \cap B = B \cap A$
(ii) $A \cup B = B \cup A$
(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof

$$(i) \quad A \cap B = B \cap A \quad (A.B + K.B)$$

$$\text{L.H.S} = A \cap B$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$$

$$= \{2, 4, 6\} \rightarrow (i)$$

$$\text{R.H.S} = B \cap A$$

$$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$$

$$= \{2, 4, 6\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A \cap B = B \cap A$$

Hence Proved

$$(ii) \quad A \cup B = B \cup A \quad (A.B + K.B)$$

Proof

$$\text{L.H.S} = A \cup B$$

$$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \rightarrow (i)$$

$$\text{R.H.S} = B \cup A$$

$$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A \cup B = B \cup A$$

Hence Proved

$$(iii) \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad (A.B + K.B)$$

Proof

$$\text{L.H.S} = A \cap (B \cup C)$$

$$B \cup C = \{2, 4, 6, 8\} \cup \{1, 4, 8\}$$

$$= \{1, 2, 4, 6, 8\}$$

$$A \cap (B \cup C)$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$$

$$= \{1, 2, 4, 6\} \rightarrow (i)$$

$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$

$$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$$

$$= \{2, 4, 6\}$$

$$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}$$

$$= \{1, 4\}$$

$$(A \cap B) \cup (A \cap C) = \{2, 4, 6\} \cup \{1, 4\}$$

$$= \{1, 2, 4, 6\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Hence Proved

$$(iv) \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(A.B + K.B)

Proof

$$\text{L.H.S} = A \cup (B \cap C)$$

$$B \cap C = \{2, 4, 6, 8\} \cap \{1, 4, 8\}$$

$$= \{4, 8\}$$

$$\text{L.H.S} = A \cup (B \cap C)$$

$$= \{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \rightarrow (i)$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Hence Proved

$$\text{Q.3} \quad \text{Given } U = \{1, 2, 3, \dots, 10\}$$

(A.B + K.B)

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{2, 3, 5, 7\}$$

To Prove

$$(i) \quad (A \cap B)' = A' \cup B'$$

$$(ii) \quad (A \cup B)' = A' \cap B'$$

$$(i) \quad (A \cap B)' = A' \cup B'$$

Proof

$$\text{L.H.S} = (A \cap B)'$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$= \{1, 2, 3, \dots, 10\} - \{3, 5, 7\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \rightarrow (i)$$

$$\text{R.H.S} = A' \cup B'$$

$$A' = U - A$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B$$

$$= \{1, 2, 3, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cap B)' = A' \cup B'$$

Hence Proved

(ii) $(A \cup B)' = A' \cap B'$ (A.B + K.B)

Proof

$$\text{L.H.S} = (A \cup B)'$$

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$$

$$= \{1, 2, 3, 5, 7, 9\}$$

$$\text{L.H.S} = (A \cup B)' = U - (A \cup B)$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 2, 3, 5, 7, 9\}$$

$$= \{4, 6, 8, 10\} \rightarrow (i)$$

$$\text{R.H.S} = A' \cap B'$$

$$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 10\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B)' = A' \cap B'$$

Hence Proved

Q.4 Given (A.B + K.B)

$$U = \{1, 2, 3, \dots, 20\}$$

$$X = \{1, 3, 7, 9, 15, 18, 20\}$$

$$Y = \{1, 3, 5, \dots, 17\}$$

To Prove

(i) $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

Proof

(i) $X - Y = X \cap Y'$ (A.B + K.B)

$$\text{L.H.S} = X - Y$$

$$= \{1, 3, 7, 9, 15, 18, 20\} - \{1, 3, 5, \dots, 17\}$$

$$= \{18, 20\} \rightarrow (i)$$

$$Y' = U - Y$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 17\}$$

$$= \{2, 4, 6, \dots, 18, 19, 20\}$$

$$\text{R.H.S} = X \cap Y'$$

$$= \{1, 3, 7, 9, 15, 18, 20\} \cap \{2, 4, 6, \dots, 18, 19, 20\}$$

$$= \{18, 20\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$X - Y = X \cap Y'$$

Hence Proved

(ii) $Y - X = Y \cap X'$ (A.B + K.B)

Proof

$$\text{L.H.S} = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\} \rightarrow (i)$$

$$\text{L.H.S} = Y \cap X'$$

$$X' = U - X$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$$

$$Y \cap X' = \{1, 3, 5, \dots, 17\}$$

$$\cap \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19\}$$

$$= \{5, 11, 13, 17\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$Y - X = Y \cap X'$$

Hence Proved

Venn Diagram

(K.B)

British mathematician John Venn introduced Venn Diagrams.

“A graphical method to represent sets in which Universal Set is represented by a rectangle and its subsets by a closed shaped (i.e. circle or oval) in it, is called Venn Diagram”.

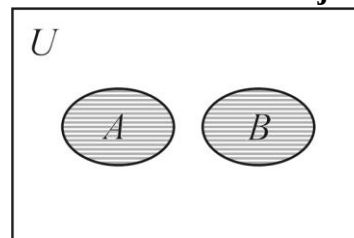
Operations on Sets Through Venn Diagrams

(K.B)

Union of Two Sets

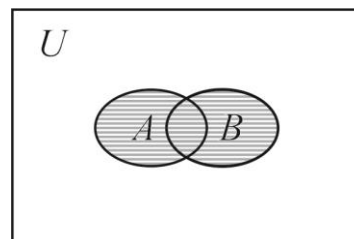
When A and B are Disjoint

(K.B)



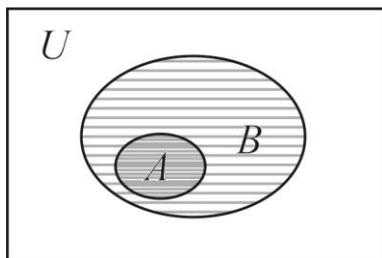
When A and B are overlapping

(K.B)

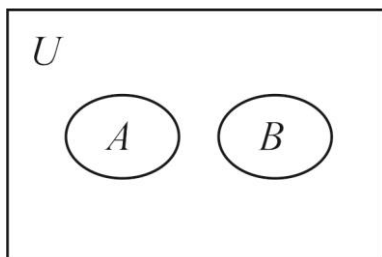


When $A \subseteq B$

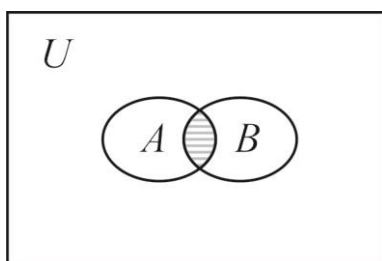
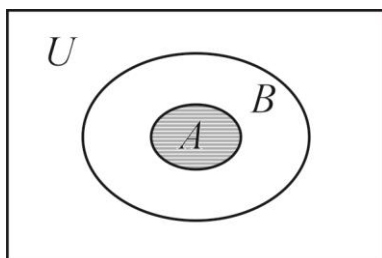
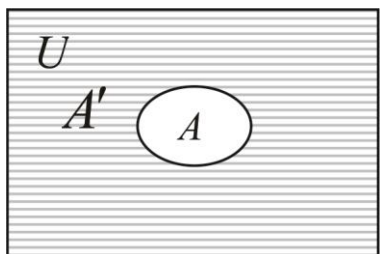
(K.B)

**Intersection of Two Sets****(K.B)**

When A and B are Disjoint



When A and B are overlapping

(K.B)When $A \subseteq B$ **Compliment of a Set****(K.B)****Exercise 5.3****Q.1** Given $U = \{1, 2, 3, 4, \dots, 10\}$

$$A = \{1, 3, 5, 7, 9\} \quad \textbf{(K.B)}$$

$$B = \{1, 4, 7, 10\} \quad \textbf{(A.B)}$$

(LHR 2017, GRW 2016, FSD 2017, SWL 2017, RWP 2016, BWP 2016, D.G.K 2016)

To Prove

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

(vi) $(B - A)' = B' \cup A$

Proof

(i) $A - B = A \cap B'$

L.H.S = $A - B$

$$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$$

$$= \{3, 5, 9\} \rightarrow \textbf{(i)}$$

R.H.S = $A \cap B'$

$$B' = U - B$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\}$$

$$= \{2, 3, 5, 6, 8, 9\}$$

$$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$$

$$= \{3, 5, 9\} \rightarrow \textbf{(ii)}$$

From equation (i) and (ii)

L.H.S = R.H.S

$$A - B = A \cap B'$$

Hence Proved

(ii) $B - A = B \cap A' \quad \textbf{(K.B + A.B)}$

L.H.S = $B - A$

$$= \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{4, 10\} \rightarrow \textbf{(i)}$$

R.H.S = $B \cap A'$

$$A' = U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B \cap A' = \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$$

$$= \{4, 10\} \rightarrow \textbf{(ii)}$$

From equation (i) and (ii)

L.H.S = R.H.S

$$B - A = B \cap A'$$

Hence Proved

(iii) $(A \cup B)' = A' \cap B' \quad \textbf{(K.B + A.B)}$

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$$

$$= \{1, 3, 4, 5, 7, 9, 10\}$$

$$\text{L.H.S} = (A \cup B)' = U - (A \cup B)$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 3, 4, 5, 7, 9, 10\}$$

$$= \{2, 6, 8\} \rightarrow (i)$$

$$\text{R.H.S} = A' \cap B'$$

$$A' = U - A = \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B = \{1, 2, 3, \dots, 10\} - \{1, 4, 7, 10\}$$

$$= \{2, 3, 5, 6, 8, 9\}$$

$$\text{R.H.S} = A' \cap B'$$

$$= \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$$

$$= \{2, 6, 8\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B)' = A' \cap B'$$

Hence Proved

$$(iv) \quad (A \cap B)' = A' \cup B' \quad (\mathbf{K.B + A.B})$$

Proof

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}$$

$$= \{1, 7\}$$

$$\text{L.H.S} = (A \cap B)' = U - (A \cap B)$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 7\}$$

$$= \{2, 3, 4, 5, 6, 8, 9, 10\} \rightarrow (i)$$

$$\text{R.H.S} = A' \cup B'$$

$$= \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\}$$

$$= \{2, 3, 4, 5, 6, 8, 9, 10\} \rightarrow (ii)$$

From equation (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cap B)' = A' \cup B'$$

Hence Proved

$$(v) \quad (A - B)' = A' \cup B \quad (\mathbf{K.B + A.B})$$

Proof

$$\text{L.H.S} = (A - B)'$$

$$A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$$

$$= \{3, 5, 9\}$$

$$\text{L.H.S} = (A - B)' = U - (A - B)$$

$$= \{1, 2, 3, \dots, 10\} - \{3, 5, 9\}$$

$$= \{1, 2, 4, 6, 7, 8, 10\} \rightarrow (i)$$

$$\text{R.H.S} = A' \cup B$$

$$A' = U - A$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$A' \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$$

$$= \{1, 2, 4, 6, 7, 8, 10\} \rightarrow (ii)$$

From (i) and (ii), we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(A - B)' = A' \cup B$$

Hence Proved

$$(vi) \quad (B - A)' = B' \cup A \quad (\mathbf{K.B + A.B})$$

(FSD 2015)

Proof

$$B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$$

$$B - A = \{4, 10\}$$

$$\text{L.H.S} = (B - A)' = U - (B - A)$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{4, 10\}$$

$$= \{1, 2, 3, 5, 6, 7, 8, 9\} \rightarrow (i)$$

$$\text{R.H.S} = B' \cup A$$

$$B' = U - B$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 4, 7, 10\}$$

$$= \{2, 3, 5, 6, 8, 9\}$$

$$B' \cup A = \{2, 3, 5, 6, 8, 9\} \cup \{1, 3, 5, 7, 9\}$$

$$= \{1, 2, 3, 5, 6, 7, 8, 9\} \rightarrow (ii)$$

From (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(B - A)' = B' \cup A$$

Hence Proved

Q.2 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$A = \{1, 3, 5, 7, 9\} \quad (\mathbf{K.B + A.B})$$

$$B = \{1, 4, 7, 10\}$$

$$C = \{1, 5, 8, 10\}$$

Then Verify

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(K.B + A.B)

Proof

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$$

$$= \{1, 3, 4, 5, 7, 9, 10\}$$

$$\text{L.H.S} = (A \cup B) \cup C$$

$$= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\}$$

$$= \{1, 3, 4, 5, 7, 8, 9, 10\} \rightarrow (i)$$

$$B \cup C = \{1, 4, 7, 10\} \cup \{1, 5, 8, 10\}$$

$$= \{1, 4, 5, 7, 8, 10\}$$

$$\text{R.H.S} = A \cup (B \cup C)$$

$$= \{1, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10\}$$

$$= \{1, 3, 4, 5, 7, 8, 9, 10\} \rightarrow (ii)$$

From (i) and (ii), we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Hence proved

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(K.B + A.B)

Proof

$$\text{L.H.S} = (A \cap B) \cap C$$

$$= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$$

$$= \{1, 7\} \cap \{1, 5, 8, 10\}$$

$$= \{1\} \rightarrow (i)$$

$$\text{R.H.S} = A \cap (B \cap C)$$

$$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$$

$$= \{1, 3, 5, 7, 9\} \cap \{1, 10\}$$

$$= \{1\} \rightarrow (ii)$$

From (i) and (ii), we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Hence proved

(iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(K.B + A.B)

Proof

$$\text{L.H.S} = A \cup (B \cap C)$$

$$= \{1, 3, 5, 7, 9\} \cup (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$$

$$= \{1, 3, 5, 7, 9\} \cup \{1, 10\}$$

$$= \{1, 3, 5, 7, 9, 10\} \rightarrow (i)$$

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$$

$$= \{1, 3, 4, 5, 7, 9, 10\}$$

$$A \cup C = \{1, 3, 5, 7, 9\} \cup \{1, 5, 8, 10\}$$

$$= \{1, 3, 5, 7, 8, 9, 10\}$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$= \{1, 3, 4, 5, 7, 9, 10\} \cap \{1, 3, 5, 7, 8, 9, 10\}$$

$$= \{1, 3, 5, 7, 9, 10\} \rightarrow (ii)$$

From (i) and (ii), we get

$$\text{L.H.S} = \text{R.H.S}$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Hence Proved

(iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(K.B + A.B)

Proof

$$B \cup C = \{1, 4, 5, 7, 8, 10\} \cup \{1, 5, 8, 10\}$$

$$= \{1, 4, 5, 7, 8, 10\}$$

$$\text{L.H.S} = A \cap (B \cup C)$$

$$= \{1, 3, 5, 7, 9\} \cap \{1, 4, 5, 7, 8, 10\}$$

$$= \{1, 5, 7\} \rightarrow (i)$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}$$

$$= \{1, 7\}$$

$$A \cap C = \{1, 3, 5, 7, 9\} \cap \{1, 5, 8, 10\}$$

$$= \{1, 5\}$$

$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$

$$= \{1, 7\} \cup \{1, 5\}$$

$$= \{1, 5, 7\} \rightarrow (ii)$$

From (i) and (ii), we get

$$L.H.S = R.H.S$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Hence Proved

Q.3 Given $U = N$, $A = \phi$, $B = P$

(K.B + A.B)

(LHR 2016, GRW 2016, RWP 2015, 17, MTN 2016)

To prove:

De-Morgan's Laws

$$(i) \quad (A \cup B)' = A' \cap B'$$

$$(ii) \quad (A \cap B)' = A' \cup B'$$

$$(i) \quad (A \cup B)' = A' \cap B'$$

Proof

$$L.H.S = (A \cup B)'$$

$$A \cup B = \phi \cup P \\ = P$$

$$L.H.S = (A \cup B)' = U - (A \cup B) \\ = N - P \rightarrow (i)$$

$$R.H.S = A' \cap B'$$

$$A' = U - A = N - \phi \\ = N$$

$$B' = U - B = N - P$$

$$R.H.S = A' \cap B' = N \cap (N - P) \\ = N - P \rightarrow (ii)$$

From equation (i) and (ii)

$$L.H.S = R.H.S$$

$$(A \cup B)' = A' \cap B'$$

Hence Proved

$$(ii) \quad (A \cap B)' = A' \cup B' \quad \text{(K.B + A.B)}$$

Proof

$$A \cap B = \phi \cap P \\ = \phi$$

$$L.H.S = (A \cap B)' \\ = U - (A \cap B) \\ = N - \phi \\ = N$$

$$R.H.S = A' \cup B' \\ = N \cup (N - P)$$

$$= N \rightarrow (ii)$$

From equation (i) and (ii)

$$L.H.S = R.H.S$$

$$(A \cap B)' = A' \cup B'$$

Hence Proved

Method-(II)

$$(i) \quad (A \cup B)' = A' \cap B' \quad \text{(K.B + A.B)}$$

Proof

$$A \cup B = \{ \} \cup \{2, 3, 5, 7, \dots\} \\ = \{2, 3, 5, 7, \dots\}$$

$$L.H.S = (A \cup B)' = U - (A \cup B) \\ = \{1, 2, 3, \dots\} - \{2, 3, 5, 7, \dots\} \\ = \{1, 4, 6, 8, 9, 10, 12, \dots\} \rightarrow (i)$$

$$A' = U - A \\ = \{1, 2, 3, \dots\} - \{ \} \\ = \{1, 2, 3, 4, \dots\}$$

$$B' = U - B \\ = \{1, 2, 3, 4, \dots\} - \{2, 3, 5, 7, \dots\} \\ = \{1, 4, 6, 8, 9, 10, \dots\}$$

Now

$$R.H.S = A' \cap B' = \{1, 2, 3, \dots\} \cap \{1, 4, 6, 8, 9, 10, \dots\} \\ = \{1, 4, 6, 8, 9, 10, \dots\} \rightarrow (ii)$$

From equation (i) and (ii)

$$L.H.S = R.H.S$$

$$(A \cup B)' = A' \cap B'$$

Hence Proved

Q.4 If $U = \{1, 2, 3, \dots, 10\}$ **(K.B + A.B)**

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{2, 3, 4, 5, 8\}$$

then prove the following question by venn diagram.

$$(i) \quad A - B = A \cap B'$$

$$(ii) \quad B - A = B \cap A'$$

$$(iii) \quad (A \cup B)' = A' \cap B'$$

$$(iv) \quad (A \cap B)' = A' \cup B'$$

$$(v) \quad (A - B)' = A' \cup B$$

$$(vi) \quad (B - A)' = B' \cup A$$

(Through Venn diagram)

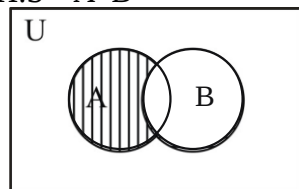
$$(i) \quad A - B = A \cap B' \quad \text{(K.B)}$$

Proof

A and B are overlapping sets because

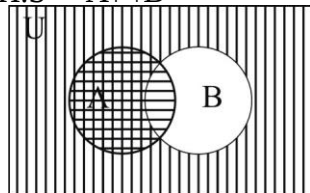
$$A \cap B = \{3, 5\}$$

$$\text{L.H.S} = A - B$$



$$B - A \equiv \text{||||}$$

$$\text{R.H.S} = A \cap B'$$



$$A \equiv \text{||||, ##}$$

$$B' \equiv \text{||||, ##}$$

$$A \cap B' \equiv \text{||||, ##}$$

Same regions represent that

$$\text{L.H.S} = \text{R.H.S}$$

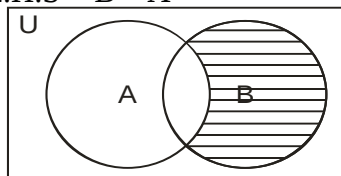
$$A - B = A \cap B'$$

Hence Proved

$$\text{(ii)} \quad B - A = B \cap A'$$

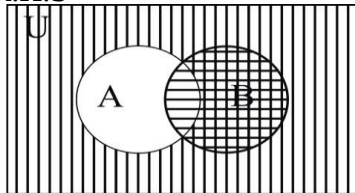
(K.B)

$$\text{L.H.S} = B - A$$



$$B - A \equiv \text{||||}$$

$$\text{R.H.S} = B \cap A'$$



$$B \equiv \text{||||, ##}$$

$$A' \equiv \text{||||, ##}$$

$$B \cap A' \equiv \text{||||, ##}$$

Same regions represent that

$$\text{L.H.S} = \text{R.H.S}$$

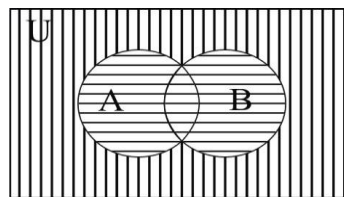
$$B - A = B \cap A'$$

Hence Proved

$$\text{(iii)} \quad (A \cup B)' = A' \cap B'$$

(K.B)

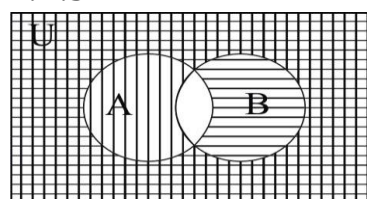
$$\text{L.H.S} = (A \cup B)'$$



$$A \cup B \equiv \text{||||}$$

$$(A \cup B)' \equiv \text{||||}$$

$$\text{R.H.S} = A' \cap B'$$



$$A' \equiv \text{||||, ##}$$

$$B' \equiv \text{||||, ##}$$

$$A' \cap B' \equiv \text{||||, ##}$$

Same regions represent that

$$\text{L.H.S} = \text{R.H.S}$$

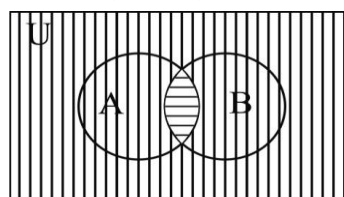
$$(A \cup B)' = A' \cap B'$$

Hence Proved

$$\text{(iv)} \quad (A \cap B)' = A' \cup B'$$

(K.B)

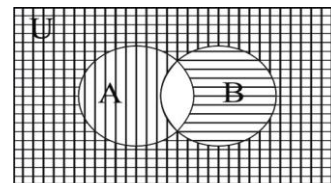
$$\text{L.H.S} = (A \cap B)'$$



$$A \cap B \equiv \text{||||}$$

$$(A \cap B)' \equiv \text{||||}$$

$$\text{R.H.S} = A' \cup B'$$



$$A' \equiv \text{||||, ##}$$

$$B' \equiv \text{||||, ##}$$

$$A' \cup B' \equiv \text{||||, ##}$$

Same regions represent that

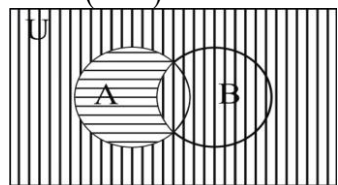
$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cap B)' = A' \cup B'$$

Hence Proved

$$(v) \quad (A - B)' = A' \cup B \quad (\text{K.B})$$

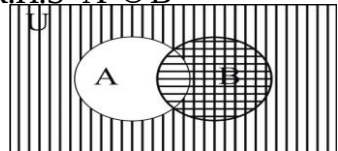
$$\text{L.H.S} = (A - B)'$$



$$A - B \equiv$$

$$(A - B)' \equiv$$

$$\text{R.H.S} = A' \cup B$$



$$A' \equiv$$

$$B \equiv$$

$$A' \cup B \equiv$$

Same regions represent that

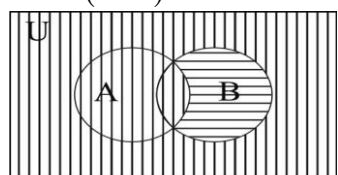
$$\text{L.H.S} = \text{R.H.S}$$

$$(A - B)' = A' \cup B$$

Hence Proved

$$(vi) \quad (B - A)' = B' \cup A \quad (\text{K.B})$$

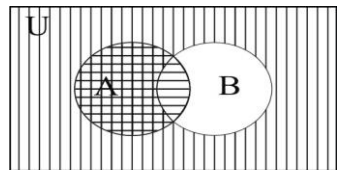
$$\text{L.H.S} = (B - A)'$$



$$B - A \equiv$$

$$(B - A)' \equiv$$

$$\text{R.H.S} = B' \cup A$$



$$A \equiv$$

$$B' \equiv$$

$$B' \cup A \equiv$$

Same regions represent that

$$\text{L.H.S} = \text{R.H.S}$$

$$(B - A)' = B' \cup A$$

Hence Proved

Ordered Pairs (K.B)

Any two numbers x and y , written in the form (x, y) is called an ordered pair. In an ordered pair (x, y) , the order of numbers is important.

For example: $(3, 2)$ is different from $(2, 3)$.

Hence $(x, y) \neq (y, x)$ unless $x = y$.

Cartesian Product (K.B)

(FSD 2015, D.G.K 2015, 17)

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all the ordered pairs (x, y) such that $x \in A$ and $y \in B$

$$\text{i.e. } A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

For example:

$$\text{If } A = \{1, 2\}, B = \{3, 4\}$$

$$\text{Then } A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

Exercise 5.4

Q.1 Given $A = \{a, b\}$ (GRW 2014) **(A.B)**

$$B = \{c, d\} \quad (\text{RWP 2015})$$

To Find

$$(i) \quad A \times B$$

$$(ii) \quad B \times A$$

Solution:

$$(i) \quad A \times B = \{a, b\} \times \{c, d\} \\ = \{(a, c), (a, d), (b, c), (b, d)\}$$

$$(ii) \quad B \times A = \{c, d\} \times \{a, b\} \\ = \{(c, a), (c, b), (d, a), (d, b)\}$$

Q.2 Given $A = \{0, 2, 4\}$ **(A.B)**

$$B = \{-1, 3\}$$

(FSD 2015, SWL 2017, BWP 2015)

To Find

$$A \times B \quad B \times A \quad A \times A \quad B \times B$$

Solution:

$$(i) \quad A \times B = \{0, 2, 4\} \times \{-1, 3\} \\ = \{(0, -1), (0, 3), (2, -1), (2, 3), (4, -1), (4, 3)\}$$

$$(ii) \quad B \times A = \{-1, 3\} \times \{0, 2, 4\} \\ = \{(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)\}$$

$$(iii) \quad A \times A = \{0, 2, 4\} \times \{0, 2, 4\} \\ = \{(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)\}$$

$$(iv) \quad B \times B = \{-1, 3\} \times \{-1, 3\} \\ = \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$$

Q.3 (A.B)

(i) **Given** $(a - 4, b - 2) = (2, 1)$
(GRW 2016, 17, FSD 2017, SWL 2015, SGD 2017, MTN 2016)

Required

Values of a and b

Solution:

Given that

$$(a - 4, b - 2) = (2, 1)$$

By comparing, we get

$$a - 4 = 2 \quad \text{and} \quad b - 2 = 1$$

$$a = 2 + 4 \quad b = 1 + 2$$

$$\Rightarrow a = 6, \quad b = 3$$

(ii) **Given** $(2a + 5, 3) = (7, b - 4)$ **(A.B)**

(SWL 2017, MTN 2017, RWP 2016, D.G.K 2015)

Required

Values of a and b

Solution:

Given that

$$(2a + 5, 3) = (7, b - 4)$$

By comparing, we get

$$2a + 5 = 7 \quad \text{and} \quad 3 = b - 4$$

$$2a = 7 - 5 \quad 3 + 4 = b$$

$$2a = 2 \quad 7 = b$$

$$a = 1 \quad b = 7$$

(iii) **Given** $(3 - 2a, b - 1) = (a - 7, 2b + 5)$
(GRW 2014) **(A.B)**

Required

Values of a and $b = ?$

Solution:

Given that

$$(3 - 2a, b - 1) = (a - 7, 2b + 5)$$

By comparing, we get

$$3 - 2a = a - 7 \quad \text{and} \quad b - 1 = 2b + 5$$

$$-2a - a = -7 - 3 \quad b - 2b = 5 + 1$$

$$-3a = -10 \quad -b = 6$$

$$a = \frac{-10}{-3} \quad b = -6$$

$$\Rightarrow a = \frac{10}{3}$$

Q.4 Given

$$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\} \quad \textbf{(A.B)}$$

Required

Set X and Y

Solution:

Given that

$$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$$

$$X = \{a, b, c, d\}$$

$$Y = \{a\}$$

Q.5 Given $X = \{a, b, c\}$ **(A.B)**

$$Y = \{d, e\}$$

Required

Number of elements in

$$(i) \quad X \times Y$$

$$(ii) \quad Y \times X$$

$$(iii) \quad X \times X$$

Solution:

$$(i) \quad X \times Y$$

$$n(X) = 3$$

$$n(Y) = 2$$

$$n(X \times Y) = n(X) \times n(Y)$$

$$= 3 \times 2$$

$$= 6$$

Method-2 (K.B)

$$(i) \quad \text{Number of elements in } X \times Y = m \times n$$

$$= 3 \times 2$$

$$= 6$$

$$(ii) \quad \text{Number of elements in } Y \times X = m \times n$$

$$= 2 \times 3$$

$$= 6$$

$$(iii) \quad \text{Number of elements in } X \times X = m \times n$$

$$= 3 \times 3$$

$$= 9$$

Binary Relation (K.B)

(GRW 2017, RWP 2016, SWL 2016, MTN 2016)

If A and B are any two non-empty sets then a subset $R \subseteq A \times B$ is called a binary relation from set A into set B .

$$\text{i.e. } A = \{1, 2\}, B = \{a\}$$

Then $A \times B = \{(1, a), (2, a)\}$ and

$$R_1 = \{(1, a)\}, \quad R_2 = \{(2, a)\}$$

$$R_3 = \{(1, a), (2, a)\}, \quad R_4 = \phi$$

Are all possible relations.

Note (K.B + U.B)

Formula to find number of binary relations is $2^{m \times n}$ where m = number of elements in set A and n = number of elements in set B.

Domain of Relation (K.B)

Domain of a relation denoted by $\text{Dom}(R)$ is a set consisting of all the first elements of each ordered pair in the relation.

For example:

$$\text{If } R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$$

$$\text{Then } \text{Dom}(R) = \{0, 2, 3\}$$

Range of a Relation (K.B)

Range of a relation is a set containing all the second elements of each ordered pair of the relation.

For Example:

$$\text{If } R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$$

$$\text{Then } \text{Range}(R) = \{2, 3, 4\}$$

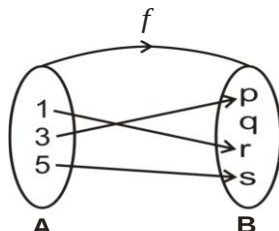
Function (K.B + U.B + A.B)

(LHR 2014)

Suppose A and B are two non-empty sets, then relation $f : A \rightarrow B$ is called a function if

- (i) $\text{Dom}(f) = A$
- (ii) Every $x \in A$ appears in one and only one ordered pair in f .

Example:



Types of Function (K.B + U.B)

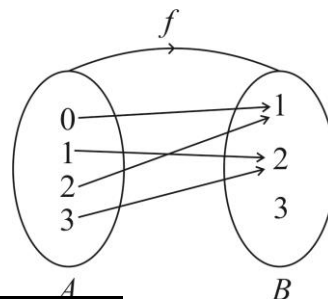
Into Function (A.B)

A function $f : A \rightarrow B$ is called an into function, if at least one element in set B is not an image of some element of set A i.e., $\text{Range of } f \subset B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \rightarrow B$ such that

$$f = \{(0, 1), (1, 2), (2, 1), (3, 2)\}.$$

$\therefore \text{Range}(f) = \{1, 2\} \subset B$. Thus f is an into function.



Onto Function (K.B + U.B + A.B)

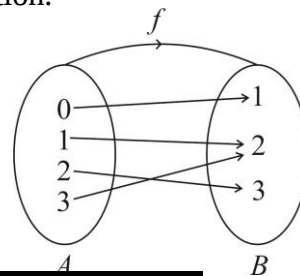
(SWL 2016, BWP 2014, 16, 17)

A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e., $\text{Range of } f = B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \rightarrow B$ such that

$$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}.$$

$\therefore \text{Range}(f) = \{1, 2, 3\} = B$. Thus f is an onto function.



One-to-One Function

(LHR 2015) (K.B + U.B + A.B)

A function $f : A \rightarrow B$ is called one-one function if all distinct elements of A have distinct images in B, i.e., $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \in A$

$$\text{or } x_1 \neq x_2 \in A \quad \forall x_1 \in A \Rightarrow f(x_1) \neq f(x_2)$$

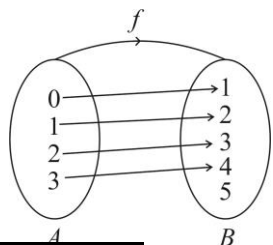
For example, if $A = \{0, 1, 2, 3\}$ and

$B = \{1, 2, 3, 4, 5\}$, then we define a function

$f : A \rightarrow B$ such that

$$f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$$

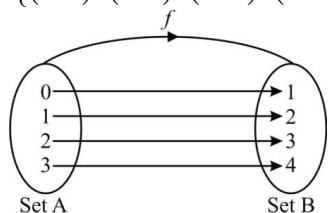
f is one-one function because no element in B is repeated.

**Bijjective Function**(GRW 2014) **(K.B + U.B + A.B)**

A function $f : A \rightarrow B$ is called bijective function iff function f is one-one and onto.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$

Then $f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

**Note****(K.B)**

- Every function is ration but converse may not be true.
- Every function may not be one-one.
- Every function may not be onto.

Exercise 5.5**Q.1 Given (LHR 2014) (A.B)**

$$L = \{a, b, c\}$$

$$M = \{3, 4\}$$

To Find

Two binary relations of $L \times M$ and $M \times L$

Solution:

$$L \times M = \{a, b, c\} \times \{3, 4\}$$

$$= \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$$

Two binary relations of $L \times M$ are:

$$R_1 = \{(a, 3), (a, 4)\}$$

$$R_2 = \{(a, 3), (b, 3), (c, 4)\}$$

$$\text{Now } M \times L = \{3, 4\} \times \{a, b, c\}$$

$$= \{(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)\}$$

Two binary relations of $M \times L$ are:

$$R_1 = \{(3, a)\}$$

$$R_2 = \{(3, a), (4, b)\}$$

Q.2 Given (A.B)

(LHR 2015, GRW 2014)

$$Y = \{-2, 1, 2\}$$

Required:

Two binary relations of $Y \times Y$ and also find their domain and range.

Solution:

$$Y \times Y = \{-2, 1, 2\} \times \{-2, 1, 2\}$$

$$= \{(-2, -2), (-2, 1), (-2, 2), (1, -2),$$

$$(1, 1), (1, 2), (2, -2), (2, 1), (2, 2)\}$$

Two Binary relations for $Y \times Y$ are:

$$R_1 = \{(-2, -2), (-2, 1), (-2, 2)\}$$

$$R_2 = \{(-2, -2), (1, 1)\}$$

Domain and Range

$$\text{Dom}(R_1) = \{-2\}$$

$$\text{Range}(R_1) = \{-2, 1, 2\}$$

$$\text{Dom}(R_2) = \{-2, 1\}$$

$$\text{Range}(R_2) = \{-2, 1\}$$

Q.3 Given (A.B)

$$L = \{a, b, c\}$$

$$M = \{d, e, f, g\}$$

Required

Two binary relation for

(i) $L \times L$ (ii) $L \times M$ (iii) $M \times M$

Solution:

$$L \times L = \{a, b, c\} \times \{a, b, c\}$$

$$= \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

$$L \times M = \{a, b, c\} \times \{d, e, f, g\}$$

$$= \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)\}$$

$$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$$

$$= \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$$

Two binary relations for $L \times L$ are:

$$R_1 = \{(a, a), (b, b), (c, c)\}$$

$$R_2 = \{(a, b)\}$$

Two Binary Relations for $L \times M$ are:

$$R_1 = \{(a, d), (a, e)\}$$

$$R_2 = \{(a, d), (b, e), (c, f)\}$$

Two Binary Relations for $M \times M$ are:

$$R_1 = \{(d, d)\}$$

$$R_2 = \{(d, e), (f, g)\}$$

Q.4 If set M has 5 elements, then find the number of binary relations in M.

(A.B)

Solution:

$$\begin{aligned}\text{No of binary relations in } M &= 2^{m \times n} \\ &= 2^{5 \times 5} \\ &= 2^{25}\end{aligned}$$

Q.5 Given $L = \{x \mid x \in \mathbb{N} \wedge x \leq 5\}$

$$M = \{y \mid y \in \mathbb{P} \wedge y < 10\}$$

(A.B + K.B)

To Find

(i) $R_1 = \{(x, y) \mid y < x\}$

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Also find the domain and range of each relation.

Solution:

Here

$$L = \{x \mid x \in \mathbb{N} \wedge x \leq 5\} = \{1, 2, 3, 4, 5\}$$

$$\text{And } M = \{y \mid y \in \mathbb{P} \wedge y < 10\} = \{2, 3, 5, 7\}$$

$$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$$

$$\begin{aligned}&= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), \\ &\quad (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), \\ &\quad (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7)\}\end{aligned}$$

(i) Relation (A.B + K.B)

$$R_1 = \{(x, y) \mid y < x\}$$

$$= \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3)\}$$

Domain and Range

$$\text{Dom}(R_1) = \{3, 4, 5\}$$

$$\text{Range}(R_1) = \{2, 3\}$$

(ii) Relation (A.B + K.B)

$$R_2 = \{(x, y) \mid y = x\}$$

$$= \{(2, 2), (3, 3), (5, 5)\}$$

Domain and Range

$$\text{Dom}(R_2) = \{2, 3, 5\}$$

$$\text{Range}(R_2) = \{2, 3, 5\}$$

(iii) Relation (A.B + K.B)

$$R_3 = \{(x, y) \mid x + y = 6\}$$

$$= \{(1, 5), (3, 3), (4, 2)\}$$

Domain and Range of R_3

$$\text{Dom}(R_3) = \{1, 3, 4\}$$

$$\text{Range}(R_3) = \{2, 3, 5\}$$

(iv) Relation (A.B + K.B)

$$R_4 = \{(x, y) \mid y - x = 2\}$$

$$= \{(1, 3), (3, 5), (5, 7)\}$$

Domain and Range of R_4

$$\text{Dom}(R_4) = \{1, 3, 5\}$$

$$\text{Range}(R_4) = \{3, 5, 7\}$$

Q.6 Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range. (In each part (i) $\rightarrow$ (vi) the co-domain set is equal to the range set)

(A.B + K.B + U.B)

(i) $R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

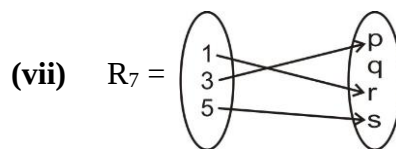
(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

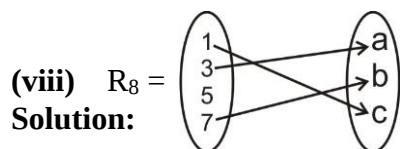
(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$





Solution:

(i) $R_1 = \{(1,1), (2,2), (3,3), (4,4)\}$

- $\text{Dom}(R_1) = \{1, 2, 3, 4\}$

There is no repetition in 1st element of any two ordered pairs, so R_1 is a function.

- $\text{Range}(R_1) = \{1, 2, 3, 4\}$

Also there is no repetition in 2nd element of any two ordered pairs. So R_1 is bijective function.

(ii) $R_2 = \{(1,2), (2,1), (3,4), (3,5)\}$

- $\text{Dom}(R_2) = \{1, 2, 3\}$

There is a repetition in first element of last two ordered pairs. So R_2 is not a function.

(iii) $R_3 = \{(b,a), (c,a), (d,a)\}$

- $\text{Dom}(R_3) = \{b, c, d\}$

There is no repetition in 1st element of any two pairs, so R_3 is a function.

- $\text{Range}(R_3) = \{a\}$

There is a repetition in second element of all ordered pairs. So R_3 is on to function.

(iv) $R_4 = \{(1,1), (2,3), (3,4), (4,3), (5,4)\}$

- $\text{Dom}(R_4) = \{1, 2, 3, 4, 5\}$

There is no repetition in 1st element of any two pairs, so R_4 is a function.

- $\text{Range}(R_4) = \{1, 3, 4\}$

There is a repetition in second element of all ordered pairs. So R_4 is on to function.

(v) $R_5 = \{(a,b), (b,a), (c,d), (d,e)\}$

- $\text{Dom}(R_5) = \{a, b, c, d\}$

There is no repetition in 1st element of any two ordered pairs, so R_5 is a function.

- $\text{Range}(R_5) = \{a, b, d, e\}$

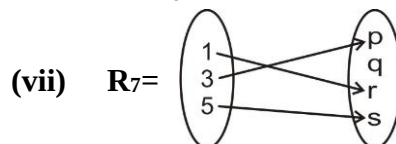
Also there is no repetition in 2nd element of any two ordered pairs. So R_5 is bijective function.

(vi) $R_6 = \{(1,2), (2,3), (1,3), (3,4)\}$

- $\text{Dom}(R_6) = \{1, 2, 3\}$

There is a repetition in first element of two ordered pairs.

So R_6 is not a function.

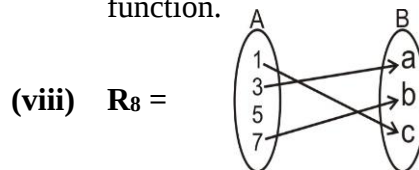


- $\text{Dom}(R_7) = \{1, 3, 5\} = A$

Also there is no repetition in 1st element of any two ordered pairs. So R_7 is a function.

- $\text{Range}(R_7) = \{p, r, s\} \subset B$

Also there is no repetition in 2nd element of any two ordered pairs. So R_7 is one-to-one (injective) function.



- $\text{Dom}(R_8) = \{1, 3, 7\} \neq A$

Therefore R_8 is not a function.

Miscellaneous Exercise 5

Q.1 Multiple choice questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

- (1) A collection of well-defined distinct objects is called; **(K.B)**
 (a) Subset (b) Power set
 (c) Set (d) None of these
- (2) A set $Q = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z} \wedge b \neq 0 \right\}$ is called a set of; **(K.B)**
 (a) Whole numbers (b) Natural numbers
 (c) Irrational numbers (d) Rational numbers
- (3) The different number of ways to describe a set are; **(K.B)**
 (a) 1 (b) 2
 (c) 3 (d) 4
- (4) A set with no element is called; **(K.B)**
 (a) Subset (b) Empty set
 (c) Singleton set (d) Super set
- (5) The set $\{x \mid x \in W \wedge x \leq 101\}$ is; **(K.B)**
 (a) Infinite set (b) Subset
 (c) Null set (d) Finite set
- (6) The set having only one element is called; **(K.B)**
 (a) Null set (b) Power set
 (c) Singleton set (d) Subset
- (7) Power set of an empty set is; **(K.B)**
 (a) ϕ (b) $\{a\}$
 (c) $\{\phi, \{a\}\}$ (d) $\{\phi\}$
- (8) The number of elements in power set $\{1, 2, 3\}$ is; **(K.B)**
 (a) 4 (b) 6
 (c) 8 (d) 9
- (9) If $A \subseteq B$, then $A \cup B$ is equal to; **(K.B)**
 (a) A (b) B
 (c) ϕ (d) None of these
- (10) If $A \subseteq B$ and $A \cap B$ is equal to; **(K.B)**
 (a) A (b) B
 (c) ϕ (d) None of these
- (11) If $A \subseteq B$ and $A - B$ is equal to; **(K.B)**
 (a) A (b) B
 (c) ϕ (d) $B - A$
- (12) $(A \cup B) \cup C$ is equal to; **(K.B)**
 (a) $A \cap (B \cup C)$ (b) $(A \cup B) \cap C$
 (c) $A \cup (B \cap C)$ (d) $A \cap (B \cap C)$

- (13) $A \cup (B \cap C)$ is equal to; **(K.B)**
 (a) $(A \cup B) \cap (A \cup C)$ (b) $(A \cup B) \cap C$
 (c) $(A \cap B) \cup (A \cap C)$ (d) $A \cap (B \cap C)$
- (14) If A and B are disjoint sets, then $A \cup B$ is equal to; **(K.B)**
 (a) A (b) B
 (c) ϕ (d) $B \cup A$
- (15) If number of elements in set A is 3 and in set B is 4, then number of elements in $A \times B$ is; **(K.B)**
 (a) 3 (b) 4
 (c) 12 (d) 7
- (16) If number of elements in set A is 3 and in set B is 2, then number of binary relations in $A \times B$ is; **(K.B)**
 (a) 2^3 (b) 2^6
 (c) 2^8 (d) 2^2
- (17) The domain of $R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$ is; **(K.B)**
 (a) $\{0, 3, 4\}$ (b) $\{0, 2, 3\}$
 (c) $\{0, 2, 4\}$ (d) $\{2, 3, 4\}$
- (18) The range of $R = \{(1, 3), (2, 2), (3, 1), (4, 4)\}$ is; **(K.B)**
 (a) $\{1, 2, 4\}$ (b) $\{3, 2, 4\}$
 (c) $\{1, 2, 3, 4\}$ (d) $\{1, 3, 4\}$
- (19) point $(-1, 4)$ lies in the quadrant; **(K.B)**
 (a) I (b) II
 (c) III (d) IV
- (20) the relation $\{(1, 2), (2, 3), (3, 3), (3, 4)\}$ is; **(K.B)**
 (a) Onto function (b) Into function
 (c) Not a function (d) One-One function

ANSWER KEY

i	c	vi	c	xi	c	xvi	b
ii	d	vii	d	xii	c	xvii	b
iii	c	viii	c	xiii	a	xviii	c
iv	b	ix	b	xiv	d	xix	b
v	d	x	a	xv	c	xx	c

Q.2 Write short answers of the following questions.

- (i) Define a subset and give one example.

(K.B + U.B)**Answer****Subset**

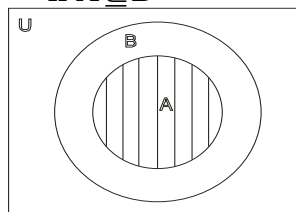
If A and B are two sets such that every element of set A is an element of set B , then set A is called subset of set B .

For example $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, \dots, 10\}$ then A is subset of B and represented by $A \subseteq B$.

- (ii) Write all the subsets of the set
- $\{a, b\}$

AnswerLet $S = \{a, b\}$ All possible subset of set S are: $\phi, \{a\}, \{b\}, \{a, b\}$

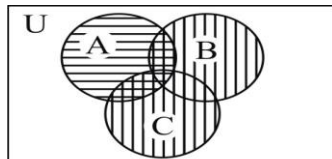
- (iii) Show
- $A \cap B$
- by Venn diagram. When

 $A \subseteq B$. (MTN 2014, SGD 2016) **(K.B)****Answer**If $A \subseteq B$  $A \cap B$

- (iv) Show by Venn diagram
- $A \cap (B \cup C)$

(K.B + A.B)**Answer**

Let A , B and C are overlapping (General Case)

 $A \cap (B \cup C)$ $B \cup C$ $A \cap (B \cup C)$

- (v) Define intersection of two sets

(K.B)**Answer****Intersection of Two Sets****(K.B)**

The intersection of two sets A and B , written as $A \cap B$ (read as 'A intersection B') is the set consisting of all the common elements of A and B . Thus

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Clearly $x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

For example, if $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$, then $A \cap B = \{c, d\}$

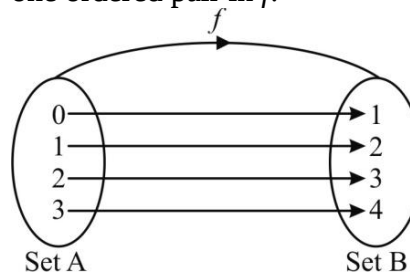
- (vi) Define a function

Answer**Function****(K.B)**

Suppose A and B are two non-empty sets, then relation $f : A \rightarrow B$ is called a function if

- (i)
- $\text{Dom } f = A$

- (ii) Every
- $x \in A$
- appears in one and only one ordered pair in
- f
- .



- (vii) Define one-one function

Answer**One - One Function****(K.B)**

A function $f : A \rightarrow B$ is called one-one function if all distinct elements of A have distinct images in B , i.e., $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \in A$ or $x_1 \neq x_2 \in A \Rightarrow f(x_1) \neq f(x_2)$.

For example, if $A = \{0, 1, 2, 3\}$ and

$B = \{1, 2, 3, 4, 5\}$, then we define a function

$f : A \rightarrow B$ such that

$$f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$$

f is one-one function because no element in B is repeated.

- (viii) Define an Onto function or Surjective function.

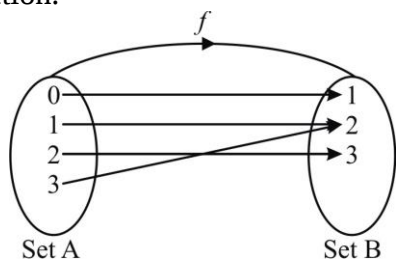
Answer**Onto (Surjective) Function****(K.B)**

A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an

image of at least one element of set A i.e.,
Range of $f = B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$,
then $f : A \rightarrow B$ such that

$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$. Here Range
 $f = \{1, 2, 3\} = B$. Thus f so defined is an onto
function.



(ix) Define a Bijective function
Answer

Bijective Function (K.B)

A function $f : A \rightarrow B$ is called bijective
function iff function f is one-one and onto.

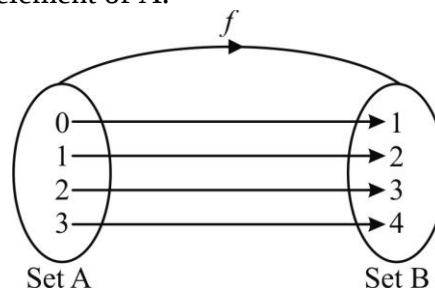
For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$

Then $f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

Q.3 Fill in the blanks

- | | |
|--|-------------|
| (i) If $A \subseteq B$, then $A \cup B =$ _____. | (K.B + A.B) |
| (ii) If $A \cap B = \phi$ then A and B are ____. | (K.B + A.B) |
| (iii) If $A \subseteq B$ and $B \subseteq A$ then ____. | (K.B + A.B) |
| (iv) $A \cap (B \cup C) =$ _____. | (K.B + A.B) |
| (v) $A \cup (B \cap C) =$ _____. | (K.B + A.B) |
| (vi) The complement of U is _____. | (K.B + A.B) |
| (vii) The complement of ϕ is _____. | (K.B + A.B) |
| (viii) $A \cap A^c =$ _____. | (K.B + A.B) |
| (ix) $A \cup A^c =$ _____. | (K.B + A.B) |
| (x) The set $\{x x \in A \text{ and } x \notin B\} =$ _____. | (K.B + A.B) |
| (xi) The point $(-5, -7)$ lies in _____ quadrant. | (K.B + A.B) |

Evidently this function is one-one because
distinct elements of A have distinct images
in B. This is an onto function also because
every element of B is the image of at least
one element of A.



(x) Write De Morgan's Laws. (K.B)

Answer

De Morgan's Laws

For any two sets A and B belonging to universal
set U,

- (i) $(A \cap B)' = A' \cup B'$
(ii) $(A \cup B)' = A' \cap B'$ are called De
Morgan's laws.

- (xii) The point $(4, -6)$ lies in _____ quadrant. **(K.B + A.B)**
- (xiii) The y co-ordinate of every point is _____ on-x-axis. **(K.B + A.B)**
- (xiv) The x co-ordinate of every point is _____ on-y-axis. **(K.B + A.B)**
- (xv) The domain of $\{(a,b), (b,c), (c,d)\}$ is _____. **(K.B + A.B)**
- (xvi) The range of $\{(a,a), (b,b), (c,c)\}$ is _____. **(K.B + A.B)**
- (xvii) Venn-diagram was first used by _____. **(K.B + A.B)**
- (xviii) A subset of $A \times A$ is called the _____ in A. **(K.B + A.B)**
- (xix) If $f : A \rightarrow B$ and range of $f = B$, then f is an _____ function. **(K.B + A.B)**
- (xx) The relation $\{(a,b), (b,c), (a,d)\}$ is _____ a function. **(K.B + A.B)**

ANSWER KEY

- | | |
|-----------------------------------|-------------------------|
| (i) B | (xiii) Zero |
| (ii) Disjoint sets | (xiv) Zero |
| (iii) $A = B$ | (xv) $\{a, b, c\}$ |
| (iv) $(A \cap B) \cup (A \cap C)$ | (xvi) $\{a, b, c\}$ |
| (v) $(A \cup B) \cap (A \cup C)$ | (xvii) John Venn |
| (vi) ϕ | (xviii) Binary relation |
| (vii) U | (xix) Onto |
| (viii) ϕ | (xx) Not |
| (ix) U | |
| (x) $A - B$ | |
| (xi) III rd quadrant | |
| (xii) IV th quadrant | |



CUT HERE

SELF TEST

Time: 40 min

Marks: 25

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. (7×1=7)

1 Power set of an empty set is:

- (A) ϕ (B) $\{a\}$
 (C) $\{0, \{a\}\}$ (D) $\{\phi\}$

2 $(A \cup B) \cup C$ is equal to:

- (A) $A \cap (B \cup C)$ (B) $(A \cup B) \cap C$
 (C) $A \cup (B \cup C)$ (D) $A \cap (B \cup C)$

3 The number of ways to describe a set are:

- (A) 1 (B) 2
 (C) 3 (D) 4

4 If $A \subseteq B$, then $A \cup B$ is equal to:

- (A) A (B) B
 (C) ϕ (D) U

5 The domain of $R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$ is:

- (A) $\{0, 3, 4\}$ (B) $\{0, 2, 3\}$
 (C) $\{0, 2, 4\}$ (D) $\{2, 3, 4\}$

6 Point $(-1, 4)$ lies in the quadrant:

- (A) I (B) II
 (C) III (D) IV

7 If the number of elements in set A is 3 and in set B is 4, then the number of elements in $A \times B$ is:

- (A) 3 (B) 4
 (C) 12 (D) 7

Q.2 Give Short Answers to following Questions.

(5×2=10)

(i) Define bijective function.

(ii) Show $A \cap B$ by Venn diagram when $A \subseteq B$

(iii) Find “a” and “b” if $(3 - 2a, b - 1) = (a - 7, 2b + 5)$.

(iv) If $P = \{-2, -1\}$ then make two binary relations for $P \times P$.

(v) If $U = \{x \mid x \in N \wedge 3 < x \leq 25\}$, $X = \{x \mid x \text{ is prime} \wedge 8 < x < 25\}$ and $Y = \{x \mid x \in W \wedge 4 \leq x \leq 17\}$. Find the $(X \cap Y)'$.

Q.3 Answer the following Questions.

(4+4=8)

(a) If $U = \{1, 2, 3, \dots, 10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 8\}$, then prove that $(A - B)' = A' \cup B$.

(b) If $L = \{x \mid x \in N \wedge x \leq 5\}$, $M = \{y \mid y \in P \wedge y < 10\}$, then make the following relations from L to M. $R = \{(x, y) \mid x + y = 6\}$ Also write the domain and range of the relation.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.