

Mathematics 10th Unit 2

Theory of Quadratics Equations

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Discriminant (BWP 2018) (U.B + K.B)

“For a standard quadratic equation $ax^2 + bx + c = 0$, the value of the expression $b^2 - 4ac$ is called discriminant.”

It is used to find the nature of roots without solving the equation.

Nature or Characteristics of the Roots (U.B + K.B)

Nature of a quadratic equation

$ax^2 + bx + c = 0$, when $a, b, c \in Q$ and $a \neq 0$ as:

- (i) If $b^2 - 4ac = 0$, then the roots are rational (real) and equal.
- (ii) If $b^2 - 4ac < 0$, then the roots are complex conjugate or imaginary.
- (iii) If $b^2 - 4ac > 0$, and is a perfect square, then the roots are rational (real) and unequal.
- (iv) If $b^2 - 4ac > 0$, and is not a perfect square, the roots are irrational (real) and unequal.

Note (K.B)

If given polynomial expression is a perfect square then discriminant is 0.

Example 2: (Page # 19)

Using discriminant, find the nature of the roots of the following equation and verify the result by solving the equation.

$$x^2 - 5x + 5 = 0$$

(LHR 2015, GRW 2016, 17, SWL 2017, RWP 2015, D.G.K 2017)

Solution:

$$x^2 - 5x + 5 = 0$$

Here $a = 1$, $b = -5$, $c = 5$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-5)^2 - 4(1)(5)$$

$$= 25 - 20 = 5$$

As discriminant > 0 but not perfect square, Roots are irrational (real) and unequal.

Verification:

Solving the equation by using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

Evidently, Roots are irrational (real) and unequal.

Example 2: (Page # 21)

Find k , if the roots of the equation $(k+3)x^2 - 2(k+1)x - (k+1) = 0$ are equal, if $k \neq -3$ (A.B)

Solution:

$$(k+3)x^2 - 2(k+1)x - (k+1) = 0$$

Here

$$a = k+3, b = -2(k+1), c = -(k+1)$$

As roots are equal, discriminant is zero.

$$\Rightarrow \text{Disc.} = b^2 - 4ac = 0$$

$$[-2(k+1)]^2 - 4(k+3)[-(k+1)] = 0$$

$$4(k+1)^2 + 4(k+3)(k+1) = 0$$

$$4(k+1)[(k+1) + (k+3)] = 0$$

$$4(k+1)(2k+4) = 0$$

Either

$$k+1=0 \quad \text{or} \quad 2k+4=0 \quad \because 4 \neq 0$$

$$k=-1 \quad \text{or} \quad 2k=-4$$

$$k=-2$$

Thus, roots will be equal if $k = -1, -2$

Exercise 2.1

Q.1 Find the discriminant of the following given quadratic equations:

Solution:

(i) $2x^2 + 3x - 1 = 0$ **(A.B)**
(GRW 2017, FSD 2016, MTN 2014, D.G.K 2016)

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 2, b = 3, c = -1$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (3)^2 - 4(2)(-1) \\ &= 9 + 8 \\ &= 17\end{aligned}$$

(ii) $6x^2 - 8x + 3 = 0$ **(A.B)**
(LHR 2016, SWL 2016, D.G.K 2015, 17)

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 6, b = -8, c = 3$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (-8)^2 - 4(6)(3) \\ &= 64 - 72 \\ &= -8\end{aligned}$$

(iii) $9x^2 - 30x + 25 = 0$ **(A.B)**
(LHR 2017, MTN 2015)

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 9, b = -30, c = 25$

$$\begin{aligned}\text{Disc} &= (-30)^2 - 4(9)(25) \\ &= 900 - 900 \\ &= 0\end{aligned}$$

(iv) $4x^2 - 7x - 2 = 0$ **(A.B)**
(GRW 2014, SGD 2017, MTN 2016)

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 4, b = -7, c = -2$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (-7)^2 - 4(4)(-2) \\ &= 49 + 32 \\ &= 81\end{aligned}$$

Q.2 Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations:

Solution:

(i) $x^2 - 23x + 120 = 0$ **(A.B)**

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 1, b = -23, c = 120$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (-23)^2 - 4(1)(120) \\ &= 529 - 480 \\ &= 49 \\ &= 7^2\end{aligned}$$

Since disc > 0 and perfect square, roots are rational (real) and unequal.

Verification:

$$\begin{aligned}x^2 - 23x + 120 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-23) \pm \sqrt{(-23)^2 - 4(1)(120)}}{2(1)} \\ &= \frac{23 \pm \sqrt{7^2}}{2} \\ x &= \frac{23 \pm 7}{2}\end{aligned}$$

Either

$$\begin{aligned}x &= \frac{23 - 7}{2} & \text{or} & & x &= \frac{23 + 7}{2} \\ &= \frac{16}{2} & \text{or} & & &= \frac{30}{2} \\ x &= 8 & & & x &= 15\end{aligned}$$

Hence roots are rational and unequal.

(ii) $2x^2 + 3x + 7 = 0$ **(A.B)**

By comparing given equation with

$ax^2 + bx + c = 0$, we get

$a = 2, b = 3, c = 7$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (3)^2 - 4(2)(7) \\ &= 9 - 56 \\ &= -47\end{aligned}$$

Since disc < 0 , roots are complex and imaginary.

Verification:

$$2x^2 + 3x + 7 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(7)}}{2(2)} \\ &= \frac{-3 \pm \sqrt{9 - 56}}{4} \\ &= \frac{-3 \pm \sqrt{-47}}{4} \\ &= \frac{-3 \pm \sqrt{47}i}{4} \end{aligned}$$

Hence roots are complex/imaginary and unequal.

(iii) $16x^2 - 24x + 9 = 0$ **(A.B)**

By comparing given equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 16, b = -24, c = 9$$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (-24)^2 - 4(16)(9) \\ &= 576 - 576 \\ &= 0 \end{aligned}$$

Since disc $= 0$, roots are rational (real) and equal.

Verification:

$$16x^2 - 24x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)} \end{aligned}$$

$$x = \frac{24 \pm \sqrt{0}}{32}$$

$$x = \frac{24 \pm 0}{32}$$

Either

$$x = \frac{24 + 0}{32} \quad \text{or} \quad x = \frac{24 - 0}{32}$$

$$x = \frac{24}{32} \quad x = \frac{24}{32}$$

Hence roots are rational and equal.

(iv) $3x^2 + 7x - 13 = 0$ **(A.B)**

By comparing given equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = 3, b = 7, c = -13$$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (7)^2 - 4(3)(-13) \\ &= 49 + 156 \\ &= 205 \end{aligned}$$

Since disc > 0 , but not a perfect sq. roots are irrational and unequal.

Verification:

$$3x^2 + 7x - 13 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7 \pm \sqrt{(7)^2 - 4(3)(-13)}}{2(3)} \\ &= \frac{-7 \pm \sqrt{49 + 156}}{6} \\ &= \frac{-7 \pm \sqrt{205}}{6} \end{aligned}$$

Either

$$x = \frac{-7 - \sqrt{205}}{6} \quad \text{or} \quad x = \frac{-7 + \sqrt{205}}{6}$$

Hence roots are irrational and unequal.

Q.3 For what value of k , the expression

$$k^2x^2 + 2(k+1)x + 4 \text{ is perfect}$$

square. (A.B + K.B)

Solution:

$$k^2x^2 + 2(k+1)x + 4 = 0$$

By comparing given equation with

$$ax^2 + bx + c = 0, \text{ we get}$$

$$a = k^2, b = 2(k+1), c = 4$$

$$\begin{aligned} \text{Discriminant} &= b^2 - 4ac \\ &= [2(k+1)]^2 - 4(k^2)(4) \\ &= 4(k+1)^2 - 16k^2 \\ &= 4(k^2 + 2k + 1) - 16k^2 \end{aligned}$$

$$\begin{aligned}
 &= 4k^2 + 8k + 4 - 16k^2 \\
 &= 4 + 8k - 12k^2 \\
 &= 4(1 + 2k - 3k^2)
 \end{aligned}$$

As expression is a perfect square, the discriminant = 0

$$\Rightarrow 4(1 + 2k - 3k^2) = 0$$

$$1 + 2k - 3k^2 = 0 \quad \because 4 \neq 0$$

$$1 + 3k - k - 3k^2 = 0$$

$$1(1 + 3k) - k(1 + 3k) = 0$$

$$(1 + 3k)(1 - k) = 0$$

Either

$$1 + 3k = 0 \quad \text{or} \quad 1 - k = 0$$

$$3k = -1 \quad \quad \quad k = 1$$

$$k = -\frac{1}{3}$$

Result

$$k = 1, -\frac{1}{3}$$

Q.4 Find the value of k , if the roots of the following equations are equal.

(A.B + K.B)

Solution:

(i) $(2k - 1)x^2 + 3kx + 3 = 0$

Here $a = 2k - 1, b = 3k, c = 3$

$$\begin{aligned}
 \text{Disc} &= b^2 - 4ac \\
 &= (3k)^2 - 4(2k - 1)(3) \\
 &= 9k^2 - 24k + 12
 \end{aligned}$$

Since roots are equal,

$$\text{Disc} = 0$$

$$\Rightarrow 9k^2 - 24k + 12 = 0$$

$$3(3k^2 - 8k + 4) = 0$$

$$3k^2 - 8k + 4 = 0 \quad \because 3 \neq 0$$

$$3k^2 - 6k - 2k + 4 = 0$$

$$3k(k - 2) - 2(k - 2) = 0$$

$$(k - 2)(3k - 2) = 0$$

Either

$$k - 2 = 0 \quad \text{or} \quad 3k - 2 = 0$$

$$\Rightarrow k = 2 \quad \text{or} \quad 3k = 2$$

$$k = \frac{2}{3}$$

Result:

$$k = 2, \frac{2}{3}$$

(ii) $x^2 + 2(k + 2)x + (3k + 4) = 0$

Here $a = 1, b = 2(k + 2), c = 3k + 4$

$$\begin{aligned}
 \text{Disc} &= b^2 - 4ac \\
 &= [2(k + 2)]^2 - 4(1)(3k + 4) \\
 &= 4(k + 2)^2 - 4(3k + 4) \\
 &= 4(k^2 + 4k + 4) - 4(3k + 4) \\
 &= 4[k^2 + 4k + 4 - 3k - 4] \\
 &= 4[k^2 + k] \\
 &= 4k(k + 1)
 \end{aligned}$$

Since roots are equal, disc = 0

$$4k(k + 1) = 0$$

Either

$$4k = 0 \quad \text{or} \quad k + 1 = 0$$

$$k = 0 \quad \text{or} \quad k = -1$$

Result:

$$k = 0, -1$$

(iii) $(3k + 2)x^2 - 5(k + 1)x + (2k + 3) = 0$

(A.B + K.B)

Here

$$a = 3k + 2, b = -5(k + 1), c = 2k + 3$$

$$\begin{aligned}
 \text{Disc} &= b^2 - 4ac \\
 &= [-5(k + 1)]^2 - 4(3k + 2)(2k + 3) \\
 &= 25(k + 1)^2 - 4(6k^2 + 9k + 4k + 6) \\
 &= 25(k^2 + 2k + 1) - 4(6k^2 + 13k + 6) \\
 &= 25k^2 + 50k + 25 - 24k^2 - 52k - 24 \\
 &= k^2 - 2k + 1
 \end{aligned}$$

Since roots are equal, disc = 0

$$\Rightarrow k^2 - 2k + 1 = 0$$

$$(k - 1)^2 = 0$$

Taking square root on both sides

$$k-1=0$$

$$k=1$$

Result:

$$k=1$$

Q.5 Show that the equation

$$x^2 + (mx + c)^2 = a^2 \text{ has equal roots,}$$

$$\text{if } c^2 = a^2(1+m^2) \quad \textbf{(A.B + K.B)}$$

Proof (FSD 2016)

$$x^2 + (mx + c)^2 = a^2$$

$$x^2 + m^2x^2 + 2mcx + c^2 = a^2$$

$$(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$$

Here

$$A = 1+m^2, B = 2mc, C = c^2 - a^2$$

$$\text{Disc} = B^2 - 4AC$$

$$= (2mc)^2 - 4(1+m^2)(c^2 - a^2)$$

$$= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - a^2m^2)$$

$$= \cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4a^2m^2$$

$$= -4c^2 + 4a^2 + 4a^2m^2$$

$$\text{Disc} = -4[a^2(1+m^2)] + 4a^2 + 4m^2$$

$$(\because c^2 = a^2(1+m^2))$$

$$= -4[a^2 + a^2m^2] + 4a^2 + 4a^2m^2$$

$$= -4a^2 - 4a^2m^2 + 4a^2 + 4a^2m^2$$

$$= 0$$

$$\text{Disc} = 0$$

$\therefore$ Roots are equal

Hence roots are equal, if $c^2 = a^2(1+m^2)$

Proved

Q.6 Find the condition that the roots of the equation $(mx + c)^2 - 4ax = 0$ are equal. (A.B + K.B + U.B)

Solution:

$$(mx + c)^2 - 4ax = 0$$

$$m^2x^2 + 2mcx - 4ax + c^2 = 0$$

$$m^2x^2 + (2mc - 4a)x + c^2 = 0$$

$$\text{Here } A = m^2, B = 2mc - 4a, C = c^2$$

$$\text{Disc} = B^2 - 4AC$$

$$= (2mc - 4a)^2 - 4(m^2)(c^2)$$

$$= 4m^2c^2 - 16amc + 16a^2 - 4m^2c^2$$

$$= -16amc + 16a^2$$

$$= -16a(mc - a)$$

Since roots are equal, disc = 0

$$\Rightarrow -16a(mc - a) = 0$$

Either

$$mc - a = 0 \quad \text{or} \quad -16a = 0$$

$$a = mc \quad a = 0$$

Roots of the given equation are equal

If $a = mc$ or $a = 0$

Q.7 If the roots of the equation.

$$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$$

are equal, then $a = 0$ or

$$a^3 + b^3 + c^3 = 3abc.$$

(A.B + K.B + U.B)

Proof

$$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$$

Here

$$A = c^2 - ab, B = -2(a^2 - bc), C = b^2 - ac$$

$$\text{Disc} = B^2 - 4AC$$

$$= [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac)$$

$$= 4(a^2 - bc)^2 - 4(b^2c^2 - ac^3 - ab^3 + a^2bc)$$

$$= 4(a^4 - 2a^2bc + b^2c^2) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc)$$

$$= 4(a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc)$$

$$= 4(a^4 + ab^3 + ac^3 - 3a^2bc)$$

$$= 4a(a^3 + b^3 + c^3 - 3abc)$$

Since roots are equal, disc = 0

$$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either

$$4a = 0 \quad \text{or} \quad a^3 + b^3 + c^3 - 3abc = 0$$

$$a = 0 \quad a^3 + b^3 + c^3 = 3abc$$

Roots are equal if either

$$a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

Q.8 Show that the roots of the following equations are rational.

Proof. (A.B + K.B + U.B)

(i) $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

Here

$$A = a(b-c), B = b(c-a), C = c(a-b)$$

$$\text{Discriminant} = B^2 - 4AC$$

$$\begin{aligned} &= [b(c-a)]^2 - 4[a(b-c)][c(a-b)] \\ &= b^2(c-a)^2 - 4ac(b-c)(a-b) \\ &= b^2(c^2 - 2ac + a^2) - 4ac(ab - b^2 - ac + bc) \\ &= b^2c^2 - 2ab^2c + a^2b^2 - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2 \\ &= b^2c^2 + a^2b^2 + 4a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2 \\ &= (bc)^2 + (ab)^2 + (-2ac)^2 + 2(bc)(ab) \\ &\quad + 2(ab)(-2ac) + 2(-2ac)(bc) \\ \therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca &= (a+b+c)^2 \\ &= (bc + ab - 2ac)^2 \end{aligned}$$

As discriminate > 0 and perfect square, roots are rational.

$$(ii) (a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0$$

Here

$$A = a+2b, B = 2(a+b+c), C = a+2c$$

$$\text{Disc} = B^2 - 4AC$$

$$\begin{aligned} &= [2(a+b+c)]^2 - 4(a+2b)(a+2c) \\ &= 4(a+b+c)^2 - 4(a^2 + 2ac + 2ab + 4bc) \\ &= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) \\ &\quad - 4(a^2 + 2ac + 2ab + 4bc) \\ &= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ &\quad - a^2 - 2ac - 2ab - 4bc) \\ &= 4(b^2 + c^2 - 2bc) \\ &= 4(b-c)^2 \\ &= [2(b-c)]^2 > 0 \end{aligned}$$

Since disc is perfect square roots are rational.

Q.9 For all values of k, prove that the roots of the equation.

$$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0, k \neq 0 \text{ are real.}$$

Proof:

(A.B + U.B)

Here

$$a = 1, b = -2\left(k + \frac{1}{k}\right), c = 3$$

$$\text{Discriminant} = b^2 - 4ac$$

$$\begin{aligned} &= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3) \\ &= 4\left(k + \frac{1}{k}\right)^2 - 12 \\ &= 4\left(k^2 + \frac{1}{k^2} + 2\right) - 12 \\ &= 4\left[\left(k^2 + \frac{1}{k^2} + 2\right) - 3\right] \\ &= 4\left[k^2 + \frac{1}{k^2} - 1\right] \\ &= 4\left[k^2 + \frac{1}{k^2} - 2 + 1\right] \\ &= 4\left[\left(k - \frac{1}{k}\right)^2 + 1\right] \end{aligned}$$

$$> 0$$

As disc. > 0 , roots are real.

Q.10 Show that the roots of the equation

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

are real. **(A.B + U.B + K.B)**

Proof:

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

$$\text{Here } A = b-c, B = c-a, C = a-b$$

$$\text{Disc} = B^2 - 4AC$$

$$\begin{aligned} &= (c-a)^2 - 4(b-c)(a-b) \\ &= c^2 - 2ac + a^2 - 4(ab - b^2 - ac + bc) \\ &= c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc \\ &= c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc \\ &= (c)^2 + (a)^2 + (2b)^2 + 2(c)(a) + 2(a)(-2b) + 2(-2b)(c) \\ \therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca &= (a+b+c)^2 \\ &= (c+a-2b)^2 > 0 \end{aligned}$$

Hence roots are real.

Derivation of Cube Roots of Unity

(A.B + K.B)

Let $x = \sqrt[3]{1}$

Taking cube on both sides

$$x^3 = 1$$

$$x^3 - 1 = 0$$

$$(x)^3 - (1)^3 = 0$$

$$(x-1)(x^2 + x + 1) = 0$$

$$(x-1)(x^2 + x + 1) = 0$$

Either

$$x-1=0 \quad \text{or} \quad x^2 + x + 1 = 0$$

$$x=1$$

Here $a=1, b=1, c=1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} \quad \because \sqrt{-1} = i$$

Either

$$x = \frac{-1 + \sqrt{3}i}{2}, \quad x = \frac{-1 - \sqrt{3}i}{2}$$

$$= \omega$$

$$= \omega^2$$

$\therefore$ Cube root of unity are $1, \omega, \omega^2$

Note

(K.B)

We can write anyone complex cube root as ω (Omega), then other will be ω^2 .

Properties of Cube Root of Unity

(i) Proving that each complex cube root of unity is the square of other.

$$\text{i.e.} \left(\frac{-1 + \sqrt{-3}}{2} \right)^2 = \frac{-1 - \sqrt{-3}}{2}$$

$$\text{and} \left(\frac{-1 - \sqrt{-3}}{2} \right)^2 = \frac{-1 + \sqrt{-3}}{2}$$

(LHR 2014, GRW 2017, FSD 2016, 17, SGD 2015, 16, BWP 2017, MTN 2017)

(K.B + U.B + A.B)

Proof:

We have to prove

$$(i) \quad \left(\frac{-1 + \sqrt{-3}}{2} \right)^2 = \frac{-1 - \sqrt{-3}}{2}$$

Consider

$$\begin{aligned} & \left(\frac{-1 + \sqrt{-3}}{2} \right)^2 \\ &= \frac{(-1)^2 + (\sqrt{-3})^2 + 2(-1)(\sqrt{-3})}{4} \\ &= \frac{1 + (-3) - 2\sqrt{-3}}{4} \\ &= \frac{1 - 3 - 2\sqrt{-3}}{4} \\ &= \frac{-2 - 2\sqrt{-3}}{4} \\ &= \frac{2(-1 - \sqrt{-3})}{4} \\ &= \frac{-1 - \sqrt{-3}}{2} \end{aligned}$$

$$(ii) \quad \left(\frac{-1 - \sqrt{-3}}{2} \right)^2 = \frac{-1 + \sqrt{-3}}{2}$$

Now consider

$$\begin{aligned} & \left(\frac{-1 - \sqrt{-3}}{2} \right)^2 \\ &= \frac{(1)^2 + (\sqrt{-3})^2 - 2(-1)(-\sqrt{3})}{4} \\ &= \frac{1 + (-3) + 2\sqrt{-3}}{4} \\ &= \frac{1 - 3 + 2\sqrt{-3}}{4} \\ &= \frac{-2 + 2\sqrt{-3}}{4} \\ &= \frac{2(-1 + \sqrt{-3})}{4} \end{aligned}$$

$$= \frac{-1 + \sqrt{-3}}{2}$$

Thus, each of the complex cube roots of unity is the square of the other.

(ii) Proving that product of three cube root of unity is one.

i.e. $1.\omega.\omega^2 = 1$ **(K.B + U.B)**

Proof:

L.H.S = $1.\omega.\omega^2$

By putting the values

$$\begin{aligned} &= (1) \times \left(\frac{-1 + \sqrt{-3}}{2} \right) \left(\frac{-1 - \sqrt{-3}}{2} \right) \\ &= \frac{(-1)^2 - (\sqrt{-3})^2}{4} \\ &= \frac{1 - (-3)}{4} \\ &= \frac{1 + 3}{4} \\ &= \frac{4}{4} \\ &= 1 \\ &= \text{R.H.S} \end{aligned}$$

Proved

(iii) Proving that sum of three cube roots of unit is zero.

i.e. $1 + \omega + \omega^2 = 0$

(LHR 2016, GRW 2017, FSD 2015, 17, SGD 2015, 16, BWP 2016, RWP 2015)

Proof:

$$\begin{aligned} \text{L.H.S} &= 1 + \omega + \omega^2 \\ &\text{(By putting values)} \\ &= 1 + \frac{-1 + \sqrt{-3}}{2} + \frac{-1 - \sqrt{-3}}{2} \\ &= \frac{2 + (-1) + \sqrt{-3} - 1 - \sqrt{-3}}{2} \\ &= \frac{2 - 1 + \sqrt{-3} - 1 - \sqrt{-3}}{2} \end{aligned}$$

$$= \frac{0}{2}$$

$$= 0$$

$$= \text{R.H.S}$$

Proved

Important Results

(K.B + U.B)

(i) $1 + \omega + \omega^2 = 0$

$$\Rightarrow 1 + \omega = -\omega^2$$

$$1 + \omega^2 = -\omega$$

$$\omega + \omega^2 = -1$$

(ii) $1.\omega.\omega^2 = 1$

$$\Rightarrow \omega^3 = 1$$

(iii) $1.\omega.\omega^2 = 1$

$$\omega.\omega^2 = 1$$

$$\Rightarrow \omega = \frac{1}{\omega^2} \quad \text{or} \quad \omega^2 = \frac{1}{\omega}$$

Note: Complex cube roots are reciprocal of each other.

(iv) $\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$$\omega^5 = \omega^3.\omega^2 = 1.\omega^2 = \omega^2$$

$$\omega^6 = (\omega^3)^2 = (1)^2 = 1 \quad \text{and so on}$$

$$\omega^{-16} = \frac{1}{\omega^{16}} = \frac{1}{\omega^{15} \omega} = \frac{1}{(\omega^3)^5 \omega} = \frac{1}{1.\omega} = \frac{1}{\omega} = \omega^2$$

Example 1: (Page # 25)

Evaluate: $(-1 + \sqrt{-3})^8 + (-1 - \sqrt{-3})^8$ **(A.B)**

Solution:

$$\begin{aligned} &(-1 + \sqrt{-3})^8 + (-1 - \sqrt{-3})^8 \\ &= \left[2 \left(\frac{-1 + \sqrt{-3}}{2} \right) \right]^8 + \left[2 \left(\frac{-1 - \sqrt{-3}}{2} \right) \right]^8 \\ &= (2\omega)^8 + (2\omega^2)^8 \\ &= 2^8 \omega^8 + 2^8 \omega^{16} \\ &= 2^8 (\omega^6.\omega^2 + \omega^{15} \omega) \\ &= 256 \left((\omega^3)^2.\omega^2 + (\omega^3)^5 \omega \right) \end{aligned}$$

$$\begin{aligned}
 &= 256 \left((1)^2 \cdot \omega^2 + (1)^5 \omega \right) \because \omega^3 = 1 \\
 &256(\omega^2 + \omega) \\
 &= 256(-1) \because 1 + \omega + \omega^2 = 0 \\
 &= -256
 \end{aligned}$$

Example 2: (Page # 25)**(A.B)****To Prove**

$$x^3 - y^3 = (x - y)(x - \omega y)(x - \omega^2 y)$$

Proof:

$$\begin{aligned}
 \text{R.H.S} &= (x - y)(x - \omega y)(x - \omega^2 y) \\
 &= (x - y)(x^2 - \omega xy - \omega^2 xy + \omega^3 y^2) \\
 &= (x - y)(x^2 - (\omega + \omega^2)xy + y^2) \\
 &= (x - y)(x^2 - (-1)xy + y^2) \because 1 + \omega + \omega^2 = 0 \\
 &= (x - y)(x^2 + xy + y^2) \\
 &= x^3 - y^3 = \text{L.H.S}
 \end{aligned}$$

Exercise 2.2**Q.1 Find the cube roots of -1, 8, -27, 64.****(LHR 2015) (K.B + A.B)****Solution:****(i) Finding cube roots of -1**

$$\text{Let } x = \sqrt[3]{-1}$$

Taking cube on both sides

$$x^3 = -1$$

$$x^3 + 1 = 0$$

$$\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$(x + 1)(x^2 - x + 1) = 0$$

Either

$$x + 1 = 0 \rightarrow (i) \text{ or } x^2 - x + 1 = 0 \rightarrow (ii)$$

Equation (i) $\Rightarrow$

$$x = -1$$

Equation (ii) $\Rightarrow$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

Either

$$x = \frac{1 - \sqrt{-3}}{2} \quad \text{or} \quad x = \frac{1 + \sqrt{-3}}{2}$$

$$x = -\left(\frac{-1 + \sqrt{-3}}{2}\right) \quad x = -\left(\frac{-1 - \sqrt{-3}}{2}\right)$$

$$= -\omega$$

$$= -\omega^2$$

 $\therefore$ **Cube roots of -1 are -1, $-\omega$, $-\omega^2$** **(ii) Finding cube roots of 8****(K.B + A.B)**

$$\text{Let } x = \sqrt[3]{8}$$

Taking cube on both sides

$$x^3 = 8$$

$$x^3 - 8 = 0$$

$$x^3 - 2^3 = 0$$

$$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(x - 2)(x^2 + 2x + 4) = 0$$

Either

$$x - 2 = 0 \rightarrow (i) \text{ or } x^2 + 2x + 4 = 0 \rightarrow (ii)$$

Equation (i) $\Rightarrow$

$$x = 2$$

Equation (ii) $\Rightarrow$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

Either

$$x = 2\left(\frac{-1 + \sqrt{-3}}{2}\right) \text{ or } x = 2\left(\frac{-1 - \sqrt{-3}}{2}\right)$$

$$= 2\omega$$

$$= 2\omega^2$$

 $\therefore$ **Cube roots of 8 are 2, 2ω , $2\omega^2$** **(iii) Finding cube roots of -27****(SGD 2014) (K.B + A.B)**

Let

$$x = \sqrt[3]{-27}$$

$$x^3 + 27 = 0$$

$$\therefore a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$x^3 + 3^3 = 0$$

$$(x+3)(x^2 - 3x + 9) = 0$$

Either

$$x+3=0 \rightarrow (i) \text{ or } x^2 - 3x + 9 = 0 \rightarrow (ii)$$

$$\text{Equation (i)} \Rightarrow$$

$$x = -3$$

$$\text{Equation (ii)} \Rightarrow$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-36}}{2}$$

$$= \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3\sqrt{-3}}{2}$$

$$= -3 \left(\frac{-1 \pm \sqrt{-3}}{2} \right)$$

Either

$$x = -3 \left(\frac{-1 + \sqrt{-3}}{2} \right) \text{ or } x = -3 \left(\frac{-1 - \sqrt{-3}}{2} \right)$$

$$x = -3\omega \quad x = -3\omega^2$$

$\therefore$ Cube roots of -27 are $-3, -3\omega, -3\omega^2$

(iv) Finding cube roots of 64.

(LHR 2015) (K.B + A.B)

Let

$$x = \sqrt[3]{64}$$

Taking cube on both sides

$$x^3 = 64$$

$$x^3 - 64 = 0$$

$$x^3 - 4^3 = 0$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-4)(x^2 + 4x + 16) = 0$$

Either

$$x-4=0 \rightarrow (i) \text{ or } x^2 + 4x + 16 = 0 \rightarrow (ii)$$

$$\text{Equation (i)} \Rightarrow$$

$$x = 4$$

$$\text{Equation (ii)} \Rightarrow$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16-64}}{2}$$

$$x = \frac{-4 \pm \sqrt{-48}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{-3}}{2}$$

$$x = 4 \left(\frac{-1 \pm \sqrt{-3}}{2} \right)$$

Either

$$x = 4 \left(\frac{-1 + \sqrt{-3}}{2} \right) \text{ or } x = 4 \left(\frac{-1 - \sqrt{-3}}{2} \right)$$

$$x = 4\omega \quad \text{or} \quad x = 4\omega^2$$

$\therefore$ Cube roots of 64 are $4, 4\omega, 4\omega^2$

Q.2 Evaluate: (K.B + A.B)

(FSD 2017, BWP 2016, RWP 2015, MTN 2014, 15, 17, D.G.K 2015, 17)

(i) $(1 - \omega - \omega^2)^7$

(ii) $(1 - 3\omega - 3\omega^2)^5$

(iii) $(9 + 4\omega + 4\omega^2)^3$

(iv) $(2 + 2\omega - 2\omega^2)(3 - 3\omega + 3\omega^2)$

(v) $(-1 + \sqrt{-3})^6 + (-1 - \sqrt{-3})^6$

(vi) $\left(\frac{1 + \sqrt{-3}}{2} \right)^9 + \left(\frac{1 - \sqrt{-3}}{2} \right)^9$

(vii) $\omega^{37} + \omega^{38} - 5$

(viii) $\omega^{-13} + \omega^{-17}$

Solution:

(i) $(1 - \omega - \omega^2)^7$

(GRW 2014, 16, 17, FSD 2016, BWP 2017, MTN 2017)

$$= [1 - (\omega + \omega^2)]^7$$

$$= [1 - (-1)]^7 \quad \because \omega + \omega^2 = -1$$

$$= (1 + 1)^7$$

$$= 2^7$$

$$= 128$$

$$\begin{aligned}
 \text{(ii)} \quad & (1-3\omega-3\omega^2)^5 \\
 &= [1-3(\omega+\omega^2)]^5 \\
 &= [1-3(-1)]^5 \quad \because \omega+\omega^2=-1 \\
 &= (1+3)^5 \\
 &= (4)^5 \\
 &= 1024
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & (9+4\omega+4\omega^2)^3 \\
 & \text{(GRW 2014, RWP 2017, FSD 2017, BWP 2016)} \\
 &= [9+4(\omega+\omega^2)]^3 \\
 &= [9+4(-1)]^3 \quad \because \omega+\omega^2=-1 \\
 &= (9-4)^3 \\
 &= (5)^3 \\
 &= 125
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & (2+2\omega-2\omega^2)(3-3\omega+3\omega^2) \\
 & \text{(SWL 2017)} \\
 &= 2(1+\omega-\omega^2)3(1-\omega+\omega^2) \\
 &= 6[(1+\omega)-\omega^2][(1+\omega^2)-\omega] \\
 &\because 1+\omega+\omega^2=0 \\
 &= 6(-\omega^2-\omega^2)(-\omega-\omega) \\
 &= 6(-2\omega^2)(-2\omega) \\
 &= 6(4\omega^3) \\
 &= 24\omega^3 \\
 &= 24(1) \quad \because \omega^3=1 \\
 &= 24
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & (-1+\sqrt{-3})^6 + (-1-\sqrt{-3})^6 \\
 &\because \omega = \frac{-1+\sqrt{-3}}{2}, \omega^2 = \frac{-1-\sqrt{-3}}{2} \\
 &= (2\omega)^6 + (2\omega^2)^6 \\
 &= 2^6\omega^6 + 2^6\omega^{12} \\
 &= 2^6(\omega^6 + \omega^{12})
 \end{aligned}$$

$$\begin{aligned}
 &= 64[(\omega^3)^2 + (\omega^3)^4] \\
 &= 64[(1)^2 + (1)^4] \quad \because \omega^3=1 \\
 &= 64(1+1) \\
 &= 64(2) \\
 &= 128
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & \left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9 \\
 &\omega = \frac{-1+\sqrt{3}}{2}, \omega^2 = \frac{-1-\sqrt{-3}}{2} \\
 &= (\omega)^9 + (\omega^2)^9 \\
 &= (\omega)^9 + (\omega)^{18} \\
 &= (\omega^3)^3 + (\omega^3)^6 \\
 &= (1)^3 + (1)^6 \\
 &= 1+1 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & \omega^{37} + \omega^{38} - 5 \\
 & \text{(LHR 2015, SWL 2016, MTN 2015, D.G.K 2016, 17)} \\
 &= \omega.\omega^{36} + \omega^2.\omega^{36} - 5 \\
 &= (\omega + \omega^2).\omega^{36} - 5 \\
 &= (-1)[(\omega^3)^{12}] - 5 \\
 &= -1[(1)^{12}] - 5 \quad \because \omega^3=1 \\
 &= -1(1) - 5 \\
 &= -1-5 \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad & \omega^{-13} + \omega^{-17} \\
 &= \frac{1}{\omega^{13}} + \frac{1}{\omega^{17}} \\
 &= \frac{1}{\omega.\omega^{12}} + \frac{1}{\omega^2.\omega^{15}} \\
 &= \frac{1}{\omega.(\omega^3)^4} + \frac{1}{\omega^2.(\omega^3)^5}
 \end{aligned}$$

$$= \frac{1}{\omega(1)^4} + \frac{1}{\omega^2(1)^5} \because \omega^3 = 1$$

$$= \frac{1}{\omega(1)} + \frac{1}{\omega^2(1)}$$

$$= \frac{1}{\omega} + \frac{1}{\omega^2}$$

$$= \omega^2 + \omega \quad \because \omega \cdot \omega^2 = 1$$

$$= -1 \quad \because 1 + \omega + \omega^2 = 0$$

Q.3 Prove that (K.B + U.B)

$$x^3 + y^3 = (x+y)(x+\omega y)(x+\omega^2 y)$$

(SGD 2015, BWP 2016)

Proof:

R.H.S

$$= (x+y)(x+\omega y)(x+\omega^2 y)$$

$$= (x+y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2)$$

$$= (x+y)[x^2 + (\omega^2 + \omega)xy + (1)y^2] \because \omega^3 = 1$$

$$= (x+y)[x^2 + (-1)xy + y^2] \because 1 + \omega + \omega^2 = 0$$

$$= (x+y)[x^2 - xy + y^2]$$

$$= x^3 + y^3$$

= L.H.S

Proved

Q.4 Prove that $x^3 + y^3 + z^3 - 3xyz$

$$= (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$$

(K.B + U.B)

Proof:

R.H.S

$$= (x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$$

$$= (x+y+z)(x^2 + \omega^2 xy + \omega xz + \omega xy + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^4 yz + \omega^3 z^2)$$

$$= (x+y+z)(x^2 + \omega^3 y^2 + \omega^3 z^2 + \omega^2 xy + \omega xy + \omega^2 yz + \omega^4 yz + \omega xz + \omega^2 xz)$$

$$= (x+y+z)(x^2 + (1)y^2 + (1)z^2 + (\omega^2 + \omega)xy + (\omega^2 + \omega^4)yz + (\omega + \omega^2)xz)$$

$$\because \omega^3 = 1$$

$$= (x+y+z)[x^2 + y^2 + z^2 + (-1)xy + (\omega^2 + \omega)yz + (-1)xz]$$

$$\because 1 + \omega + \omega^2 = 0, \omega^4 = \omega \cdot \omega^3 = \omega$$

$$= (x+y+z)(x^2 + y^2 + z^2 - xy + (-1)yz - xz)$$

$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$$= x^3 + y^3 + z^3 - 3xyz$$

= L.H.S

Proved

Q.5 Prove that

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) \dots 2n$$

(K.B + A.B + U.B)

Proof:

L.H.S

$$= (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) \dots 2n \text{ factors}$$

$$\because \omega^4 = \omega \times \omega^3 = \omega, \omega^8 = \omega^2 \times \omega^6 = \omega^2 \times (\omega^3)^2 = \omega^2 (1) = \omega^2$$

$$= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2) \dots 2n \text{ factors}$$

$$= [(1+\omega)(1+\omega)(1+\omega) \dots n \text{ factors}]$$

$$[(1+\omega^2)(1+\omega^2)(1+\omega^2) \dots n \text{ factors}]$$

$$= (1+\omega)^n (1+\omega^2)^n$$

$$= [(1+\omega)(1+\omega^2)]^n$$

$$= (1+\omega+\omega^2+\omega^3)^n$$

$$= (0+1)^n \quad \because 1+\omega+\omega^2=0, \omega^3=1$$

$$= (1)^n$$

$$= 1$$

= R.H.S

Relation between Roots and Co-efficient of a Quadratic Equation (K.B + U.B)

Roots of standard quadratic equation

$ax^2 + bx + c = 0$ are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\text{If } \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots

$$S = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} + (-b) - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$\Rightarrow S = -\frac{b}{a}$$

Product of roots

$$P = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

$$\because (a+b)(a-b) = a^2 - b^2$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2}$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$\Rightarrow P = \frac{c}{a}$$

Note**(K.B + U.B)**

$$(i) \quad S = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$(ii) \quad P = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Quadratic Equation with Given Roots**(K.B + U.B)**

A quadratic equation whose roots are given can be obtained by using formula

$$x^2 - Sx + P = 0 \quad \text{Or}$$

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

Example 1: (Page # 26)**(A.B)**

Without solving, find the sum and product of roots of the equation

$$3x^2 - 5x + 7 = 0$$

Solution:

$$3x^2 - 5x + 7 = 0$$

$$\text{Here } a = 3, b = -5, c = 7$$

$$\text{Sum of roots} = S = -\frac{b}{a}$$

$$= -\frac{-5}{3} = \frac{5}{3}$$

$$\text{Product of roots} = P = \frac{c}{a}$$

$$= \frac{7}{3}$$

To find unknown values involved in a given Quadratic Equation**Example 1: (Page # 27)****(A.B)**

Find the value of h, if the sum of roots is equal to 3-times the product of roots of the equation:

$$3x^2 + (9 - 6h)x + 5h = 0.$$

Solution:

$$3x^2 + (9 - 6h)x + 5h = 0$$

$$\text{Here } a = 3, b = 9 - 6h, c = 5h$$

Let α, β be the roots of given equation

$$\text{Then } \alpha + \beta = -\frac{b}{a} = -\frac{9 - 6h}{3} = \frac{6h - 9}{3}$$

$$\text{And } \alpha\beta = \frac{c}{a} = \frac{5h}{3}$$

According to given condition

$$\text{Sum of roots} = 3(\text{Product of roots})$$

$$\alpha + \beta = 3\alpha\beta$$

$$\frac{6h - 9}{3} = 3\left(\frac{5h}{3}\right)$$

$$6h - 9 = 15h$$

$$6h - 15h = 9$$

$$-9h = 9$$

$$h = -1$$

Exercise 2.3

Q.1 Without solving, find the sum and the product of the roots of the following quadratic equations.

$$(i) \quad x^2 - 5x + 3 = 0 \quad (\text{MTN 2017}) \quad \textbf{(A.B)}$$

$$\text{Here } a = 1, b = -5, c = 3$$

$$\begin{aligned}\text{Sum of roots} &= -\frac{b}{a} \\ &= -\frac{(-5)}{1} \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{3}{1} \\ &= 3\end{aligned}$$

(ii) $3x^2 + 7x - 11 = 0$ **(A.B)**
(LHR 2017, SWL 2017, SGD 2016, D.G.K 2014)

Here

$$a = 3, b = 7, c = -11$$

$$\begin{aligned}\text{Sum of roots} &= -\frac{b}{a} \\ &= -\frac{7}{3}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{-11}{3} \\ &= -\frac{11}{3}\end{aligned}$$

(iii) $px^2 - qx + r = 0$ **(A.B)**
(LHR 2014, GRW 2014, SWL 2016, MTN 2017, SGD 2016)

Here $a = p, b = -q, c = r$

$$\begin{aligned}\text{Sum of roots} &= -\frac{b}{a} \\ &= -\frac{(-q)}{p} \\ &= \frac{q}{p}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{r}{p}\end{aligned}$$

(iv) $(a+b)x^2 - ax + b = 0$ **(A.B)**
(BWP 2014, 17)

Here

$$A = a+b, B = -a, C = b$$

$$\begin{aligned}\text{Sum of roots} &= -\frac{B}{A} \\ &= -\frac{-a}{a+b} \\ &= \frac{a}{a+b}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{C}{A} \\ &= \frac{b}{a+b}\end{aligned}$$

(v) $(l+m)x^2 + (m+n)x + n-1 = 0$

Here

$$a = l+m, b = m+n, c = n-1$$

$$\begin{aligned}\text{Sum of roots} &= S = -\frac{b}{a} \\ &= -\frac{m+n}{l+m}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= P = \frac{c}{a} \\ &= \frac{n-1}{l+m}\end{aligned}$$

(vi) $7x^2 - 5mx + 9n = 0$ **(A.B)**

Here

$$a = 7, b = -5m, c = 9n$$

$$\begin{aligned}\text{Sum of roots} &= -\frac{b}{a} \\ &= -\frac{-5m}{7} \\ &= \frac{5m}{7}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{9n}{7}\end{aligned}$$

Q.2 Find the value of k , if

(i) **Sum of the roots of the equation $2kx^2 - 3x + 4k = 0$ is twice the product of the roots.** **(A.B)**

Solution:

Let α, β be the roots of equation

$$2kx^2 - 3x + 4k = 0$$

Here

$$a = 2k, b = -3, c = 4k$$

$$S = \alpha + \beta = -\frac{b}{a}$$

$$= \frac{-(-3)}{2k}$$

$$= \frac{3}{2k}$$

$$P = \alpha\beta = \frac{c}{a}$$

$$= \frac{4k}{2k}$$

$$= 2$$

According to given condition

$$S = 2P$$

$$\frac{3}{2k} = 2 \times 2$$

$$\frac{3}{2k} = \frac{4}{1}$$

$$3 \times 1 = 4 \times 2k$$

$$3 = 8k$$

$$\frac{3}{8} = k$$

$$k = \frac{3}{8}$$

- (ii) **Sum of the roots of the equation $x^2 + (3k - 7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots. (A.B)**

Solution:

$$x^2 + (3k - 7)x + 5k = 0$$

$$\text{Here, } a = 1, b = 3k - 7, c = 5k$$

$$\text{Sum of roots} = S = \frac{-b}{a}$$

$$= -\frac{3k - 7}{1}$$

$$= -3k + 7$$

$$\text{Product of roots} = P = \frac{c}{a}$$

$$= \frac{5k}{1}$$

$$= 5k$$

According to given condition:

$$S = \frac{3}{2}P$$

$$-3k + 7 = \frac{3}{2}(5k)$$

$$2(-3k + 7) = 3(5k)$$

$$-6k + 14 = 15k$$

$$14 = 15k + 6k$$

$$14 = 21k$$

$$\frac{14}{21} = k$$

$$\frac{2}{3} = k$$

$$\text{Or } k = \frac{2}{3}$$

Q.3 Find k,

- (i) **If sum of squares of the roots of the equation $4kx^2 + 3kx - 8 = 0$ is 2.**

Solution (FSD 2015) (A.B)

$$4kx^2 + 3kx - 8 = 0$$

$$\text{Here } a = 4k, b = 3k, c = -8$$

Let α, β be the roots

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$= -\frac{3k}{4k}$$

$$\Rightarrow \alpha + \beta = -\frac{3}{4}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

$$= \frac{-8}{4k}$$

$$\Rightarrow \alpha\beta = \frac{-2}{k}$$

According to given condition

$$\alpha^2 + \beta^2 = 2$$

$$(\alpha + \beta)^2 - 2\alpha\beta = 2$$

Putting the values

$$\left(-\frac{3}{4}\right)^2 - 2\left(-\frac{2}{k}\right) = 2$$

$$\frac{9}{16} + \frac{4}{k} = 2$$

$$\begin{aligned}\frac{4}{k} &= 2 - \frac{9}{16} \\ \frac{4}{k} &= \frac{32-9}{16} \\ \frac{4}{k} &= \frac{23}{16} \\ \Rightarrow \frac{k}{4} &= \frac{16}{23} \\ k &= \frac{64}{23}\end{aligned}$$

- (ii) Sum of the squares of the roots of the equation $x^2 - 2kx + (2k+1) = 0$ is 6. **(A.B)**

Solution:

Here

$$a=1, b=-2k, c=2k+1$$

Let, α, β be the roots of given equation,

Then

$$\begin{aligned}\text{Sum of roots} &= \alpha + \beta = -\frac{b}{a} \\ &= -\left(\frac{-2k}{1}\right) \\ &= 2k\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= \alpha\beta = \frac{c}{a} \\ &= \frac{2k+1}{1} \\ &= 2k+1\end{aligned}$$

According to given condition

$$\alpha^2 + \beta^2 = 6$$

$$\text{Or } (\alpha + \beta)^2 - 2\alpha\beta = 6$$

Putting the values

$$(2k)^2 - 2(2k+1) = 6$$

$$4k^2 - 4k - 2 - 6 = 0$$

$$4k^2 - 4k - 8 = 0$$

$$4(k^2 - k - 2) = 0$$

$$\text{Or } k^2 - k - 2 = 0$$

$$k^2 - 2k + k - 2 = 0$$

$$k(k-2) + 1(k-2) = 0$$

$$(k-2)(k+1) = 0$$

Either

$$k-2=0 \quad \text{or} \quad k=2$$

$$k+1=0$$

$$k=-1$$

Result

$$k=2, -1$$

Q.4 Find p, if

- (i) The roots of the equation $x^2 - x + p^2 = 0$ differ by unity.

(FSD 2015) (A.B)

Let α, β be the roots of given equation.

$$\text{Here } a=1, b=-1, c=p^2$$

$$\begin{aligned}\text{Then } \alpha + \beta &= -\frac{b}{a} \\ &= -\left(\frac{-1}{1}\right) \\ &= 1\end{aligned}$$

$$\begin{aligned}\alpha\beta &= \frac{c}{a} \\ &= \frac{p^2}{1} \\ &= p^2\end{aligned}$$

According to given condition

$$\alpha - \beta = 1$$

Taking square on both sides

$$(\alpha - \beta)^2 = 1$$

$$\therefore (a+b)^2 - (a-b)^2 = 4ab$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 1$$

Putting the values

$$(1)^2 - 4p^2 = 1$$

$$1 - 4p^2 = 1$$

$$1 - 1 = 4p^2$$

$$0 = 4p^2$$

Or

$$p^2 = 0$$

By taking square root

$$p = 0$$

Result:

$$p = 0$$

(ii) The roots of the equation

$$x^2 + 3x + P - 2 = 0 \text{ differ by 2.}$$

Solution: (A.B)

$$x^2 + 3x + P - 2 = 0$$

$$\text{Here } a = 1, b = 3, c = P - 2$$

Let roots of given equation are α, β

$$\text{Then sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$= -\frac{3}{1}$$

$$\Rightarrow \alpha + \beta = -3$$

$$\begin{aligned} \text{Product of roots} &= \alpha\beta = \frac{c}{a} \\ &= \frac{P-2}{1} \end{aligned}$$

According to given condition

$$\alpha - \beta = 2$$

Taking square of both sides

$$(\alpha - \beta)^2 = 4$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 4$$

Putting the values

$$(-3)^2 - 4(P - 2) = 4$$

$$9 - 4P + 8 = 4$$

$$9 + 8 - 4 = 4P$$

$$13 = 4P$$

$$\text{Or } P = \frac{13}{4}$$

$$\text{Result: } P = \frac{13}{4}$$

Q.5 Find m, if

(i) The roots of the equation

$$x^2 - 7x + 3m - 5 = 0 \text{ satisfy the}$$

$$\text{relation } 3\alpha + 2\beta = 4 \quad \text{(A.B)}$$

Solution:

$$x^2 - 7x + 3m - 5 = 0$$

$$\text{Here } a = 1, b = -7, c = 3m - 5$$

Let α, β be the roots of given equation

$$\begin{aligned} \text{Then sum of roots} &= \alpha + \beta = -\frac{b}{a} \\ &= -\frac{-7}{1} \end{aligned}$$

$$\Rightarrow \alpha + \beta = 7 \rightarrow (i)$$

$$\begin{aligned} \text{Product of roots} &= \alpha\beta = \frac{c}{a} \\ &= \frac{3m-5}{1} \end{aligned}$$

$$\Rightarrow \alpha\beta = 3m - 5 \rightarrow (ii)$$

According to given condition

$$3\alpha + 2\beta = 4 \rightarrow (iii)$$

Multiply equation (i) by '2'

$$2\alpha + 2\beta = 14 \rightarrow (iv)$$

Sub. Equation (iii) & (iv)

$$3\alpha + 2\beta = 4$$

$$\underline{2\alpha + 2\beta = 14}$$

$$\alpha = -10$$

Put in equation (i)

$$\alpha + \beta = 7$$

$$-10 + \beta = 7$$

$$\beta = 7 + 10$$

$$\beta = 17$$

Putting the values of α and β in equation (ii)

$$\alpha\beta = 3m - 5$$

$$-10(17) = 3m - 5$$

$$-170 + 5 = 3m$$

$$\frac{-165}{3} = m$$

$$\text{Or } m = -55$$

Result:

$$m = -55$$

(ii) The roots of the equation

$$x^2 + 7x + 3m - 5 = 0 \text{ satisfy the}$$

$$\text{relation } 3\alpha - 2\beta = 4 \quad \text{(A.B)}$$

Let α, β be the roots of given equation

Here

$$a = 1, \quad b = 7, \quad c = 3m - 5$$

Then

$$\alpha + \beta = -\frac{b}{a}$$

$$= \frac{-7}{1}$$

$$\alpha + \beta = -7 \longrightarrow (i)$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{3m-5}{1}$$

$$\alpha\beta = 3m-5 \longrightarrow (ii)$$

Also given

$$3\alpha - 2\beta = 4 \longrightarrow (iii)$$

Multiply equation (i) by 2

$$2\alpha + 2\beta = -14 \longrightarrow (iv)$$

Adding equation (iii) and (iv)

$$3\alpha - 2\beta = 4$$

$$2\alpha + 2\beta = -14$$

$$5\alpha = -10$$

$$\alpha = -\frac{10}{5}$$

$$\Rightarrow \alpha = -2$$

Put in equation (i)

$$\alpha + \beta = -7$$

$$-2 + \beta = -7$$

$$\beta = -7 + 2$$

$$\beta = -5$$

Now putting the values in equation (ii)

$$\alpha\beta = 3m-5$$

$$-2(-5) = 3m-5$$

$$10+5 = 3m$$

$$15 = 3m$$

$$\frac{15}{3} = m$$

$$5 = m$$

Result:

$$m = 5$$

$$(iii) \quad 3x^2 - 2x + 7m + 2 = 0 \quad (\mathbf{A.B})$$

Here $a = 3, b = -2, c = 7m + 2$

Let α, β be the roots of given equation

$$\text{Then sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$= -\frac{-2}{3}$$

$$\Rightarrow \alpha + \beta = \frac{2}{3} \longrightarrow (i)$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

$$= \frac{7m+2}{3} \longrightarrow (ii)$$

Also given

$$7\alpha - 3\beta = 18 \longrightarrow (iii)$$

Multiply equation (i) by '3'

$$3\alpha + 3\beta = 2 \longrightarrow (iv)$$

Adding equation (iii) and (iv)

$$7\alpha - 3\beta = 18$$

$$3\alpha + 3\beta = 2$$

$$10\alpha = 20$$

$$\Rightarrow \alpha = 2$$

Put in equation (i)

$$\alpha + \beta = \frac{2}{3}$$

$$2 + \beta = \frac{2}{3}$$

$$\beta = \frac{2}{3} - 2$$

$$= \frac{2-6}{3}$$

$$\beta = \frac{-4}{3}$$

Now putting the values of α and β in equation (ii)

$$2\left(\frac{-4}{3}\right) = \frac{7m+2}{3}$$

$$-8 = 7m + 2$$

$$-8 - 2 = 7m$$

$$-10 = 7m$$

$$\frac{-10}{7} = m$$

Result:

$$m = \frac{-10}{7}$$

Q.6 Find m, if sum and product of the roots of the following equations is equal to a given number λ .

(A.B)

(i) $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Here

$$a = 2m+3, b = 7m-5 \text{ and } c = 3m-10$$

Let α, β be the roots of given equation,

$$\begin{aligned} \text{Then } \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{7m-5}{2m+3} \\ &= \frac{5-7m}{2m+3} \end{aligned}$$

$$\begin{aligned} \text{And } \alpha\beta &= \frac{c}{a} \\ &= \frac{3m-10}{2m+3} \end{aligned}$$

According to given condition

$$\alpha + \beta = \lambda = \alpha\beta$$

(By using transitive property)

$$\Rightarrow \alpha + \beta = \alpha\beta$$

Putting the values

$$\frac{5-7m}{2m+3} = \frac{3m-10}{2m+3}$$

Or

$$5-7m = 3m-10$$

(By using cancellation property)

$$-7m-3m = -10-5$$

$$-10m = -15$$

$$m = \frac{-15}{-10}$$

$$\Rightarrow m = \frac{3}{2}$$

Result: $m = \frac{3}{2}$

(ii) $4x^2 - (3+5m)x - (9m-17) = 0$

(A.B)

Here

$$a = 4, b = -(3+5m), c = -(9m-17)$$

Let α, β be the roots

$$\begin{aligned} \text{Then } \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{-(3+5m)}{4} \\ &= \frac{3+5m}{4} \end{aligned}$$

$$\begin{aligned} \text{And } \alpha\beta &= \frac{c}{a} \\ &= \frac{-(9m-17)}{4} \end{aligned}$$

According to given condition:

$$\alpha + \beta = \lambda \quad \text{and} \quad \alpha\beta = \lambda$$

$$\Rightarrow \alpha + \beta = \alpha\beta$$

$\therefore$ Transitive property of equality

Putting the values

$$\frac{3+5m}{4} = \frac{-(9m-17)}{4}$$

$$\Rightarrow 3+5m = -9m+17$$

$$5m+9m = 17-3$$

$$14m = 14$$

$$\Rightarrow m = 1$$

Result: $m = 1$

Symmetric Function of the Roots of Quadratic Equation **(K.B + U.B)**

(MTN 2014, FSD 2014)

A function in which the roots involved are such that the value of the expression remains same, when roots are interchanged is called symmetric function. i.e. $f(\alpha, \beta) = f(\beta, \alpha)$

Some symmetric functions are:

$$\alpha^2 + \beta^2, \quad \alpha^3 + \beta^3, \quad \frac{1}{\alpha} + \frac{1}{\beta}$$

Example: (Page # 30) **(K.B + U.B)**

Verify that

$$\alpha^2 + \beta^2 + 2\alpha\beta \text{ is Symmetric}$$

Verification:

$$\text{Let } f(\alpha, \beta) = \alpha^2 + \beta^2 + 2\alpha\beta \rightarrow (i)$$

$$f(\beta, \alpha) = (\beta)^2 + (\alpha)^2 + 2\beta\alpha$$

$$= \beta^2 + \alpha^2 + 2\alpha\beta$$

$$= \alpha^2 + \beta^2 + 2\alpha\beta \rightarrow (ii)$$

From equation (i) and (ii) we get

$$f(\alpha, \beta) = f(\beta, \alpha)$$

Hence $\alpha^2 + \beta^2 + 2\alpha\beta$ is symmetric

Example: (Page # 30) (A.B + U.B)

Find the value of $\alpha^3 + \beta^3 + 3\alpha\beta$, if $\alpha = 2, \beta = 1$. Also find the value of $\alpha^3 + \beta^3 + 3\alpha\beta$ if $\alpha = 1, \beta = 2$.

Solution:

When $\alpha = 2, \beta = 1$

$$\begin{aligned}\alpha^3 + \beta^3 + 3\alpha\beta &= (2)^3 + (1)^3 + 3(2)(1) \\ &= 8 + 1 + 6 = 15\end{aligned}$$

When $\alpha = 1, \beta = 2$

$$\begin{aligned}\alpha^3 + \beta^3 + 3\alpha\beta &= (1)^3 + (2)^3 + 3(1)(2) \\ &= 1 + 8 + 6 = 15\end{aligned}$$

Note

Expression $\alpha^3 + \beta^3 + 3\alpha\beta$ represents a symmetric function.

Exercise 2.4

Q.1 If α, β are roots of the equation $x^2 + px + q = 0$, then evaluate

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3\beta + \alpha\beta^3$

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(i) Solution: (A.B + U.B)

$$x^2 + px + q = 0$$

Here $a = 1, b = p, c = q$

Roots of given equation are α, β

Then

$$\begin{aligned}\alpha + \beta &= -\frac{b}{a} \\ &= -\frac{p}{1} \\ &= -p\end{aligned}$$

$$\begin{aligned}\alpha\beta &= \frac{c}{a} \\ &= \frac{q}{1} \\ &= q\end{aligned}$$

(i) Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\alpha^2 + \beta^2 = (-p)^2 - 2(q)$$

$$\begin{aligned}&= p^2 - 2q \\ \text{(ii)} \quad \alpha^3\beta + \alpha\beta^3 &= \alpha\beta(\alpha^2 + \beta^2) \quad \text{(A.B)}\end{aligned}$$

$$= \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= q[(-p)^2 - 2(q)]$$

$$\Rightarrow \alpha^3\beta + \alpha\beta^3 = q(p^2 - 2q)$$

$$\text{(iii)} \quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

(LHR 2015) **(K.B + U.B)**

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(-p)^2 - 2(q)}{q}$$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{p^2 - 2q}{q}$$

Q.2 If α, β are the roots of the equation $4x^2 - 5x + 6 = 0$, then find the value of

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ **(K.B + A.B)**

(ii) $\alpha^2\beta^2$ **(A.B + U.B)**

(iii) $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ **(U.B + A.B)**

(iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ **(K.B + U.B)**

(LHR 2016, GRW 2014, SWL 2016, MTN 216, SGD 2015, D.G.K 2014)

Solution:

$$4x^2 - 5x + 6 = 0$$

Here $a = 4, b = -5, c = 6$

Since α, β be the roots of the given equation

Then

$$\begin{aligned}\alpha + \beta &= -\frac{b}{a} \\ &= -\frac{(-5)}{4}\end{aligned}$$

$$= \frac{5}{4}$$

$$\alpha\beta = \frac{c}{a}$$

$$\begin{aligned}
 &= \frac{6}{4} \\
 &= \frac{3}{2} \\
 \text{(i)} \quad \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\beta + \alpha}{\alpha\beta} \\
 &= \frac{\alpha + \beta}{\alpha\beta}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{5}{\frac{4}{3}} \\
 &= \frac{5}{4} \times \frac{3}{1} \\
 &= \frac{5}{4} \times \frac{2}{3}
 \end{aligned}$$

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{5}{6}$$

$$\begin{aligned}
 \text{(ii)} \quad \alpha^2\beta^2 &= (\alpha\beta)^2 \\
 &= \left(\frac{3}{2}\right)^2
 \end{aligned}$$

$$\Rightarrow \alpha^2\beta^2 = \frac{9}{4}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} &= \frac{\beta + \alpha}{\alpha^2\beta^2} \\
 &= \frac{\alpha + \beta}{(\alpha\beta)^2} \\
 &= \frac{5}{\left(\frac{3}{2}\right)^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{5}{\frac{9}{4}} \\
 &= \frac{5}{9} \times \frac{4}{1}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{5}{9} \times \frac{4}{1} \\
 &= \frac{5}{9} \times \frac{4}{1}
 \end{aligned}$$

$$\Rightarrow \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{5}{9}$$

$$\text{(iv)} \quad \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

Putting the values

$$\begin{aligned}
 &= \frac{\left(\frac{5}{4}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{4}\right)}{\frac{3}{2}} \\
 &= \frac{\frac{125}{64} - \frac{45}{8}}{\frac{3}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{125}{64} - \frac{45}{8}}{\frac{3}{2}} \\
 &= \frac{125 - 360}{64} \times \frac{2}{3}
 \end{aligned}$$

$$= \frac{-235}{64} \times \frac{2}{3}$$

$$= \frac{-235}{64} \times \frac{2}{3}$$

$$\Rightarrow \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = -\frac{235}{96}$$

Q.3 If α, β are the roots of the equation $lx^2 + mx + n = 0$ ($l \neq 0$), then find the values of

$$\text{(i)} \quad \alpha^3\beta^2 + \alpha^2\beta^3 \quad \text{(A.B + U.B)}$$

$$\text{(ii)} \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} \quad \text{(K.B + U.B)}$$

Solution:

$$lx^2 + mx + n = 0$$

Roots of given equation are α, β

$$\text{Then } \alpha + \beta = -\frac{b}{a} = -\frac{m}{l}$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{n}{l}$$

$$\text{(i)} \quad \text{Now } \alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha + \beta)$$

$$= (\alpha\beta)^2(\alpha + \beta)$$

$$= \left(\frac{n}{l}\right)^2 \times \left(-\frac{m}{l}\right)$$

$$= \frac{n^2}{l^2} \times \left(-\frac{m}{l}\right)$$

$$\Rightarrow \alpha^3\beta^2 + \alpha^2\beta^3 = -\frac{mn^2}{l^3}$$

$$\begin{aligned}
 \text{(ii)} \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2 + \alpha^2}{\alpha^2 \beta^2} \\
 &\text{(FSD 2017, SWL 2017, BWP 2014, D.G.K 2017)} \\
 &= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} \\
 &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \\
 &= \frac{\left(-\frac{m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2} \\
 &= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}} \\
 &= \frac{m^2 - 2ln}{l^2} \times \frac{l^2}{n^2} \\
 \Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{m^2 - 2ln}{n^2}
 \end{aligned}$$

Exercise 2.5

Q.1 Write the quadratic equations having following roots.

- (a) 1, 5 **(K.B + A.B)**
 (b) 4, 9 **(K.B + A.B)**
 (c) -2, 3 **(K.B + A.B)**
 (d) 0, -3 **(K.B + A.B)**
 (e) 2, -6 **(K.B + A.B)**
 (f) -1, -7 **(K.B + A.B)**
 (g) $1+i, 1-i$ **(K.B + A.B)**
 (h) $3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**

Solution:

- (a) Roots of required equation are 1, 5
 Then sum of roots = $S = 1 + 5 = 6$
 And product of roots = $P = 1 \times 5 = 5$
 $\therefore$ Required quadratic equation is:
 $x^2 - Sx + P = 0$
 $x^2 - 6x + 5 = 0$
 (b) **(FSD 2016, 17, RWP 2017, RWP 2017)**
 Roots of required equation are 4, 9
 Then sum of roots = $S = 4 + 9 = 13$
 And product of roots = $P = 4 \times 9 = 36$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 13x + 36 = 0$$

- (c) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**

Roots of required equation are -2, 3

Then sum of roots = $S = -2 + 3 = 1$

And product of roots = $P = -2(3) = -6$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 1x + (-6) = 0 \quad \textbf{(K.B + A.B)}$$

$$x^2 - x - 6 = 0$$

- (d) **(SGD 2014, BWP 2017)**

Roots of required equation are 0, -3

Then sum of roots = $S = 0 + (-3) = -3$

And product of roots = $P = 0(-3) = 0$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 0 = 0 \quad \textbf{(K.B + A.B)}$$

$$x^2 + 3x = 0$$

- (e) **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**

Roots of required equation are 2, -6

Sum of roots = $S = 2 + (-6) = -4$

Product of roots = $P = 2(-6) = -12$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 2$$

$$x^2 - (-4)x + (-12) = 0$$

$$x^2 + 4x - 12 = 0 \quad \textbf{(K.B + A.B)}$$

- (f) **(LHR 2015, 17, RWP 2016)**

Roots of required equation are -1, -7

Sum of roots = $S = -1 + (-7) = -8$

Product of roots = $P = -1(-7) = 7$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - (-8)x + 7 = 0 \quad \textbf{(K.B + A.B)}$$

$$x^2 + 8x + 7 = 0$$

- (g) **(D.G.K 2014, SGD 2017)**

$$1+i, 1-i \quad \textbf{(K.B + A.B)}$$

Roots of the required equation are

$$1+i, 1-i$$

$$\text{Sum of roots} = S = (1+i) + (1-i)$$

$$= 1+i+1-i$$

- $= 2$
- Product of roots $= P = (1+i)(1-i)$
- $$= (1)^2 - (i)^2$$
- $$= 1 - i^2$$
- $$= 1 - (-1)$$
- $$= 1 + 1$$
- $$= 2$$
- Required quadratic equation is
- $$x^2 - Sx + P = 0$$
- $$x^2 - 2x + 2 = 0$$
- (h) Roots of required equation are $3 + \sqrt{2}, 3 - \sqrt{2}$ **(K.B + A.B)**
- Sum of roots $= (3 + \sqrt{2}) + (3 - \sqrt{2})$
- $$= 3 + \sqrt{2} + 3 - \sqrt{2}$$
- $$= 6$$
- Product of roots $= (3 + \sqrt{2})(3 - \sqrt{2})$
- $$= (3)^2 - (\sqrt{2})^2$$
- $$= 9 - 2$$
- $$= 7$$
- $\therefore$ Required quadratic equation is
- $$x^2 - Sx + P = 0$$
- $$x^2 - 6x + 7 = 0$$
- Q.2** If α, β are the roots of the equation $x^2 - 3x + 6 = 0$. Form equations whose roots are
- (a) $2\alpha + 1, 2\beta + 1$
- (b) α^2, β^2
- (c) $\frac{1}{\alpha}, \frac{1}{\beta}$
- (d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$
- (e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution: **(K.B + A.B)**

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

Roots of given equations are α, β

Then $\alpha + \beta = -\frac{b}{a}$

$$= -\frac{-3}{1}$$

$$= 3$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{6}{1}$$

$$= 6$$

- (a) Roots of required equation are $2\alpha + 1, 2\beta + 1$

(FSD 2015) (K.B + A.B)

Sum of roots $= S = (2\alpha + 1) + (2\beta + 1)$

$$= 2\alpha + 1 + 2\beta + 1$$

$$= 2\alpha + 2\beta + 2$$

$$= 2(\alpha + \beta) + 2$$

$$= 2(3) + 2$$

$$= 6 + 2$$

$$= 8$$

Product of roots $= P = (2\alpha + 1)(2\beta + 1)$

$$= 4\alpha\beta + 2\alpha + 2\beta + 1$$

$$= 4(6) + 6 + 1$$

$$= 24 + 7 = 31$$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 8x + 31 = 0$$

- (b) Roots of required equation are α^2, β^2

Sum of roots $= S = \alpha^2 + \beta^2$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (3)^2 - 2(6)$$

$$= 9 - 12$$

$$= -3 \quad \textbf{(K.B + A.B)}$$

Product of roots $= P = \alpha^2\beta^2$

$$= (\alpha\beta)^2$$

$$= (6)^2$$

$$= 36$$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 36 = 0$$

$$x^2 + 3x + 36 = 0$$

- (c) Roots of required equation are $\frac{1}{\alpha}, \frac{1}{\beta}$
(K.B + A.B)

$$\begin{aligned}\text{Sum of roots} = S &= \frac{1}{\alpha} + \frac{1}{\beta} \\ &= \frac{\beta + \alpha}{\alpha\beta} \\ &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{3}{6} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{Product of roots} = P &= \frac{1}{\alpha} \cdot \frac{1}{\beta} \\ &= \frac{1}{\alpha\beta} \\ &= \frac{1}{6}\end{aligned}$$

∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiply by '6'

$$6x^2 - 3x + 1 = 0$$

- (d) Roots of required equation are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$
(K.B + A.B)

$$\begin{aligned}S &= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \\ &= \frac{(3)^2 - 2(6)}{6} \\ &= \frac{9 - 12}{6} = \frac{-3}{6} \\ \Rightarrow S &= -\frac{1}{2}\end{aligned}$$

$$P = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$$

$$\Rightarrow P = 1$$

Required quadratic equation is

$$x^2 - Sx + P = 0$$

Or $x^2 - \left(-\frac{1}{2}\right)x + 1 = 0$

$$x^2 + \frac{1}{2}x + 1 = 0 \quad (\text{Multiplying by 2})$$

$$2x^2 + x + 2 = 0$$

- (e) Roots of required equation,

$$\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta} \quad (\text{K.B + A.B})$$

$$S = (\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta) + \left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$= (\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$= 3 + \frac{3}{6} = \frac{18 + 3}{6}$$

$$= \frac{21}{6} = \frac{7}{2}$$

$$P = (\alpha + \beta) \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta) \left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$= 3 \left(\frac{3}{6}\right)$$

$$= (3) \left(\frac{1}{2}\right)$$

$$P = \frac{3}{2}$$

Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying by (2)

$$2x^2 - 7x + 3 = 0$$

Q.3 If α, β are the roots of the equation $x^2 + px + q = 0$ form equations whose roots are:

(a) α^2, β^2 (FSD 2015)

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

$$x^2 + px + q = 0$$

Here

$$a = 1, b = p, c = q$$

Roots of given equation are α, β

$$\text{Then } \alpha + \beta = -\frac{b}{a} = -p$$

$$\alpha\beta = \frac{c}{a} = q$$

(a) Roots of required equation are α^2, β^2

(K.B + A.B)

$$\text{Sum of roots} = S = \alpha^2 + \beta^2$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-p)^2 - 2q$$

$$= p^2 - 2q$$

$$\text{Product of roots} = P = \alpha^2\beta^2$$

$$= (\alpha\beta)^2$$

$$= (q)^2$$

$$= q^2$$

$\therefore$ Required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

(b) Roots of required equation are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(K.B + A.B)

$$\text{Sum of roots} = S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{p^2 - 2q}{q}$$

$$\text{Product of roots} = P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$\Rightarrow x^2 - \frac{p^2 - 2q}{q}x + 1 = 0$$

Multiply by 'q'

$$qx^2 - (p^2 - 2q)x + q = 0$$

Synthetic Division

(K.B)

It is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial.

It is a short cut of long division.

Example 3: (Page # 35)

(A.B)

Use synthetic division, divide the polynomial $P(x) = 5x^4 + x^3 - 3x$ by $x - 2$.

Solution:

$$P(x) = 5x^4 + x^3 + 0x^2 - 3x + 0$$

$$\text{Here } x - a = x - 2 \Rightarrow x = 2$$

2	5	1	0	-3	0
	↓	10	22	44	82
	5	11	22	41	82

$$\therefore Q(x) = 5x^3 + 11x^2 + 22x + 41$$

$$R = 82$$

Example 3: (Page # 36)

(A.B)

Using synthetic division, find the value of h . If the zero of polynomial $p(x) = 3x^2 + 4x - 7h$ is 1.

Solution:

$$p(x) = 3x^2 + 4x - 7h \text{ and its zero is } 1.$$

Then by the synthetic division.

1	3	4	-7h
	3	7	7
	3	7	7-7h

$$\text{Remainder} = 7 - 7h$$

Since 1 is the zero of the polynomial, therefore,

Remainder = 0, that is

$$7 - 7h = 0$$

$$7 = 7h$$

$$\Rightarrow h = 1$$

Example 4: (Page # 36)

(A.B)

Using synthetic division, find the values of l and m , if $x - 1$ and $x + 1$

are the factors of the polynomial

$$P(x) = x^3 + 3lx^2 + mx - 1$$

Solution:

Since $x-1$ and $x+1$ are the factors of $P(x) = x^3 + 3lx^2 + mx - 1$

Therefore, 1 and -1 are zeros of polynomial $P(x)$.

Now by synthetic division

1	1	3l	m	-1
	↓	-1	-3l+1	3l+m+1
	1	3l-1	3l+m+1	3l+m

Since 1 is the zero of polynomial, therefore, remainder is zero, that is, $3l+m=0 \rightarrow (i)$

And

-1	1	3l	m	-1
	↓	-1	-3l+1	3l-m-1
	1	3l-1	3l+m+1	3l-m-2

Since -1 is the zero of polynomial, therefore, remainder is zero, that is, $3l-m-2=0 \rightarrow (ii)$

Adding equations (i) and (ii), we get $6l-2=0$

$$6l = 2 \Rightarrow l = \frac{2}{6} = \frac{1}{3}$$

Put the value of l in equation (i)

$$3\left(\frac{1}{3}\right) + m = 0 \text{ or}$$

$$1 + m = 0 \Rightarrow m = -1$$

$$\text{Thus } l = \frac{1}{3} \text{ and } m = -1$$

Example 6: (Page # 38) (A.B)

By synthetic division, solve the equation $x^4 - 49x^2 + 36x + 252 = 0$ having roots -2 and 6.

Solution:

Since -2 and 6 are the roots of the given equation $x^4 - 49x^2 + 36x + 252 = 0$.

Then by synthetic division, we get

	1	0	-49	36	252
-2	↓	-2	4	90	-252
	1	-2	-45	126	0
6		6	24	-126	
	1	4	-21	0	

$\therefore$ The depressed equation is

$$x^2 + 4x - 21 = 0$$

$$x^2 + 7x - 3x - 21 = 0$$

$$x(x+7) - 3(x+7) = 0$$

$$(x+7)(x-3) = 0$$

Either $x+7=0$ or $x-3=0$

$$x = -7 \text{ or } x = 3$$

Thus -2, -7 and 3 are the roots of the given equation.

Exercise 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

Solution:

(i) $P(x) = x^2 + 7x - 1$

(FSD 2016, 17) (A.B)

-1	1	7	-1
	↓	-1	-6
	1	6	-7

$$\therefore Q(x) = x + 6$$

$$R = -7$$

(ii) $P(x) = 4x^3 - 5x + 15$ (A.B)

(SWL 2016, SGD 2017, MTN 2016)

$$= 4x^3 + 0x^2 - 5x + 15$$

-3	4	0	-5	15
	↓	-12	36	-93
	4	-12	31	-78

$$\therefore Q(x) = 4x^2 - 12x + 31$$

$$R = -78$$

(iii) $P(x) = x^3 + x^2 - 3x + 2$ (A.B)

(GRW 2017, FSD 2015, MTN 2017, D.G.K 2015)

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

$$\therefore Q(x) = x^2 + 3x + 3$$

$$R = 8$$

Q.2 Find the value of h using synthetic division, if (A.B)

- (i) 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$
 (ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$
 (iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$

Solution:

(i) $P(x) = 2x^3 - 3hx^2 + 9$

(SWL 2014) (A.B)

$$= 2x^3 - 3hx^2 + 0x + 9$$

3	2	-3h	0	9
	↓	6	-9h+18	-27+54
		2	-3h+6	-9h+18
				-27h+63

Since 3 is the zero of given polynomial, $R = 0$

$$\Rightarrow -27h + 63 = 0$$

$$-27h = -63$$

$$h = \frac{-63}{-27}$$

$$\Rightarrow h = \frac{7}{3}$$

(ii) $P(x) = x^3 - 2hx^2 + 11$ (A.B)

$$= x^3 - 2hx^2 + 0x + 11$$

1	1	-2h	0	11
1	↓	1	-2h+1	-2h+1
		1	-2h+1	-2h+1
				-2h+12

Since 1 is zero of given polynomial, $R = 0$

$$\Rightarrow -2h + 12 = 0$$

$$-2h = -12$$

$$h = \frac{-12}{-2}$$

$$h = 6$$

Result:

$$h = 6$$

(iii) $P(x) = 2x^3 + 5hx - 23$ (A.B)

$$= 2x^3 + 0x^2 + 5hx - 23$$

-1	2	0	5h	-23
	↓	-2	2	-5h-2

1	-2h+1	-2h+1	-5h-25
---	-------	-------	--------

Since 1 is zero of given polynomial, $R = 0$

$$\Rightarrow -5h - 25 = 0$$

$$-5h = 25$$

$$h = -\frac{25}{5} = -5$$

Result:

$$h = -5$$

Q.3 Use synthetic division to find the values of l and m, if (A.B)

- (i) $(x + 3)$ and $(x - 2)$ are the factors of the polynomial $x^3 + 4x^2 + 2lx + m$
 (ii) $(x - 1)$ and $(x + 1)$ are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

Solution:

(i) $P(x) = x^3 + 4x^2 + 2lx + m$ (A.B)

-3	1	4	2l	m
	↓	-3	-3	-6l-9
		1	1	2l-3
2	↓	2	6	-6l+m+9
		1	3	2l+3

Since $x - 2$ is a factor, $R = 0$

$$\Rightarrow 2l + 3 = 0$$

$$2l = -3$$

$$\Rightarrow l = \frac{-3}{2}$$

Also $x + 3$ is a factor, $R = 0$

$$\Rightarrow -6l + m + 9 = 0$$

$$-6\left(\frac{-3}{2}\right) + m + 9 = 0 \quad \therefore l = \frac{-3}{2}$$

$$9 + m + 9 = 0$$

$$m + 18 = 0$$

$$\Rightarrow m = -18$$

Result

$$l = \frac{-3}{2}, \quad m = -18$$

(ii) $P(x) = x^3 - 3lx^2 + 2mx + 6$ (A.B)

1	1	-3l	2m	6
	↓	1	-3l+1	-3l+2m+1

$$\begin{array}{r|rrrr}
 -1 & 1 & -3l+1 & -3l+2m+1 & -3l+2m+7 \\
 & \downarrow & -1 & 3l & \\
 \hline
 & 1 & -3l & 2m+1 &
 \end{array}$$

Since $x+1$ is a factor, $R = 0$

$$\Rightarrow 2m+1=0$$

$$2m = -1$$

$$\Rightarrow m = -\frac{1}{2}$$

Also $x-1$ is a factor, $R = 0$

$$-3l+2m+7=0$$

$$-3l+2\left(-\frac{1}{2}\right)+7=0 \therefore m = -\frac{1}{2}$$

$$-3l-1+7=0$$

$$-3l+6=0$$

$$-3l = -6$$

$$\Rightarrow l = 2$$

Result

$$l = 2, m = -\frac{1}{2}$$

Q.4 Solve by using synthetic division, if

(i) 2 is the root of the equation

$$x^3 - 28x + 48 = 0$$

(ii) 3 is the root of the equation

$$2x^3 - 3x^2 - 11x + 6 = 0$$

(iii) -1 is the root of the equation

$$4x^3 - x^2 - 11x - 6 = 0$$

Solution:

(i) $P(x) = x^3 - 28x + 48$ **(A.B)**

$$= x^3 + 0x^2 - 28x + 48$$

$$\begin{array}{r|rrrr}
 2 & 1 & 0 & -28 & 48 \\
 & \downarrow & 2 & 4 & -48 \\
 \hline
 & 1 & 2 & -24 & 0
 \end{array}$$

$\therefore$ Depressed equation is:

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

Either

$$x+6=0 \quad \text{or} \quad x-4=0$$

$$x = -6$$

$$x = 4$$

Thus 2, 4 and -6 are the roots of the given equation.

$$\therefore \text{Solution Set} = \{2, 4, -6\}$$

(ii) $P(x) = 2x^3 - 3x^2 - 11x + 6$ **(A.B)**

$$\begin{array}{r|rrrr}
 3 & 2 & -3 & -11 & 6 \\
 & \downarrow & 4 & 9 & -6 \\
 \hline
 & 2 & 3 & -2 & 0
 \end{array}$$

$\therefore$ Depressed equation is:

$$2x^2 + 3x - 2 = 0$$

$$2x^2 + 4x - x - 2 = 0$$

$$2x(x+2) - 1(x+2) = 0$$

$$(x+2)(2x-1) = 0$$

Either

$$x+2=0$$

$$\text{Or} \quad 2x-1=0$$

$$x = -2$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\therefore \text{Solution Set} = \left\{3, -2, \frac{1}{2}\right\}$$

(iii) $P(x) = 4x^3 - x^2 - 11x - 6$ **(A.B)**

$$\begin{array}{r|rrrr}
 -1 & 4 & -1 & -11 & -6 \\
 & \downarrow & -4 & 5 & 6 \\
 \hline
 & 4 & -5 & -6 & 0
 \end{array}$$

$\therefore$ Depressed equation is

$$4x^2 - 5x - 6 = 0$$

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x-2) + 3(x-2) = 0$$

$$(x-2)(4x+3) = 0$$

Either

$$x-2=0$$

$$\text{or} \quad 4x+3=0$$

$$x = 2$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

Thus $-1, 2, -\frac{3}{4}$ are the roots of the given equation.

$$\therefore \text{Solution Set} = \left\{-1, 2, -\frac{3}{4}\right\}$$

Q.5 Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

- (ii) 3 and -4 are the roots of the equation
 $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Solution: (A.B)

(i)	$P(x) = x^4 + 0x^3 - 10x^2 + 0x + 9$				
1	1	0	-10	0	9
		1	1	-9	-9
3	1	1	-9	-9	0
		3	12	9	
	1	4	3	0	

∴ Depressed equation is

$$x^2 + 4x + 3 = 0$$

$$x^2 + 3x + x + 3 = 0$$

$$x(x+3) + 1(x+3) = 0$$

$$(x+3)(x+1) = 0$$

Either

$$x+3=0 \quad \text{or} \quad x+1=0$$

$$x=-3 \quad \quad \quad x=-1$$

$$\therefore \text{Solution Set} = \{\pm 3, \pm 1\}$$

- (ii) $P(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$

	1	2	-13	-14	24
3	↓	3	15	6	-24
	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

∴ Depressed equation is:

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x+2)(x-1) = 0$$

Either

$$x+2=0 \quad \text{or} \quad x-1=0$$

$$x=-2 \quad \quad \quad x=1$$

Thus -2, -4, 1 and 3 are the roots of the given equation.

$$\therefore \text{Solution Set} = \{-2, -4, 1, 3\}$$

Simultaneous Equation (K.B)

A system of equations having a common solution is called a system of simultaneous equations.

For example $x + 2y = 3$, $2x - y = 1$ having same solution (1,1).

Solution Set (K.B)

The set of all the ordered pairs (x, y) , which satisfies the system of equations is called the solution set of the system.

Ordered Pair (K.B)

An ordered pair of real numbers x and y is a pair (x, y) in which elements are written in specific order.

For example: (x, y) is an ordered pair in which first elements (abscissa) is x and second element (ordinate) is y .

Note (K.B)

$(x, y) \neq (y, x)$. For example (2,3) and (3,2) are two different ordered pairs.

Example 1 (Page # 39) (A.B)

Solve the system of equations

$$3x + y = 4 \quad \text{and} \quad 3x^2 + y^2 = 52.$$

Solution:

The given equations are

$$3x + y = 4 \rightarrow (i)$$

$$3x^2 + y^2 = 52 \rightarrow (ii)$$

$$\text{From equation (i) } y = 4 - 3x \rightarrow (iii)$$

Put value of y in equation (ii)

$$3x^2 + (4 - 3x)^2 = 52$$

$$3x^2 + 16 - 24x + 9x^2 - 52 = 0$$

$$12x^2 - 24x - 36 = 0$$

$$12(x^2 - 2x - 3) = 0$$

$$x^2 - 2x - 3 = 0 \quad \because 12 \neq 0$$

By factorization

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x-3)(x+1) = 0$$

Either

$$x-3=0 \quad \text{or} \quad x+1=0$$

$$x=3 \quad \text{or} \quad x=-1$$

Put the values of x in equation (iii)

When $x = 3$ when $x = -1$

$$y = 4 - 3x \quad y = 4 - 3x$$

$$y = 4 - 3(3) \quad y = 4 - 3(-1)$$

$$= 4 - 9 \quad = 4 + 3$$

$$y = -5 \quad y = 7$$

$\therefore$ ordered pairs are $(3, -5)$ and $(-1, 7)$

Thus, the solution set is $\{(3, -5), (-1, 7)\}$

Example 2 (Page # 40)**(A.B)**

Solve the equations

$$x^2 + y^2 + 2x = 8 \text{ and } (x-1)^2 + (y+1)^2 = 8$$

Solution:

The given equation are

$$x^2 + y^2 + 2x = 8 \rightarrow (i)$$

$$(x-1)^2 + (y+1)^2 = 8 \rightarrow (ii)$$

From equation (ii), we get

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 8$$

$$\text{Or } x^2 + y^2 - 2x + 2y = 6 \rightarrow (iii)$$

Subtracting equation (iii) from equation (i) we have

$$4x - 2y = 2 \quad \text{or} \quad 2x - y = 1$$

$$\Rightarrow y = 2x - 1$$

Put the value of y in equation (ii)

$$(x-1)^2 + (2x-1+1)^2 = 8$$

$$x^2 - 2x + 1 + 4x^2 - 8 = 0$$

$$5x^2 - 2x - 7 = 0$$

$$5x^2 - 7x + 5x - 7 = 0$$

$$x(5x-7) + 1(5x-7) = 0$$

$$(5x-7)(x+1) = 0$$

Either

$$5x-7=0 \quad \text{or} \quad x+1=0$$

$$5x=7 \quad \text{or} \quad x=-1$$

$$\Rightarrow x = \frac{7}{5}$$

Now putting the values of x in equation (iv), we have

$$\text{When } x = \frac{7}{5} \quad \text{when } x = -1$$

$$y = 2\left(\frac{7}{5}\right) - 1 \quad y = 2(-1) - 1$$

$$y = \frac{14}{5} - 1 = \frac{14-5}{5} = \frac{9}{5} \quad y = -3$$

Thus, the solution set is

$$\left\{(-1, -3), \left(\frac{7}{5}, \frac{9}{5}\right)\right\}$$

Example 3 (Page # 41)**(A.B)**

Solve the equations

$$x^2 + y^2 = 7 \text{ and } 2x^2 + 3y^2 = 18$$

Solution:

Given equations are

$$x^2 + y^2 = 7 \quad (i)$$

$$2x^2 + 3y^2 = 18 \quad (ii)$$

Multiply equation (i) with 3

$$3x^2 + 3y^2 = 21 \quad (iii)$$

Subtracting equations (ii) from (iii)

$$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

When $x = \sqrt{3}$, then from equation (i)

$$x^2 + y^2 = 7 \text{ or}$$

$$3 + y^2 = 7 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$$

When $x = -\sqrt{3}$, then $y = \pm 2$

Thus, the required solution set is

$$\{(\pm\sqrt{3}, \pm 2)\}.$$

Example 4 (Page # 41)**(A.B)**

Solve the equations

$$x^2 + y^2 = 20 \quad (i)$$

$$6x^2 + xy - y^2 = 0 \quad (ii)$$

The equation (ii) can be written as

$$y^2 - xy - 6x^2 = 0$$

$$\begin{aligned} \Rightarrow y &= \frac{-(-x) \pm \sqrt{(-x)^2 - 4 \times 1 \times (-6x^2)}}{2 \times 1} \\ &= \frac{x \pm \sqrt{x^2 + 24x^2}}{2} = \frac{x \pm \sqrt{25x^2}}{2} \\ &= \frac{x \pm 5x}{2} \end{aligned}$$

$$\text{We have } y = \frac{x+5x}{2} = \frac{6x}{2} = 3x$$

$$\text{Or } y = \frac{x-5x}{2} = \frac{-4x}{2} = -2x$$

Substituting $y = 3x$ in the equation (i), we get

$$x^2 + (3x)^2 = 20$$

$$x^2 + 9x^2 = 20$$

$$\Rightarrow 10x^2 = 20$$

$$\Rightarrow x^2 = 4 \Rightarrow x = \pm\sqrt{2}$$

$$\text{When } x = \sqrt{2}, y = 3(\sqrt{2}) = 3\sqrt{2} \text{ and}$$

$$\text{when } x = -\sqrt{2}, y = 3(-\sqrt{2}) = -3\sqrt{2}$$

Substituting $y = -2x$ in the equation (i), we have

$$x^2 + (-2x)^2 = 20 \text{ or } x^2 + 4x^2 = 20$$

$$\Rightarrow 5x^2 = 20 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

$$\text{When } x = 2 \Rightarrow y = -2(2) = -4$$

$$\text{when } x = -2 \Rightarrow y = -2(-2) = 4$$

Thus, the solution is

$$\{(\sqrt{2}, 3\sqrt{2}), (-\sqrt{2}, -3\sqrt{2}), (2, -4), (-2, 4)\}.$$

Example 5 (Page # 42)

(A.B)

Solve the equations

$$x^2 + y^2 = 40 \text{ and } 3x^2 - 2xy - y^2 = 80$$

Solution:

Given equations are

$$x^2 + y^2 = 40 \rightarrow (i)$$

$$3x^2 - 2xy - y^2 = 80 \rightarrow (ii)$$

Multiplying equation (i) by 2, we have

$$2x^2 + 2y^2 = 80 \rightarrow (iii)$$

Subtracting the equation (iii) from equation (ii), we get

$$x^2 - 2xy - 3y^2 = 0 \rightarrow (iv)$$

The equation (iv) can be written as

$$x^2 - 3xy + xy - 3y^2 = 0$$

$$x(x - 3y) + y(x - 3y) = 0$$

$$(x - 3y)(x + y) = 0$$

$$\text{Either } x - 3y \text{ or } x + y = 0$$

$$\Rightarrow x = 3y \rightarrow (v) \text{ or } x = -y \rightarrow (vi)$$

Put in equation (i),

$$\text{Equation (i)} \Rightarrow (3y)^2 + y^2 = 40$$

$$10y^2 = 40$$

$$y^2 = 4$$

$$y = \pm 2$$

$$y = 2$$

$$y = -2$$

$$\text{eq. (v)} \Rightarrow$$

$$x = 3(2)$$

$$x = 3(-2)$$

$$x = 6$$

$$x = -6$$

$$\text{Equation (i)} \Rightarrow (-y)^2 + y^2 = 40$$

$$2y^2 = 40$$

$$y^2 = 20$$

$$y = \pm 2\sqrt{5}$$

$$y = 2\sqrt{5}$$

$$y = -2\sqrt{5}$$

$$\text{eq. (vi)} \Rightarrow$$

$$x = -(2\sqrt{5})$$

$$x = -(-2\sqrt{5})$$

$$x = -2\sqrt{5}$$

$$= 2\sqrt{5}$$

$\therefore$ The solution set is

$$\{(6, 2), (-6, -2), (2\sqrt{5}, -2\sqrt{5}), (-2\sqrt{5}, 2\sqrt{5})\}$$

Exercise 2.7

Solve the following simultaneous equations.

Q.1 $x + y = 5$

(A.B)

$$x^2 - 2y - 14 = 0$$

Solution:

$$x + y = 5 \rightarrow (i)$$

$$x^2 - 2y - 14 = 0 \rightarrow (ii)$$

From equation (i)

$$y = 5 - x \rightarrow (iii)$$

Put in equation (ii)

$$x^2 - 2y - 14 = 0$$

$$x^2 - 2(5 - x) - 14 = 0$$

$$x^2 - 10x + 2x - 14 = 0$$

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

Either

$$x+6=0 \quad \text{or} \quad x-4=0$$

$$\Rightarrow x = -6 \quad \text{or} \quad x = 4$$

Put in equation (iii)

$$y = 5 - x \quad \text{or} \quad y = 5 - x$$

$$= 5 - (-6) \quad \text{or} \quad = 5 - 4$$

$$= 5 + 6 \quad \text{or} \quad = 1$$

$$= 11$$

$$\therefore \text{Solution Set} = \{(-6, 11), (4, 1)\}$$

Q.2 $3x - 2y = 1$ **(A.B)**

$$x^2 + xy - y^2 = 1$$

Solution:

$$3x - 2y = 1 \rightarrow (i)$$

$$\text{and } x^2 + xy - y^2 = 1 \rightarrow (ii)$$

From equation (i)

$$3x - 2y = 1$$

$$3x = 2y + 1$$

$$x = \frac{2y+1}{3} \rightarrow (iii)$$

Now put in equation (ii)

$$x^2 + xy - y^2 = 1$$

$$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)y - y^2 = 1$$

$$\frac{1+4y+4y^2}{9} + \frac{y+2y^2}{3} - y^2 = 1$$

Multiplying both sides by LCM

We get

$$1+4y+4y^2+3(y+2y^2)-9y^2=9$$

$$1+4y+4y^2+3y+6y^2-9y^2-9=0$$

$$y^2+7y-8=0$$

$$y^2+8y-y-8=0$$

$$\Rightarrow y(y+8)-1(y+8)=0$$

$$(y+8)(y-1)=0$$

Either

$$y+8=0 \quad \text{or} \quad y-1=0$$

$$y=-8 \quad \text{or} \quad y=1$$

Putting values in equation (iii)

When $y = -8$

$$x = \frac{1+2y}{3}$$

$$x = \frac{1+2(-8)}{3}$$

$$x = \frac{1-16}{3}$$

$$x = \frac{-15}{3}$$

$$x = -5$$

When $y = 1$

$$x = \frac{1+2y}{3}$$

$$x = \frac{1+2(1)}{3}$$

$$x = \frac{3}{3}$$

$$x = 1$$

$$\therefore \text{Solution Set} = \{(-5, -8), (1, 1)\}$$

Q.3 $x - y = 7$ **(A.B)**

$$\frac{2}{x} - \frac{5}{y} = 2$$

Solution:

$$x - y = 7 \rightarrow (i)$$

$$\frac{2}{x} - \frac{5}{y} = 2 \rightarrow (ii)$$

From equation (ii)

$$\frac{2}{x} - \frac{5}{y} = 2$$

Multiple by xy

$$2y - 5x = 2xy$$

$$2y - 5x - 2xy = 0$$

$$2y - 2xy - 5x = 0 \rightarrow \text{(iii)}$$

From equation (i)

$$x = 7 + y \rightarrow \text{(iv)}$$

Put in equation (iii)

$$2y - 2(7 + y)(y) - 5(7 + y) = 0$$

$$2y - 14y - 2y^2 - 5y - 35 = 0$$

$$-2y^2 - 17y - 35 = 0$$

$$-(2y^2 + 17y + 35) = 0$$

$$\Rightarrow 2y^2 + 17y + 35 = 0$$

$$2y^2 + 10y + 7y + 35 = 0$$

$$2y(y + 5) + 7(y + 5) = 0$$

$$(y + 5)(2y + 7) = 0$$

Either

$$y + 5 = 0 \quad \text{or} \quad 2y + 7 = 0$$

$$y = -5 \quad \quad \quad y = \frac{-7}{2}$$

Now put in equation (i)

when $y = -5$

$$x - (-5) = 7$$

$$x + 5 = 7$$

$$x = 7 - 5$$

$$x = 2$$

when

$$y = \frac{-7}{2}$$

$$x - \left(\frac{-7}{2}\right) = 7$$

$$x + \frac{7}{2} = 7$$

$$x = 7 - \frac{7}{2}$$

$$x = \frac{14 - 7}{2}$$

$$x = \frac{7}{2}$$

$$\therefore \text{Solution Set} = \left\{ (2, -5), \left(\frac{7}{2}, \frac{-7}{2}\right) \right\}$$

$$\text{Q.4} \quad x + y = a - b \quad \quad \quad \text{(A.B)}$$

$$\frac{a}{x} - \frac{b}{y} = 2$$

Solution:

$$x + y = a - b \rightarrow \text{(i)}$$

$$\frac{a}{x} - \frac{b}{y} = 2 \rightarrow \text{(ii)}$$

From equation (ii)

$$\frac{a}{x} - \frac{b}{y} = 2$$

Multiply by 'xy'

$$ay - bx = 2xy \rightarrow \text{(iii)}$$

From equation (i)

$$\Rightarrow x + y = a - b$$

$$y = a - b - x \rightarrow \text{(iv)}$$

Put in equation (iii)

$$a(a - b - x) - bx = 2x(a - b - x)$$

$$a^2 - ab - ax - bx = 2ax - 2bx - 2x^2$$

$$2x^2 - ax - 2ax - bx + 2bx + a^2 - ab = 0$$

$$2x^2 - 3ax + bx + a^2 - ab = 0$$

$$2x^2 - (3a - b)x + (a^2 - ab) = 0$$

$$\text{Here } A = 2, B = -(3a - b), C = a^2 - ab$$

Using quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{-[-(3a - b)] \pm \sqrt{[-(3a - b)]^2 - 4(2)(a^2 - ab)}}{2(2)}$$

$$x = \frac{3a - b \pm \sqrt{(3a - b)^2 - 8(a^2 - ab)}}{4}$$

$$= \frac{3a - b \pm \sqrt{9a^2 - 6ab + b^2 - 8a^2 + 8ab}}{4}$$

$$= \frac{3a - b \pm \sqrt{a^2 + 2ab + b^2}}{4}$$

$$= \frac{3a - b \pm \sqrt{(a+b)^2}}{4}$$

$$= \frac{3a - b \pm (a+b)}{4}$$

Either

$$x = \frac{3a - b + a + b}{4} \quad \text{or} \quad x = \frac{3a - b - a - b}{4}$$

$$= \frac{4a}{4}$$

$$x = a$$

$$= \frac{2a - 2b}{4}$$

$$x = \frac{a - b}{2}$$

Put in equation (iv)

$$y = a - b - a \quad \left| \quad y = a - b - \frac{a - b}{2} \right.$$

$$y = -b \quad \left| \quad y = \frac{2a - 2b - (a - b)}{2} \right.$$

$$y = \frac{a - b}{2}$$

$$\therefore \text{Solution Set} = \left\{ (a, -b), \left(\frac{a-b}{2}, \frac{a-b}{2} \right) \right\}$$

Q.5 $x^2 + (y-1)^2 = 10$ **(A.B)**

$$x^2 + y^2 + 4x = 1$$

Solution:

$$x^2 + (y-1)^2 = 10 \rightarrow (i)$$

$$x^2 + y^2 + 4x = 1 \rightarrow (ii)$$

Equation (i) $\Rightarrow$

$$x^2 + y^2 - 2y + 1 = 10$$

$$x^2 + y^2 - 2y = 9 \rightarrow (iii)$$

Subtract equation (ii) and (iii)

$$x^2 + y^2 + 4x = 1$$

$$\underline{-x^2 \pm y^2 \mp 2y = -9}$$

$$4x + 2y = -8$$

$$2(2x + y) = -8$$

$$2x + y = -4$$

$$y = -4 - 2x \rightarrow (iv)$$

Put in equation (ii)

$$x^2 + (-4 - 2x)^2 + 4x = 1$$

$$x^2 + [-(4 + 2x)]^2 + 4x = 1$$

$$x^2 + (4 + 2x)^2 + 4x = 1$$

$$x^2 + 16 + 16x + 4x^2 + 4x - 1 = 0$$

$$5x^2 + 20x + 15 = 0$$

$$5(x^2 + 4x + 3) = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0 \quad \because 5 \neq 0$$

$$x^2 + 3x + x + 3 = 0$$

$$x(x+3) + 1(x+3) = 0$$

$$(x+3)(x+1) = 0$$

Either

$$x+3=0 \quad \text{or} \quad x+1=0$$

$$x = -3$$

$$x = -1$$

Put in equation (iv)

$$y = -4 - 2(-3) \quad y = -4 - 2(-1)$$

$$y = -4 + 6 \quad y = -4 + 2$$

$$y = 2 \quad y = -2$$

$$\therefore \text{Solution Set} = \{(-3, 2), (-1, -2)\}$$

Q.6 $(x+1)^2 + (y+1)^2 = 5$ **(A.B)**

$$(x+2)^2 + y^2 = 5$$

Solution:

$$(x+1)^2 + (y+1)^2 = 5 \rightarrow (i)$$

$$(x+2)^2 + y^2 = 5 \rightarrow (ii)$$

From equation (i) $\Rightarrow$

$$x^2 + 2x + 1 + y^2 + 2y + 1 = 5$$

$$x^2 + y^2 + 2x + 2y = 3$$

$$x^2 + y^2 + 2x + 2y = 3 \rightarrow (iii)$$

$$(x+2)^2 + y^2 = 5$$

$$x^2 + 4x + 4 + y^2 = 5$$

$$x^2 + y^2 + 4x = 1 \rightarrow (iv)$$

Sub equation (iii) and (iv)

$$x^2 + y^2 + 2x + 2y = -3$$

$$\underline{-x^2 \pm y^2 \pm 4x = -1}$$

$$-2x + 2y = 2$$

$$-x + y = 1$$

$$y = x + 1 \rightarrow (v)$$

Put in equation (iv)

$$x^2 + (x+1)^2 + 4x = 1$$

$$x^2 + x^2 + 2x + 1 + 4x - 1$$

$$2x^2 + 6x = 0$$

$$2x(x+3) = 0$$

Either

$$2x=0 \quad \text{or} \quad x+3=0$$

$$x=0 \quad \quad \quad x=-3$$

Put in equation (v)

$$\text{When } x=0$$

$$y = 1 + 0$$

$$y = 1$$

$$\text{When } x=-3$$

$$y = 1 - 3$$

$$y = -2$$

$$\therefore \text{Solution Set} = \{(0,1), (-3,-2)\}$$

$$\text{Q.7} \quad x^2 + 2y^2 = 22 \quad (\text{A.B})$$

$$5x^2 + y^2 = 29$$

Solution:

$$x^2 + 2y^2 = 22 \rightarrow (i)$$

$$5x^2 + y^2 = 29 \rightarrow (ii)$$

Multiply equation (ii) by '2'

$$10x^2 + 2y^2 = 58 \rightarrow (iii)$$

Subtract equation (i) and (iii)

$$10x^2 + 2y^2 = 58$$

$$\underline{-x^2 \pm 2y^2 = -22}$$

$$9x^2 = 36$$

$$x^2 = 4 \quad (\text{Div both sides by 9})$$

Taking square root on both sides

$$x = \pm 2$$

Put $x^2 = 4$ in equation (ii)

$$5(4) + y^2 = 29$$

$$20 + y^2 = 29$$

$$y^2 = 29 - 20$$

$$y^2 = 9$$

Taking square root

$$y = \pm 3$$

$$\therefore \text{Solution Set} = \{(\pm 2, \pm 3)\}$$

$$\text{Q.8} \quad 4x^2 - 5y^2 = 6 \quad (\text{A.B})$$

$$3x^2 + y^2 = 14$$

Solution:

$$4x^2 - 5y^2 = 6 \rightarrow (i)$$

$$3x^2 + y^2 = 14 \rightarrow (ii)$$

Multiply equation (ii) by 5

$$15x^2 + 5y^2 = 70 \rightarrow (iii)$$

Adding equation (i) and (iii)

$$4x^2 - 5y^2 = 6$$

$$15x^2 + 5y^2 = 70$$

$$\underline{19x^2 = 76}$$

$$x^2 = \frac{76}{19}$$

$$x^2 = 4$$

Taking square root

$$x = \pm 2$$

Put $x^2 = 4$ in equation (ii)

$$3(4) + y^2 = 14$$

$$y^2 = 14 - 12$$

$$y^2 = 2$$

Taking square root

$$y = \pm \sqrt{2}$$

$$\therefore \text{Solution Set} = \{(\pm 2, \pm \sqrt{2})\}$$

$$\text{Q.9} \quad 7x^2 - 3y^2 = 4 \quad (\text{A.B})$$

$$2x^2 + 5y^2 = 7$$

Solution:

$$7x^2 - 3y^2 = 4 \rightarrow (i)$$

$$2x^2 + 5y^2 = 7 \rightarrow (ii)$$

Multiply equation (i) by '5'

$$35x^2 - 15y^2 = 20 \rightarrow (iii)$$

Multiply equation (ii) by '3'

$$6x^2 + 15y^2 = 21 \rightarrow (iv)$$

Add equation (iii) and (iv)

$$35x^2 - 15y^2 = 20$$

$$\underline{6x^2 + 15y^2 = 21}$$

$$41x^2 = 41$$

$$x^2 = 1$$

On taking square root, we get

$$x = \pm 1$$

Put $x^2 = 1$ in equation (ii)

$$2x^2 + 5y^2 = 7$$

$$2(1) + 5y^2 = 7$$

$$2 + 5y^2 = 7$$

$$5y^2 = 5$$

$$y^2 = \frac{5}{5}$$

$$y^2 = 1$$

On taking the square root, we get

$$y = \pm 1$$

$$\therefore \text{Solution Set} = \{(\pm 1, \pm 1)\}$$

Q.10 $3x^2 - y^2 = 3$ **(A.B)**

$$x^2 + 4xy - 5y^2 = 0$$

Solution:

$$3x^2 - y^2 = 3 \rightarrow (i)$$

$$x^2 + 4xy - 5y^2 = 0 \rightarrow (ii)$$

Equation (ii) $\Rightarrow$

$$x^2 + 4xy - 5y^2 = 0$$

$$x^2 + 5xy - xy - 5y^2 = 0$$

$$x(x + 5y) - y(x + 5y) = 0$$

$$(x + 5y)(x - y) = 0$$

Either

$$x + 5y = 0 \quad \text{or} \quad x - y = 0$$

$$x = -5y \rightarrow (iii) \quad x = y \rightarrow (iv)$$

Put $x = -5y$ in equation (i)

$$x^2 + 2y^2 = 3$$

$$(-5y)^2 + 2y^2 = 3$$

$$25y^2 + 2y^2 = 3$$

$$27y^2 = 3$$

$$y^2 = \frac{3}{27}$$

$$y^2 = \frac{1}{9}$$

On taking square root, we get

$$y = \pm \frac{1}{3}$$

Putting in equation (iii)

When

$$y = \frac{1}{3}$$

$$x = -5\left(\frac{1}{3}\right)$$

$$x = -\frac{5}{3}$$

When $y = -\frac{1}{3}$

$$x = -5\left(-\frac{1}{3}\right)$$

$$x = \frac{5}{3}$$

Put $x = y$ in equation (i)

$$x^2 + 2y^2 = 3$$

$$y^2 + 2y^2 = 3$$

$$3y^2 = 3$$

$$y^2 = 1$$

On taking square root, we get

$$y = \pm 1$$

Put in equation (iv)

When $y = 1$

$$x = 1$$

When $y = -1$

$$x = -1$$

$\therefore$ **Solution Set**

$$= \left\{ (1, 1), (-1, -1), \left(-\frac{5}{3}, \frac{1}{3}\right), \left(\frac{5}{3}, -\frac{1}{3}\right) \right\}$$

Q.11 $3x^2 - y^2 = 26$ **(A.B)**

$$3x^2 - 5xy - 12y^2 = 0$$

Solution:

$$3x^2 - y^2 = 26 \rightarrow (i)$$

$$3x^2 - 5xy - 12y^2 = 0 \rightarrow (ii)$$

$$\text{Equation (ii)} \Rightarrow 3x^2 - 5xy - 12y^2 = 0$$

$$3x^2 - 9xy + 4xy - 12y^2 = 0$$

$$3x(x-3y) + 4y(x-3y) = 0$$

$$(x-3y)(3x+4y) = 0$$

Either

$$x-3y=0 \rightarrow (a) \quad \text{or} \quad 3x+4y=0 \rightarrow (b)$$

$$\text{Equation (a)} \Rightarrow x=3y \rightarrow (iii)$$

Put in equation (i)

$$3(3y)^2 - y^2 = 26$$

$$3(9y^2) - y^2 = 26$$

$$27y^2 - y^2 = 26$$

$$26y^2 = 26$$

$$y^2 = 1$$

$$\Rightarrow y = \pm 1$$

Put in equation (iii)

$$\text{When } y = -1$$

$$x = 3(-1)$$

$$x = -3$$

$$\text{When } y = 1$$

$$x = 3(1)$$

$$x = 3$$

$$\text{Equation (b)} \Rightarrow 3x + 4y = 0$$

$$3x = -4y$$

$$x = \frac{-4}{3}y \rightarrow (iv)$$

Put in equation (i)

$$3\left(\frac{-4}{3}y\right)^2 - y^2 = 26$$

$$\left(\frac{16}{3}y^2\right) - y^2 = 26$$

$$\frac{16y^2 - 3y^2}{3} = 26$$

$$13y^2 = 26 \times 3$$

$$y^2 = 6$$

$$\Rightarrow y = \pm\sqrt{6}$$

Put in equation (iv)

$$\text{When } y = -\sqrt{6}$$

$$x = \frac{-4}{3}(-\sqrt{6})$$

$$x = \frac{4\sqrt{6}}{3}$$

$$\text{When } y = \sqrt{6}$$

$$x = \frac{-4}{3}(\sqrt{6})$$

$$x = \frac{-4\sqrt{6}}{3}$$

$\therefore$ **Solution Set**

$$= \left\{ (-3, -1), (3, 1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6} \right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6} \right) \right\}$$

$$\text{Q.12} \quad \begin{aligned} x^2 + xy &= 5 \\ y^2 + xy &= 3 \end{aligned} \quad \text{(A.B)}$$

Solution:

$$x^2 + xy = 5 \rightarrow (i)$$

$$y^2 + xy = 3 \rightarrow (ii)$$

Multiply equation (i) by '3' and equation (ii) '5'

$$3x^2 + 3xy = 15 \rightarrow (iii)$$

$$5y^2 + 5xy = 15 \rightarrow (iv)$$

Subtraction equation (iii) and (iv)

$$3x^2 + 3xy = 15$$

$$\underline{5y^2 + 5xy = 15}$$

$$3x^2 - 2xy - 5y^2 = 0 \rightarrow (v)$$

Equation (v)

$$3x^2 - 5xy + 3xy - 5y^2 = 0$$

$$x(3x - 5y) + y(3x - 5y) = 0$$

$$(3x - 5y)(x + y) = 0$$

Either

$$3x - 5y = 0 \quad \text{or} \quad x + y = 0$$

$$x = \frac{5y}{3} \rightarrow (vi) \quad x = -y \rightarrow (vii)$$

$$\text{Put } x = \frac{5y}{3} \text{ in equation (i)}$$

$$\left(\frac{5y}{3}\right)^2 + \left(\frac{5y}{3}\right)y = 5$$

$$\frac{25}{9}y^2 + \frac{5y^2}{3} = 5$$

$$\frac{25y^2 + 15y^2}{9} = 5$$

$$25y^2 + 15y^2 = 45$$

$$40y^2 = 45$$

$$y^2 = \frac{9}{8}$$

On taking square root, we get.

$$y = \pm \frac{3}{2\sqrt{2}}$$

Put $y = \frac{3}{2\sqrt{2}}$ in equation (vi)

$$x = \frac{5}{3} \times \frac{3}{2\sqrt{2}}$$

$$x = \frac{5}{2\sqrt{2}}$$

Now put $y = \frac{-3}{2\sqrt{2}}$ in equation (vi)

$$x = \frac{5}{3} \left(\frac{-3}{2\sqrt{2}} \right)$$

$$x = \frac{-5}{2\sqrt{2}}$$

Now put $x = -y$ in equation (i)

$$(-y)^2 + (-y)(y) = 5$$

$$y^2 - y^2 = 5$$

$$0 = 5$$

But $0 \neq 5$

$$\therefore \text{Solution Set} = \left\{ \left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}} \right) \right\}$$

Alternate Method

$$x^2 + xy = 5 \rightarrow (i)$$

$$y^2 + xy = 3 \rightarrow (ii)$$

$$eq(i) \Rightarrow$$

$$x(x+y) = 5 \rightarrow (iii)$$

$$eq(ii) \Rightarrow$$

$$y(x+y) = 3 \rightarrow (iv)$$

Dividing equation (iii) and (iv)

$$\frac{x(x+y)}{y(x+y)} = \frac{5}{3}$$

$$\frac{x}{y} = \frac{5}{3}$$

$$x = \frac{5y}{3}$$

Put $x = \frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3} \right)^2 + \left(\frac{5y}{3} \right)y = 5$$

$$\frac{25}{9}y^2 + \frac{5y^2}{3} = 5$$

$$\frac{25y^2 + 15y^2}{9} = 5$$

$$25y^2 + 15y^2 = 45$$

$$40y^2 = 45$$

$$y^2 = \frac{9}{8}$$

On taking square root, we get.

$$y = \pm \frac{3}{2\sqrt{2}}$$

Put $y = \frac{3}{2\sqrt{2}}$ in equation (vi)

$$x = \frac{5}{3} \times \frac{3}{2\sqrt{2}}$$

$$x = \frac{5}{2\sqrt{2}}$$

Now put $y = \frac{-3}{2\sqrt{2}}$ in equation (vi)

$$x = \frac{5}{3} \left(\frac{-3}{2\sqrt{2}} \right)$$

$$x = \frac{-5}{2\sqrt{2}}$$

$$\therefore \text{Solution Set} = \left\{ \left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}} \right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}} \right) \right\}$$

Q.13 $x^2 - 2xy = 7$ **(A.B)**

$$xy + 3y^2 = 2$$

Solution:

$$x^2 - 2xy = 7 \rightarrow (i)$$

$$xy + 3y^2 = 2 \rightarrow (ii)$$

Multiply equation (i) by '2' and equation (ii) by '7'

$$2x^2 - 4xy = 14 \rightarrow \text{(iii)}$$

$$7xy + 21y^2 = 14 \rightarrow \text{(iv)}$$

Subtract equation (iii) and (iv)

$$2x^2 - 4xy = 14$$

$$\underline{-7xy + 21y^2 = -14}$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(x - 7y)(2x + 3y) = 0$$

Either

$$x - 7y = 0$$

or $2x + 3y = 0$

$$x = 7y \rightarrow \text{(v)}$$

$$x = \frac{-3}{2}y \rightarrow \text{(vi)}$$

Put in equation (ii)

Put in equation (ii)

$$(7y)y + 3y^2 = 2$$

$$\left(\frac{-3}{2}y\right)y + 3y^2 = 2$$

$$7y^2 + 3y^2 = 2$$

$$\frac{-3y^2 + 6y^2}{2} = 2$$

$$10y^2 = 2$$

$$3y^2 = 4$$

$$5y^2 = 1$$

$$y^2 = \frac{4}{3}$$

$$y^2 = \frac{1}{5}$$

$$y = \pm \frac{2}{\sqrt{3}}$$

$$y = \pm \frac{1}{\sqrt{5}}$$

Put in equation (vi)

Put in equation (v)

$$\text{when } y = \frac{-2}{\sqrt{3}}$$

$$\text{when } y = -\frac{1}{\sqrt{5}}$$

$$x = \frac{-3}{2}\left(-\frac{2}{\sqrt{3}}\right)$$

$$x = 7\left(-\frac{1}{\sqrt{5}}\right)$$

$$x = \frac{3}{\sqrt{3}}$$

$$x = \frac{-7}{\sqrt{5}}$$

$$x = \sqrt{3}$$

when

$$y = \frac{1}{\sqrt{5}}$$

$$\text{when } y = \frac{2}{\sqrt{3}}$$

$$x = 7\left(\frac{1}{\sqrt{5}}\right)$$

$$x = \frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$$

$$x = \frac{7}{\sqrt{5}}$$

$$x = -\sqrt{3}$$

∴ Solution Set

$$= \left\{ \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right), \left(\frac{-7}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}} \right), \left(\sqrt{3}, \frac{-2}{3} \right) \right\}$$

Problems Leading to Quadratic Equations (A.B + K.B + U.B)

Example 1: (Page # 43)

Three less than a certain number multiplied by 9 less than twice the number is 104. Find the number.

Solution:

Let the required number = x

Then, three less than the number = $x - 3$

And, 9 less than twice the number = $2x - 9$

According to given condition

$$(x - 3)(2x - 9) = 104$$

$$2x^2 - 9x - 6x + 27 - 104 = 0$$

$$2x^2 - 15x - 77 = 0$$

$$2x^2 - 22x + 7x - 77 = 0$$

$$2x(x - 11) + 7(x - 11) = 0$$

$$(x - 11)(2x + 7) = 0$$

Either

$$x - 11 = 0 \text{ or } 2x + 7 = 0$$

$$\Rightarrow x = 11 \text{ or } x = -\frac{7}{2}$$

Result:

Thus, required number is either 11 or $-\frac{7}{2}$

Example 2: (Page # 44)

(A.B)

The length of rectangle is 4cm more than its breadth. If the area of rectangle is 45cm^2 . Find its sides.

Solution:

Let breadth of rectangle = x

Then, length of rectangle = $x + 4$

Area of rectangle = 45cm^2

According to given condition

$$x(x + 4) = 45$$

$$x^2 + 4x - 45 = 0$$

$$x^2 + 9x - 5x - 45 = 0$$

$$x(x + 9) - 5(x + 9) = 0$$

$$(x + 9)(x - 5) = 0$$

Either

$$x + 9 = 0 \quad \text{or} \quad x - 5 = 0$$

$$\Rightarrow x = -9 \quad \text{or} \quad x = 5$$

(Neglecting -ve value)

$$\therefore x + 4 = 5 + 4 = 9$$

Result:

Thus, the breadth is 5cm and length is 9cm

Exercise 2.8

Q.1 The product of two positive consecutive numbers is 182. Find the numbers. **(A.B)**

Solution:

Let two positive consecutive numbers are $x, x + 1$

According to given condition:

$$x(x + 1) = 182$$

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x + 14) - 13(x + 14) = 0$$

$$(x + 14)(x - 13) = 0$$

Either

$$x + 14 = 0 \quad \text{or} \quad x - 13 = 0$$

$$x = -14 \quad \quad \quad x = 13$$

(Ignore negative value)

Therefore,

$$x = 13$$

$$\Rightarrow x + 1 = 13 + 1 = 14$$

Result:

Thus, required Numbers are 13 and 14.

Q.2 The sum of squares of three positive consecutive numbers is 77. Find the numbers. **(A.B + K.B)**
(SWL 2015)

Solution:

Let three consecutive numbers are

$$x, x + 1, x + 2$$

According to given condition:

$$x^2 + (x + 1)^2 + (x + 2)^2 = 77$$

$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 - 77 = 0$$

$$3x^2 + 6x - 72 = 0$$

$$3(x^2 + 2x - 24) = 0$$

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x + 6) - 4(x - 6) = 0$$

$$(x + 6)(x - 4) = 0$$

Either

$$x + 6 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = -6 \quad \quad \quad x = 4$$

(Ignore negative value)

Therefore, $x = 4$

$$\Rightarrow x + 1 = 4 + 1 = 5$$

$$\& \quad x + 2 = 4 + 2 = 6$$

Result:

Thus required numbers are 4, 5 and 6.

Q.3 The sum of five times a number and the square of the numbers is 204. **(A.B + K.B)**

Solution:

Let required number = x

Five times of the number = $5x$

According to given condition:

$$x^2 + 5x = 204$$

$$x^2 + 5x - 204 = 0$$

$$x^2 + 17x - 12x - 204 = 0$$

$$x(x + 17) - 12(x + 17) = 0$$

$$(x + 17)(x - 12) = 0$$

Either

$$x + 17 = 0 \quad \text{or} \quad x - 12 = 0$$

$$x = -17 \quad \quad \quad x = 12$$

Result:

Thus required number is either -17 or 12.

Q.4 The product of five less than three times a certain number and one less than four times the number is 7. Find the number. **(A.B + K.B)**

Solution:

Let the required number is x
Five less than three times the
number $= 3x - 5$

One less than four times a
number $= 4x - 1$

According to given condition

$$(3x - 5)(4x - 1) = 7$$

$$12x^2 - 3x - 20x + 5 - 7 = 0$$

$$12x^2 - 23x - 2 = 0$$

$$12x^2 - 24x + x - 2 = 0$$

$$12x(x - 2) + 1(x - 2) = 0$$

$$12x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(12x + 1) = 0$$

Either

$$x - 2 = 0 \quad \text{or} \quad 12x + 1 = 0$$

$$x = 2 \quad 12x = -1$$

$$x = \frac{-1}{12}$$

Result:

Thus, required number is either 2

$$\text{or } -\frac{1}{12}.$$

Q.5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

(A.B + K.B)

Solution:

Let, required number is x

$$\text{Reciprocal of the number} = \frac{1}{x}$$

$$\text{Difference of the numbers} = \frac{15}{4}$$

According to given condition

$$x - \frac{1}{x} = \frac{15}{4}$$

$$\frac{x^2 - 1}{x} = \frac{15}{4}$$

By cross multiplication

$$4x^2 - 4 = 15x$$

$$4x^2 - 15x - 4 = 0$$

$$4x^2 - 16x + x - 4 = 0$$

$$4x(x - 4) + 1(x - 4) = 0$$

$$(x - 4)(4x + 1) = 0$$

Either

$$x - 4 = 0$$

$$\text{or } 4x + 1 = 0$$

$$x = 4$$

$$4x = -1$$

$$x = \frac{-1}{4}$$

Result:

Thus, required number is either 4 or $-\frac{1}{4}$

Q.6 The sum of a number of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

(A.B + K.B)

Solution:

Let unit's digit $= x$

And ten's digit $= y$

$$\therefore \text{Required number} = 10y + x$$

According to given condition (I)

$$x^2 + y^2 = 65 \longrightarrow (i)$$

According to condition (II)

$$10y + x = 9(x + y)$$

$$10y + x = 9x + 9y$$

$$10y - 9y = 9x - x$$

$$y = 8x \longrightarrow (ii)$$

Put in equation (i)

$$x^2 + (8x)^2 = 65$$

$$x^2 + 64x^2 = 65$$

$$65x^2 = 65$$

$$x^2 = 1$$

Taking positive square root

$$x = 1$$

Put in equation (ii)

$$y = 8(1)$$

$$y = 8$$

$$\text{Required number} = 10y + x$$

$$= 10(8) + 1$$

$$= 80 + 1$$

$$= 81$$

Result:

Thus, required number is 81

Q.7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

(A.B + K.B)

Solution:

Let required point is (x, y)

According to condition I

$$x + y = 9 \longrightarrow (i)$$

According to condition II

$$x^2 + y^2 = 45 \longrightarrow (ii)$$

Equation (i) $\Rightarrow$

$$x = 9 - y \longrightarrow (iii)$$

Put in equation (ii)

$$(9 - y)^2 + y^2 = 45$$

$$81 - 18y + y^2 + y^2 = 45$$

$$2y^2 - 18y + 81 - 45 = 0$$

$$2y^2 - 18y + 36 = 0$$

$$2(y^2 - 9y + 18) = 0$$

$$y^2 - 9y + 18 = 0$$

$$y^2 - 6y - 3y + 18 = 0$$

$$y(y - 6) - 3(y - 6) = 0$$

$$(y - 6)(y - 3) = 0$$

Either

$$y - 6 = 0 \quad \text{or} \quad y - 3 = 0$$

$$y = 6 \quad \quad \quad y = 3$$

Put in equation (iii)

$$x = 9 - 6 \quad \quad \quad x = 9 - (3)$$

$$x = 3 \quad \quad \quad x = 9 - 3$$

$$x = 6$$

Result:

Thus, required point is either

(3,6) or (6,3)

Q.8 Find two integers whose sum is 9 and the difference of their squares is also 9. (A.B + K.B)

Solution:

Let two integer are x and y

According to condition I

$$x + y = 9 \longrightarrow (i)$$

According to condition (ii)

$$x^2 - y^2 = 9 \longrightarrow (ii)$$

$$\text{Equation (i)} \Rightarrow y = 9 - x \longrightarrow (iii)$$

Put in equation (ii)

$$x^2 - (9 - x)^2 = 9$$

$$x^2 - (81 - 18x + x^2) = 9$$

$$x^2 - 81 + 18x - x^2 = 9$$

$$18x = 9 + 81$$

$$\Rightarrow x = 5$$

Put in equation (iii)

$$y = 9 - 5 = 4$$

Result:

Thus, required numbers are 5 and 4.

Q.9 Find two integers whose difference is 4 and whose squares differ by 72. (A.B + K.B)

Solution:

Let the integers are x and y

According to condition-I

$$x - y = 4 \longrightarrow (i)$$

According to condition-II

$$x^2 - y^2 = 72 \longrightarrow (ii)$$

$$\text{Equation (i)} \Rightarrow x = y + 4 \longrightarrow (iii)$$

Put in equation (ii)

$$(y + 4)^2 - y^2 = 72$$

$$y^2 + 8y + 16 - y^2 = 72$$

$$8y = 72 - 16$$

$$8y = 56$$

$$\Rightarrow y = 7$$

Put in equation (iii)

$$x = 7 + 4$$

$$x = 11$$

Result:

Thus, required integers are 11 and 7.

Q.10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm² (K.B + A.B)

Solution:

Let length of rectangle = x cm

And width of rectangle = y cm

According to condition-I

$$2(x + y) = 80$$

$\therefore$ perimeter = 2(length + width)

$$x + y = 40 \longrightarrow (i)$$

$\therefore$ Area = length $\times$ width

According to condition-II

$$xy = 375 \longrightarrow (ii)$$

From equation (i)

$$y = 40 - x \longrightarrow (iii)$$

Put in (ii)

$$x(40 - x) = 375$$

$$40x - x^2 - 375 = 0$$

$$-x^2 - 40 - 375 = 0$$

$$x^2 - 40x + 375 = 0$$

$$x^2 - 25x - 15x + 375 = 0$$

$$x(x - 25) - 15(x - 25) = 0$$

$$(x - 25)(x - 15) = 0$$

$$\text{Either } x - 25 = 0 \quad \text{or} \quad x - 15 = 0$$

$$x = 25$$

$$x = 15$$

Put in equation (iii)

$$y = 40 - 25$$

$$y = 40 - 15$$

$$y = 15$$

$$y = 25$$

Result:

Dimension of rectangle are either 25cm by 15cm or 15cm by 25cm.

Miscellaneous Exercise 2

Q.1 Multiple Choice Questions

Four possible answers are given for the following question. Tick ($\checkmark$) the correct answer.

- (i) If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is; **(K.B + A.B)**

(LHR 2017, SWL 2014, MTN 2015, 17, SGD 2015, 17, RWP 2016, D.G.K 2016)

(a) $\frac{5}{3}$

(b) $\frac{3}{5}$

(c) $\frac{-5}{3}$

(d) $\frac{-2}{3}$

- (ii) If α, β are the roots of $7x^2 - x + 4 = 0$, then $\alpha\beta$ is; **(K.B + U.B)**

(LHR 2014, GRW 2014, 15, FSD 2016, BWP 2016)

(a) $\frac{-1}{7}$

(b) $\frac{4}{7}$

(c) $\frac{7}{4}$

(d) $\frac{-4}{7}$

- (iii) Roots of the equation $4x^2 - 5x + 2 = 0$ are; **(K.B + A.B)**

(a) Irrational

(b) Imaginary

(c) Rational

(d) None of these

- (iv) Cube roots of -1 are; **(K.B + U.B)**

(LHR 2017, GRW 2017, SWL 2017, MTN 2014, 17, SGD 2015, 16, D.G.K 2017)

(a) $-1, -\omega, -\omega^2$

(b) $-1, \omega, -\omega^2$

(c) $-1, -\omega, \omega^2$

(d) $1, -\omega, -\omega^2$

- (v) Sum of the cube roots of unity is; **(K.B + A.B)**

- (a) 0 (b) 1
(c) -1 (d) 3
- (vi) Product of cube roots of unity is; **(K.B + A.B)**
(LHR 2016, GRW 2014, 16, SGD 2015, 17, BWP 2016, 17, RWP 2017)
- (a) 0 (b) 1
(c) -1 (d) 3
- (vii) If $b^2 - 4ac < 0$, then the roots of $ax^2 + bx + c = 0$ are; (GRW 2014) **(K.B + A.B)**
(a) Irrational (b) Rational
(c) Imaginary (d) None of these
- (viii) If $b^2 - 4ac > 0$, but not a perfect square then roots of $ax^2 + bx + c = 0$ are; **(K.B)**
(LHR 2014, BWP 2017)
(a) Imaginary (b) Rational
(c) Irrational (d) None of these
- (ix) $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to; **(K.B + U.B)**
(LHR 2014, 15, GRW 2016, FSD 2017, BWP 2017, RWP 2016, SGD 2017)
- (a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha} - \frac{1}{\beta}$
(c) $\frac{\alpha - \beta}{\alpha\beta}$ (d) $\frac{\alpha + \beta}{\alpha\beta}$
- (x) $\alpha^2 + \beta^2$ is equal to; **(U.B + A.B)**
(LHR 2014, 15, GRW 2014, 17, FSD 2016, BWP 2015, RWP 2016, 17)
- (a) $\alpha^2 - \beta^2$ (b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
(c) $(\alpha + \beta)^2 - 2\alpha\beta$ (d) $\alpha + \beta$
- (xi) Two square roots of unity are; **(U.B + A.B)**
(LHR 2015, 16, GRW 2014, FSD 2015, 16, MTN 2016, SGD 2016, D.G.K 2015, 16, 17)
- (a) 1, -1 (b) 1, ω
(c) 1, $-\omega$ (d) ω, ω^2
- (xii) Roots of the equation $4x^2 - 4x + 1 = 0$ are; **(U.B + A.B)**
(LHR 2015, GRW 2017, FSD 2016, BWP 2015, MTN 2017, SGD 2016)
- (a) Real equal (b) Real unequal
(c) Imaginary (d) Irrational
- (xiii) If α, β are the roots of $px^2 + qx + r = 0$, then sum of the roots 2α and 2β is; **(K.B)**
- (a) $\frac{-q}{p}$ (b) $\frac{r}{p}$
(c) $\frac{-2q}{p}$ (d) $-\frac{q}{2p}$
- (xiv) If α, β are the roots of $x^2 - x - 1 = 0$, then product of the roots 2α and 2β is; **(U.B)**
(FSD 2014, 17, BWP 2016, D.G.K 2015, 16, 17)
- (a) -2 (b) 2
(c) 4 (d) -4

- (xv) The nature of the roots of equation $ax^2 + bx + c = 0$ is determined by; **(A.B)**
 (GRW 2016, SWL 2015, 2017, MTN 2015)
 (a) Sum of the roots (b) Product of the roots
 (c) Synthetic division (d) Discriminant
- (xvi) The discriminant of $ax^2 + bx + c = 0$ is; **(K.B + A.B)**
 (LHR 2016, FSD 2017, SWL 2016, 17, RWP 2014, 16, SGD 2016, MTN 2015, D.G.K 2016)
 (a) $b^2 - 4ac$ (b) $b^2 + 4ac$
 (c) $-b^2 + 4ac$ (d) $-b^2 - 4ac$

ANSWER KEY

(i)	c	(v)	a	(ix)	d	(xiii)	c
(ii)	b	(vi)	b	(x)	c	(xiv)	d
(iii)	b	(vii)	c	(xi)	a	(xv)	d
(iv)	a	(viii)	c	(xii)	a	(xvi)	a

Q.2

- (i) Discuss the nature of the roots of the following equations. (A.B)

Solution:

(a) $x^2 + 3x + 5 = 0$

Here $a = 1, b = 3, c = 5$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (3)^2 - 4(1)(5) \\ &= 9 - 20 \\ &= -11 \\ &< 0\end{aligned}$$

∴ Roots are complex conjugate or imaginary.

(b) $2x^2 - 7x + 3 = 0$ (A.B)

(GRW 2016, SGD 2014, RWP 2017, D.G.K 2016)

Here $a = 2, b = -7, c = 3$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (-7)^2 - 4(2)(3) \\ &= 49 - 24 \\ &= 25\end{aligned}$$

Since disc > 0 and perfect square roots are rational and unequal.

(c) $x^2 + 6x - 1 = 0$ (A.B)

Here $a = 1, b = 6, c = -1$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (6)^2 - 4(1)(-1) \\ &= 36 + 4 \\ &= 40\end{aligned}$$

Since Disc. > 0 and not a perfect square roots are irrational and unequal.

(d) $16x^2 - 8x + 1 = 0$ (FSD 2017) (A.B)

Here $a = 16, b = -8, c = 1$

$$\begin{aligned}\text{Disc.} &= b^2 - 4ac \\ &= (-8)^2 - 4(16)(1) \\ &= 64 - 64 \\ &= 0\end{aligned}$$

Since, Disc. $= 0$, roots are rational and equal.

(ii) Find ω^2 , if $\omega = \frac{-1 + \sqrt{-3}}{2}$ (A.B)

(GRW 2017, SGD 2014, BWP 2016)

Solution:

Here

$$\omega = \frac{-1 + \sqrt{-3}}{2}$$

Square both sides

$$(\omega)^2 = \left(\frac{-1 + \sqrt{-3}}{2} \right)^2$$

$$\omega^2 = \frac{(-1)^2 + (\sqrt{-3})^2 + 2(-1)(\sqrt{-3})}{(2)^2}$$

$$\omega^2 = \frac{1 + (-3) - 2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{-2 - 2\sqrt{-3}}{4}$$

$$\omega^2 = 2 \left(\frac{-1 - \sqrt{-3}}{4} \right)$$

$$\omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

- (iii) Prove that the sum of all cube roots of unity is zero. (A.B + K.B)

Ans. See property of cube roots Page # 45

- (iv) Find the product of complex cube roots of unity. (A.B + K.B)

Ans. See property of cube roots Page # 44

- (v) Show that:

$$x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$$

(A.B + K.B)

Ans. See Exe-2.2 Q.3 Page # 47

- (vi) Evaluate: $\omega^{37} + \omega^{38} + 1$

(A.B + K.B)

Solution:

$$\begin{aligned}\omega^{37} + \omega^{38} + 1 &= \omega^{36} \cdot \omega + \omega^{36} \cdot \omega^2 + 1 \\ &= (\omega^3)^{12} \cdot \omega + (\omega^3)^{12} \cdot \omega^2 + 1 \\ &= (1)^{12} \cdot \omega + (1)^{12} \cdot \omega^2 + 1 \\ &= \omega + \omega^2 + 1 \\ &= 0\end{aligned}$$

$$\therefore \omega^{37} + \omega^{38} + 1 = 0$$

- (vii) Evaluate $(1 - \omega + \omega^2)^6$

(A.B + K.B)

Solution:

$$\begin{aligned}(1 - \omega + \omega^2)^6 &= [1 + \omega^2 - \omega]^6 \\ &= (-\omega - \omega)^6 \\ &= (-2\omega)^6 \\ &= (-2)^6 \omega^6 \\ &= 64(\omega^3)^2 \\ &= 64(1)^2 \\ &= 64(1) \\ &= 64\end{aligned}$$

- (viii) If ω is cube root of unity, form an equation whose roots are 3ω and $3\omega^2$. **(A.B + K.B)**

Solution:

Roots of required equation are 3ω and $3\omega^2$.

$$\begin{aligned}\text{Sum of roots} = S &= 3\omega + 3\omega^2 \\ &= 3(\omega + \omega^2) \\ &= 3(-1) \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{Product of roots} = P &= (3\omega)(3\omega^2) \\ &= 9\omega^3 \\ &= 9(1) \\ &= 9\end{aligned}$$

$\therefore$ Required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 9 = 0$$

$$x^2 + 3x + 9 = 0$$

- (ix) Use synthetic division, find the remainder and quotient when $(x^3 + 3x^2 + 2) \div (x - 2)$. **(A.B)**

Solution:

$$P(x) = x^3 + 3x^2 + 2$$

$$= x^3 + 3x^2 + 0x + 2$$

2	1	3	0	2
	↓	2	10	20
	1	5	10	22

$\therefore$ Remainder = 22

$$Q(x) = x^2 + 5x + 10$$

- (x) Use synthetic division, show that $x - 2$ is the factor $x^3 + x^2 - 7x + 2$. **(A.B + K.B)**

Solution:

$$P(x) = x^3 + x^2 - 7x + 2$$

2	1	1	-7	2
	↓	2	6	-2
	1	3	-1	0

Since remainder is zero, $x - 2$ is a factor of given polynomial.

- (xi) Find the sum and product of the roots of the equation $2Px^2 + 3qx - 4r = 0$ **(A.B + K.B)**

Solution:

$$2Px^2 + 3qx - 4r = 0$$

$$\text{Here } a = 2P, b = 3q, c = -4r$$

$$\text{Sum of roots} = \frac{-b}{a}$$

$$= -\frac{3q}{2P}$$

$$\begin{aligned}\text{Product of roots} &= \frac{c}{a} \\ &= \frac{-4r}{2P} \\ &= -\frac{2r}{P}\end{aligned}$$

- (xii) Find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ when α, β are of the roots of the equation $x^2 - 4x + 3 = 0$. **(A.B + K.B)**

Solution:

$$x^2 - 4x + 3 = 0$$

$$\text{Here } a = 1, b = -4, c = 3$$

Let roots of given equation are α, β

$$\begin{aligned}\text{Then sum of roots} = \alpha + \beta &= -\frac{b}{a} \\ &= -\frac{-4}{1} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{Product of roots} = \alpha\beta &= \frac{c}{a} \\ \alpha\beta &= \frac{3}{1} \\ \alpha\beta &= 3\end{aligned}$$

Consider

$$\begin{aligned}\frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} \\ &= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \\ &= \frac{(4)^2 - 2(3)}{(3)^2} \\ &= \frac{16 - 6}{9} \\ \Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{10}{9}\end{aligned}$$

(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$, find **(A.B)**

(a) $\alpha^2 + \beta^2$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(c) $\alpha - \beta$

Solution:

$$4x^2 - 3x + 6 = 0$$

Roots of given equation are α, β

$$\alpha + \beta = -\frac{-3}{4}$$

$$= \frac{3}{4}$$

$$\alpha\beta = \frac{6}{4}$$

$$= \frac{3}{2}$$

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **(A.B)**

$$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)$$

$$= \frac{9}{16} - 3$$

$$= \frac{9 - 48}{16}$$

$$\Rightarrow \alpha^2 + \beta^2 = \frac{-39}{16}$$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ **(A.B)**

$$= \frac{-39/16}{3/2}$$

$$= \frac{-39}{16} \times \frac{2}{3}$$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{-13}{8}$$

(c) $\alpha - \beta = \sqrt{(\alpha - \beta)^2}$ **(A.B)**

$$= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)}$$

$$= \sqrt{\frac{9}{16} - 6}$$

$$= \sqrt{\frac{9 - 96}{16}}$$

$$= \sqrt{\frac{-87}{16}}$$

$$\Rightarrow \alpha - \beta = \frac{\sqrt{-87}}{4}$$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are

(a) $-\alpha, -\beta$

(b) $2\alpha, 2\beta$

Solution:

$$x^2 - 5x + 7 = 0$$

Roots of given equation are α, β

$$\alpha + \beta = -\frac{-5}{1}$$

$$= 5$$

$$\alpha\beta = \frac{7}{1}$$

$$\alpha\beta = 7$$

(a) **Roots of required equation are $-\alpha, -\beta$** **(A.B)**

$$\text{Sum of roots} = S = -\alpha + (-\beta)$$

$$= -(\alpha + \beta)$$

$$= -5$$

$$\text{Prod. Of roots} = P = (-\alpha)(-\beta)$$

$$= \alpha\beta$$

$$= 7$$

$\therefore$ Required equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - (-5)x + 7 = 0$$

$$x^2 + 5x + 7 = 0$$

(b) **Roots of required equation are $2\alpha, 2\beta$** **(A.B)**

$$\text{Sum of roots} = S = 2\alpha + 2\beta$$

$$= 2(\alpha + \beta)$$

$$= 2(5)$$

$$= 10$$

$$\text{Prod. Of roots} = P = (2\alpha)(2\beta)$$

$$= 4\alpha\beta$$

$$= 4(7)$$

$$= 28$$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 10x + 28 = 0$$

Q.3 Fill in the blanks

- (i) The discriminant of $ax^2 + bx + c = 0$ is _____. **(K.B)**
- (ii) If $b^2 - 4ac = 0$, then roots of $ax^2 + bx + c = 0$ are _____. **(K.B)**
- (iii) If $b^2 - 4ac > 0$, then the roots of $ax^2 + bx + c = 0$ are _____. **(K.B)**
- (iv) If $b^2 - 4ac < 0$, then the root of $ax^2 + bx + c = 0$ are _____. **(K.B)**
- (v) If $b^2 - 4ac < 0$ and perfect square, then the roots of $ax^2 + bx + c = 0$ are _____. **(K.B)**
- (vi) If $b^2 - 4ac < 0$ and not a perfect square, then roots of $ax^2 + bx + c = 0$ are _____. **(K.B)**
- (vii) If α, β are the roots of $ax^2 + bx + c = 0$, then sum of the roots is _____. **(K.B)**
- (viii) If α, β are the roots of $ax^2 + bx + c = 0$, then product of the roots is _____. **(K.B)**
- (ix) If α, β are the roots of $7x^2 - 5x + 3 = 0$, then sum of the roots is _____. **(K.B)**
- (x) If α, β are the roots of $5x^2 + 3x - 9 = 0$, then product of the roots is _____. **(K.B)**
- (xi) For a quadratic equation $ax^2 + bx + c = 0$, $\frac{1}{\alpha\beta}$ is equal to _____. **(K.B)**
- (xii) Cube roots of unity are _____. **(K.B)**
- (xiii) Under usual notation sum of the roots of unity is _____. **(K.B)**
- (xiv) If $1, \omega, \omega^2$ are the cube roots of unity, then ω^{-7} is equal to _____. **(K.B)**
- (xv) If α, β are the roots of the quadratic equation, then the quadratic equation is written as _____. **(K.B)**
- (xvi) If 2ω and $2\omega^2$ are the roots of an equation, then equation is _____. **(K.B)**

ANSWER KEY

- | | |
|------------------------|--|
| (i) $b^2 - 4ac$ | (x) $\frac{-9}{5}$ |
| (ii) Equal | (xi) $\frac{a}{c}$ |
| (iii) Real | (xii) $1, \omega, \omega^2$ |
| (iv) Imaginary | (xiii) Zero |
| (v) Rational | (xiv) ω^2 |
| (vi) Irrational (real) | (xv) $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ |
| (vii) $-\frac{b}{a}$ | (xvi) $x^2 + 2x + 4 = 0$ |
| (viii) $\frac{c}{a}$ | |
| (ix) $\frac{5}{7}$ | |

**SELF TEST**

Time: 40 min

Marks: 25

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. (7×1=7)

1 $\omega^{-7} = \underline{\hspace{2cm}}$ is:

(A) ω^2 (B) ω

(C) 1

(D) 0

2 Which is not a symmetric function?

(A) $\alpha^2 - \beta^2$ (B) $\alpha^2 + \beta^2$ (C) $\alpha^3 + \beta^3$ (D) $\frac{1}{\alpha} + \frac{1}{\beta}$

3 If $\frac{3}{2}, \frac{1}{2}$ are the roots of a quadratic equation, then required quadratic equation is:

(A) $2x^2 + 2x + 3 = 0$ (B) $4x^2 + 8x + 3 = 0$ (C) $x^2 + 4x + 3 = 0$ (D) $4x^2 - 8x + 3 = 0$

4 If roots of a quadric equation $x^2 + qx + p = 0$ are the additive inverse of each other, then:

(A) $p = 0, q = 0$ (B) $p = 0$ (C) $q = 0$ (D) $p = 1, q = 1$

5 What will be the remainder if $4x^3 - 5x + 15$ is divided by $x + 3$?

(A) 78

(B) 139

(C) -78

(D) 125

6 Cube roots of -1 are:

(A) $-1, -\omega, -\omega^2$ (B) $-1, \omega, -\omega^2$ (C) $-1, -\omega, \omega^2$ (D) $1, \omega, \omega^2$

7 Roots of the equation $4x^2 - 5x + 2 = 0$ are:

(A) Irrational

(B) Imaginary

(C) Rational

(D) None

Q.2 Give Short Answers to following Questions. (5×2=10)

(i) Evaluate: $(1 - 3\omega - 3\omega^2)^5$.

(ii) Prove that each complex cube root of unity is reciprocal of the other.

(iii) Show that the roots of the equation $(p + q)x^2 - px - q = 0$ are rational.

(iv) If ω is a cube root of unity, form an equation whose roots are 2ω and $2\omega^2$.

(v) Use synthetic division to find the quotient and the remainder when the polynomial $x^4 - 10x^2 - 2x$ is divided by $x + 3$.

Q.3 Answer the following Questions. (4+4=8)

(a) Prove that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

(b) Find two integers whose sum is 9 and the difference of their squares is also 9.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.