EXERCISE # 2.1

(Question # 1)

Express the following numbers in scientific notation:

(i) 2000000

Sol. 2000000.

$\xleftarrow{+}$.

. $\xrightarrow{-}$

$= 2.000000 \times 10^{6}$

$= \boxed{2 \times 10^{6}}$ Ans.

(ii) 48900

Sol. 48900.

$= 4.8900 \times 10^{4}$

$= \boxed{4.89 \times 10^{4}}$ Ans.

(iii) 0.0042

Sol. 0.0042

$= 0004.2 \times 10^{-3}$

$= \boxed{4.2 \times 10^{-3}}$ Ans.

(iv) 0.0000009

Sol. 0.0000009

$= 00000009. \times 10^{-7}$

$= 9. \times 10^{-7}$

$= \boxed{9 \times 10^{-7}}$ Ans.

(v) 73×10^3

Sol. 73×10^3

$= 7.3 \times 10^3$

$= 7.3 \times 10^1 \times 10^3$

$= 7.3 \times 10^{1+3}$

$= \boxed{7.3 \times 10^4}$ Ans.

(vi) 0.65×10^2

Sol. 0.65×10^2

$= 0.65 \times 10^2$

$= 06.5 \times 10^{-1} \times 10^2$

$= 06.5 \times 10^{-1+2}$

$= \boxed{6.5 \times 10^1}$ Ans.

Question # 2

Express the following numbers in ordinary notation:

Power ⊕ → Power 10

Power ⊖ ←

ordinary notation

(i) 8.04×10^2

Sol. 8.04×10^2

$= 804.$

$= \boxed{804}$ Ans.

(ii) 3×10^5

Sol. 3×10^5

$= 3. \times 10^5$

$= 3\,00000.$

$= \boxed{300000}$ Ans.

(iii) 1.5×10^{-2}

Sol. 1.5×10^{-2}

$= .01\,5$

$= .015$

$= \boxed{0.015}$ Ans.

(iv) 1.77×10^7

Sol. 1.77×10^7

$= 1\,7700000.$

$= \boxed{17700000}$ Ans.

(v) 5.5×10^{-6}

Sol. 5.5×10^{-6}

$= .000005\,5$

$= \boxed{0.0000055}$ Ans.

Solved by : Bashir Ahmad Mubashar (M.Phil, M.Ed)

(vi) 4×10^{-5}

Sol. 4×10^{-5}

$= 4. \times 10^{-5}$

$= .00004$

$= \boxed{0.00004}$ Ans.

EXERCISE # 2.1

(Question # 3)

The speed of light is approximately 3×10^8 metres per second. Express it in standard form.

Sol.

$$3 \times 10^8 \ \frac{m}{s}$$

$$= 3 \times 100000000 \ \frac{m}{s}$$

$$= \boxed{300000000 \ \frac{m}{s}}$$ Ans.

(Question # 4)

The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.

Sol.

$$40075000 \text{ metres}$$

$$= 40075000. \ m$$

$$= 4.0075000 \times 10^7 \ m$$

OR

$$= \boxed{4.0075 \times 10^7 \ m}$$

(Question #5)

The diameter of Mars is 6.7779×10^3 Km. Express this number in standard form.

Sol. 6.7779×10^3 km

$= 6.7779 \times 10^3$ Km

$= \boxed{6777.9 \text{ Km}}$ Ans.

(Question #6)

The diameter of Earth is about 1.2756×10^4 Km. Express this number in standard form.

Sol. 1.2756×10^4 km

$= 1.2756 \times 10^4$ Km

$= 12756.$ Km

$= \boxed{12756 \text{ Km}}$ Ans.

EXERCISE # 2.2

Question # 1

Express each of the following in logarithmic form:

(i) $10^3 = 1000$

Sol. $1000 = 10^3$

$$\boxed{\log_{10} 1000 = 3}$$ Ans.

(ii) $2^8 = 256$

Sol. $258 = 2^8$

$$\boxed{\log_2 258 = 8}$$ Ans.

(iii) $3^{-3} = \frac{1}{27}$

Sol. $\frac{1}{27} = 3^{-3}$

$$\boxed{\log_3 \frac{1}{27} = -3}$$ Ans.

(iv) $20^2 = 400$

Sol.

$$400 = 20^2$$

$$\boxed{\log_{20} 400 = 2}$$ Ans.

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$

Sol.

$$\frac{1}{2} = 16^{-\frac{1}{4}}$$

$$\boxed{\log_{16} \frac{1}{2} = -\frac{1}{4}}$$ Ans.

(vi) $11^2 = 121$

Sol.

$$121 = 11^2$$

$$\boxed{\log_{11} 121 = 2}$$ Ans.

(vii) $p = q^r$

Sol.

$$p = q^r$$

$$\boxed{\log_q p = r}$$ Ans.

(viii) $(32)^{-\frac{1}{5}} = \frac{1}{2}$

Sol. $\frac{1}{2} = (32)^{-\frac{1}{5}}$

$$\boxed{\log_{32}\frac{1}{2} = -\frac{1}{5}}$$ Ans.

Question # 2

Express each of the following in exponential form:

(i) $\log_5 125 = 3$

Sol. $\log_5 125 = 3$

$$\boxed{125 = 5^3}$$ Ans.

(ii) $\log_2 16 = 4$

Sol. $\log_2 16 = 4$

$$\boxed{16 = 2^4}$$ Ans.

(iii) $\log_{23} 1 = 0$

Sol. $\log_{23} 1 = 0$

$\boxed{1 = 23^0}$ Ans.

(iv) $\log_5 5 = 1$

Sol. $\log_5 5 = 1$

$\boxed{5 = 5^1}$ Ans.

(v) $\log_2 \frac{1}{8} = -3$

Sol.

$\log_2 \frac{1}{8} = -3$

$\boxed{\frac{1}{8} = 2^{-3}}$ Ans.

(vi) $\frac{1}{2} = \log_9 3$

Sol. $\log_9 3 = \frac{1}{2}$

$\boxed{3 = 9^{\frac{1}{2}}}$ Ans.

(vii) $5 = \log_{10} 100000$

Sol. $\log_{10} 100000 = 5$

$\boxed{100000 = 10^5}$ Ans.

(vii) $\log_4 \frac{1}{16} = -2$

Sol. $\log_4 \frac{1}{16} = -2$

$\boxed{\frac{1}{16} = 4^{-2}}$ Ans.

Question # 3

Find the value of x in each of the following

(i) $\log_x 64 = 3$

Sol $\log_x 64 = 3$

$64 = x^3$

$4^3 = x^3$

$\boxed{4 = x}$ Ans.

4	64
4	16
4	4
	1

$64 = 4 \times 4 \times 4$

$\boxed{64 = 4^3}$

(ii) $\log_5 1 = x$

Sol. $\log_5 1 = x$

$1 = 5^x$

put $\boxed{5^0 = 1}$

$5^0 = 5^x$

$\boxed{0 = x}$ Ans.

(iii) $\log_x 8 = 1$

Sol. $\log_x 8 = 1$

$8 = x^1$

$8^1 = x^1$ $\quad \because 8 = 8^1$

$\boxed{8 = x}$ Ans.

(iv) $\log_{10} x = -3$

Sol. $\log_{10} x = -3$

$x = 10^{-3}$

$x = \frac{1}{10^3}$ $\quad \because 10^3 = 10 \times 10 \times 10$ (100, 1000)

$\boxed{x = \frac{1}{1000}}$ Ans

(v) $\log_4 x = \frac{3}{2}$

Sol. $\log_4 x = \frac{3}{2}$

$$x = (4)^{\frac{3}{2}}$$

$$x = (2^2)^{\frac{3}{2}} \qquad \because \; 4 = 2 \times 2, \quad 4 = 2^2$$

$$x = (2)^3$$

$$x = \underbrace{\underbrace{2 \times 2}_{4} \times 2}_{8}$$

$\boxed{x = 8}$ Ans.

(vi) $\log_2 1024 = x$

Sol. $\log_2 1024 = x$

$$1024 = 2^x$$

$$2^{10} = 2^x$$

$\boxed{10 = x}$ Ans.

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\boxed{1024 = 2^{10}}$

EXERCISE # 2.3

Question # 1

Find characteristic of the following number :

(i) 5287

Sol. 5287.

Rule 1 For a number greater than 1 (+)

Characteristic = number of digits to the left of decimal point − 1

Characteristic = 4 − 1

$= \boxed{3}$ Ans.

(ii) 59.28

Sol. 59.28

Rule 1 For a number greater than 1 (+)

Characteristic = number of digits to the left of decimal point − 1

Characteristic = 2 − 1

$= \boxed{1}$ Ans.

(iii) 0.0567

Sol. 0.0567

Rule 2 For a number less than 1 (−)

Characteristic = −(number of zeros between the decimal point and the first non-zero digit +1)

Characteristic = −(1+1)

$= \boxed{-2}$ Ans.

(iv) 234.7

Sol. 234.7

Apply Rule # 1 (number greater than 1)

Characteristic = Number of digits to the left of the decimal point − 1

Characteristic = 3 − 1

$= \boxed{2}$ Ans.

Solved by : Bashir Ahmad Mubashar (M.Phil, M.Ed)

(v) 0.000049

Sol. 0.000049

Apply Rule 2 (For number less than 1)

Characteristic = −(number of zeros between the decimal point and the first non-zero digit + 1)

Characteristic = − (4 + 1)

$= \boxed{-5}$ Ans.

(vi) 145000

Sol. 145000.

Apply Rule #1 (number greater than 1)

Characteristic = Number of digits to the left of the decimal point − 1

Characteristic = 6 − 1

$= \boxed{5}$

Ans.

Question # 2

Find logarithm of the following numbers:

(i) 43

Sol. Characteristic $= 2 - 1$

$= 1$

Mantissa $= 0.6335$

Look for 43 in the row and 0 in the column of the log table

$\log(43) = 1 + 0.6335$

$\log 43 = 1.6335$ Ans.

(ii) 579

Sol. 579.

Characteristic $= 3 - 1$

$= 2$

Mantissa $= 0.7627$

look for 57 in the row and 9 in the column of the log table

$\log 579 = 2 + 0.7627$

$\log 579 = 2.7627$ Ans.

((iii)) 1.982

Sol. Characteristic $= 1 - 1$

$= 0$

Mantissa $= 0.2967 + 4$

$= 0.2971$

Look for 19 in the row and 8 in the column and then 2 in Mean difference column and finally add these values.

$\log(1.982) = 0 + 0.2971$

$\log(1.982) = 0.2971$

((iv)) 0.0876

Sol. 0.0876

Characteristic $= -(1+1)$ Apply Rule#2

$= -2$

Mantissa $= 0.9425$

look for 87 in the row and 6 in the column.

$\log(0.0876) = -2 + 0.9425$

$= -1.0575$

$= \bar{1}.0575$ Ans.

Q(v) 0.047

Sol. 0.047

Characteristic $= -(1+1)$ Apply Rule # 2

$= -2$

look for 47 in the row and 0 in the column.

Mantissa $= 0.6721$

$\log(0.047) = -2 + 0.6721$

$= -1.3279$

$= \bar{1}.3279$ Ans.

Q(vi) 0.000354

Sol. 0.000354

Characteristis $= -(3+1)$ Apply Rule # 2

$= -4$

look for 35 in the row and 4 in the column.

Mantissa $= 0.5490$

$\log(0.000354) = -4 + 0.5490$

$= -3.4510$

$= \bar{3}.4510$ Ans.

EXERCISE # 2.3

(Question # 3)

If $\log 3.177 = 0.5019$,
then find :

(i) $\log 3177$

Sol. $\log 3177.$

Characteristic $= 4 - 1$ Rule # 1
$= 3$

Mantissa $= 0.5019$ (Given in Question)

$\log 3177 = 3 + 0.5019$

$\log 3177 = \boxed{3.5019}$ Ans.

(ii) $\log 31.77$

Sol. $\log 31.77$

Characteristics $= 2 - 1$ Rule # 1
$= 1$

Mantissa $= 0.5019$ (Give in Question)

$\log 31.77 = 1 + 0.5019$

$\log 31.77 = \boxed{1.5019}$ Ans.

(iii) $\log 0.03177$

Sol. $\log 0.03177$

Characteristic $= -(1+1)$ Rule#2

$= -2$

Mantissa $= 0.5019$ (Given in Question)

$\log(0.03177) = -2 + 0.5019$

$\log(0.03177) = \boxed{-1.4981}$ Ans.

Question # 4

Find the value of x.

(i) $\log x = 0.0065$

Sol. $\log x = 0.0065$

Characteristic $= 0$

Mantissa $= .0065$

Table value of $.0065 = 1014 + 1$

$= 1015$

$\log x = 0.0065$

$x = \text{anti}\log(0.0065)$

$$\boxed{x = 1.015}$$ Ans.

Insert the decimal point at reference position because charateristic is 0.

Question # 4(ii)

Find the value of x

$$\log x = 1.192$$

Sol. $\log x = 1.192$

Characteristic = 1

Mantissa = .192

Table value of .192 = 1556

$$\log x = 1.192$$

$$x = \text{antilog}(1.192)$$

$$x = 1.556$$

due to characteristic = 1

$$x = \boxed{15.56}$$

move the decimal point one digit to right side.

Ans.

Question # 4(iii)

Find the value of x.

$$\log x = -3.434$$

Sol. Since mantissa is negative, so we make it positive by adding and subtracting 4

$$\log x = -4 + 4 - 3.434$$

$$\log x = -4 + 0.566$$

$$\log x = \bar{4}.566$$

Here characteristic $= \bar{4}$

mantissa $= 0.566$

Table value of $0.566 = 3681$

So $\log x = \bar{4}.566$

$$x = \text{antilog}(\bar{4}.566)$$

$$x = 0.0003681$$

$$\boxed{x = 0.0003681}$$

Ans.

Question # 4(iv)

Find the value of x.

$$\log x = -1.5726$$

Sol. Since mantissa is negative, so we make it positive by adding and subtracting 2

$$\log x = -2 + 2 - 1.5726$$

$$\log x = -2 + 0.4274$$

$$\log x = \bar{2}.4274$$

Here Characteristic $= \bar{2}$

Mantissa $= 0.4274$

Table value of 0.4274

$= 2673 + 2$

$= 2675$

So $\log x = \bar{2}.4274$

$x = \text{antilog}(\bar{2}.4274)$

$x = 0.02675$

$$\boxed{x = 0.02675}$$

Ans.

Question # 4 (v)

Find the value of x.

$$\log x = 4.3561$$

Sol.

$$\log x = 4.3561$$

Characteristic $= 4$

Mantissa $= 0.3561$

Table value of $0.3561 = 2270 + 1$

$= 2271$

$$\log x = 4.3561$$

$$x = \text{anti}\log(4.3561)$$

$$x = 2\overset{\wedge}{}271$$

$$x = 22710.$$

$$\boxed{x = 22710}$$

Ans.

Question # 4(vi)

Find the value of x.

$$\log x = -2.0184$$

Sol.

Since mantissa is negative, so we make it positive by adding and subtracting 3

$$\log x = -3 + 3 - 2.0184$$

$$\log x = -3 + 0.9816$$

$$\log x = \bar{3}.9816$$

Here Characteristic $= \bar{3}$

Mantissa $= .9816$

Table value of $.9816 = 9572 + 13$

$= 9585$

So $\log x = \bar{3}.9816$

$$x = \text{anti}\log(\bar{3}.9816)$$

$$x = 0.009585$$

$$\boxed{x = 0.009585}$$

Ans.

Solved by : Bashir Ahmad Mubashar (M.Phil, M.Ed)

EXERCISE # 2.4

Question # 1

Without using calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$

Sol. $\log_2 18 - \log_2 9$

$$= \log_2(2\times 9) - \log_2 9$$

$$\begin{array}{r|l} 2 & 18 \\ \hline 9 & 9 \\ \hline & 1 \end{array}$$

$$= \log_2 2 + \log_2 9 - \log_2 9$$

$\because$ Applying product law

$\log(m\times n) = \log m + \log n$

$$= \log_2 2 + \cancel{\log_2 9} - \cancel{\log_2 9}$$

$$= \log_2 2$$

$$= \boxed{1}$$

Answer.

Ex #2.4 — Question #1 (ii)

Evaluate

$$\log_2 64 + \log_2 2$$

Sol.

$$\log_2 64 + \log_2 2$$

$$= \log_2 2^6 + \log_2 2$$

Apply Power Law

$$= \log_2 2^6 + \log_2 2$$

$$= 6 \log_2 2 + \log_2 2$$

$$= 6 \log_2 2 + 1 \log_2 2$$

$$= (6 + 1) \log_2 2$$

$$= 7 \log_2 2$$

$$= 7(1)$$

$$= \boxed{7}$$ Ans.

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$64 = 2^6$

Ex#2.4 — (Question # 1(iii)) —

Evaluate

$$\frac{1}{3}\log_3 8 - \log_3 18$$

Sol.

$$\frac{1}{3}\log_3 8 - \log_3 18$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$8 = 2\times2\times2$

$8 = 2^3$

$$= \frac{1}{3}\log_3 2^3 - \log_3 (2\times3^2)$$

Apply Power law and product law

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$18 = 2\times3\times3$

$18 = 2\times3^2$

$$= \frac{1}{\cancel{3}}\cancel{3}\log_3 2 - \left[\log_3 2 + \log_3 3^2\right]$$

$$= \cancel{\log_3 2} - \cancel{\log_3 2} - \underbrace{\log_3 3^2}_{\text{Again Apply power law}}$$

$$= -\log_3 3^2$$

$$= -2\log_3 3$$

$$= -2(1)$$

$$= \boxed{-2} \text{ Ans}$$

Ex#2.4 — (Question#1(iv)) —

Evaluate

$$2\log 2 + \log 25$$

Sol.

$$2\log 2 + \log 25$$

$$= 2\log 2 + \underbrace{\log 5^2}$$

$$\begin{array}{c|c} 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$25 = 5 \times 5$

$25 = 5^2$

Apply power law

$$= 2\log 2 + 2\log 5$$

$$= 2(\log 2 + \log 5)$$

Apply product Law (Inverse)

$$\log m + \log n = \log(m \times n)$$

$$= 2[\log(2 \times 5)]$$

$$= 2\log 10$$

$$= 2(1) \qquad \because \log 10 = 1$$

$$= \boxed{2} \text{ Ans.}$$

Ex#2.4 — Question # 1 (v) —

Evaluate

$$\frac{1}{3}\log_4 64 + 2\log_5 25$$

Sol.

$$\frac{1}{3}\log_4 64 + 2\log_5 25$$

4	64
4	16
4	4
	1

$64 = 4\times4\times4$

$64 = 4^3$

$$= \frac{1}{3}\log_4 4^3 + 2\log_5 5^2$$

Apply Power Law

$$= \frac{1}{\cancel{3}}\cancel{3}\log_4 4 + 2(2)\log_5 5$$

5	25
5	5
	1

$25 = 5\times5$

$25 = 5^2$

$$= \log_4 4 + 4\log_5 5$$

$$= 1 + 4(1)$$

$$= 1 + 4$$

$$= \boxed{5}$$ Ans.

Ex # 2.4 Question # 1 (vi)

Evaluate

$$\log_3 12 + \log_3 0.25$$

Sol.

$$\log_3 12 + \log_3 0.25$$

Apply Product law (Inverse)

$$\log_a m + \log_a n = \log_a (m \times n)$$

$$= \log_3 (12 \times 0.25)$$

$$= \log_3 \left(12 \times \frac{0.25}{100}\right)$$

$$= \log_3 \left(12 \times \frac{1}{4}\right)$$

$$= \log_3 (3)$$

$$= \log_3 3$$

$$= \boxed{1} \quad \text{Ans.}$$

EXERCISE # 2.4 Maths 9th PTB

Question # 2

Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 + 2\log 3$

Sol. $\frac{1}{2}\log 25 + 2\log 3$

$= \log(25)^{\frac{1}{2}} + \log(3)^2$

$= \log(5^2)^{\frac{1}{2}} + \log(3)^2$ $\quad \because 25 = 5\times5$, $25 = 5^2$

$= \log 5 + \log 9$ $\quad \because 3^2 = 3\times3$, $3^2 = 9$

$= \log(5\times9)$ $\quad \because \log m + \log n = \log(m\times n)$

$= \boxed{\log 45}$

Ans.

Ex # 2.4 — (Question # 2(ii)) —

Write as a single logarithm:

$$\log 9 - \log \frac{1}{3}$$

Sol.

$$\log 9 - \log \frac{1}{3}$$

$\because \log m - \log n = \log \frac{m}{n}$

$$= \log\left(\frac{9}{\frac{1}{3}}\right)$$

$$= \log\left(9 \div \frac{1}{3}\right)$$

$$= \log\left(9 \times \frac{3}{1}\right)$$

$$= \log\left(\frac{27}{1}\right)$$

$$= \boxed{\log 27}$$

Ans.

Ex #2.4 — (Question # 2(iii))

Write as a single logarithm:

$$\log_5 b^2 \cdot \log_a 5^3$$

Sol.

$$\log_5 b^2 \cdot \log_a 5^3$$

$$= 2\log_5 b \times 3\log_a 5$$

$$= 2\frac{\log b}{\cancel{\log 5}} \times 3\frac{\cancel{\log 5}}{\log a}$$

$$= 2\times 3\,\frac{\log b}{\log a}$$

$$= \boxed{6\log_a b}$$ Ans.

Ex # 2.4 (Question # 2(iv))

Write as a single logarithm:

$$2\log_3 x + \log_3 y$$

Sol.

$$2\log_3 x + \log_3 y$$

$$= \log_3 x^2 + \log_3 y$$

$$= \log_3 (x^2 \times y) \qquad \because \log_a m + \log_a n = \log_a (m \times n)$$

$$= \boxed{\log_3 x^2 y}$$

Ans.

Ex # 2.4 (Question # 2 (v))

Write as a single logarithm:

$$4\log_5 x - \log_5 y + \log_5 z$$

Sol.

$$4\log_5 x - \log_5 y + \log_5 z$$

$$= \log_5 x^4 - \log_5 y + \log_5 z$$

$$= \underbrace{\log_5 x^4 + \log_5 z} - \log_5 y$$

$\because \log_a m + \log_a n = \log_a (m \times n)$

$$= \log_5 (x^4 \times z) - \log_5 y$$

$$= \log_5 x^4 z - \log_5 y$$

$\because \log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

$$= \boxed{\log_5 \frac{x^4 z}{y}}$$

Ans.

Ex # 2.4 —(Question # 2 (vi))—

Write as a single logarithm:

$$2\ln a + 3\ln b - 4\ln c$$

Sol.

$$2\ln a + 3\ln b - 4\ln c$$

$$= \underbrace{\ln a^2 + \ln b^3} - \ln c^4$$

$\because \log m + \log n = \log(m \times n)$

$$= \ln(a^2 \times b^3) - \ln c^4$$

$$= \underbrace{\ln a^2 b^3 - \ln c^4}$$

$\because \log m - \log n = \log \frac{m}{n}$

$$= \boxed{\ln \frac{a^2 b^3}{c^4}}$$

Ans.

EXERCISE # 2.4 — (Question # 3)

Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$

Sol.

$$\log\left(\frac{11}{5}\right) = \log 11 - \log 5$$

(ii) $\log_5 \sqrt{8a^6}$

Sol.

$$\log_5 \sqrt{8a^6}$$

$$= \log_5 (8a^6)^{\frac{1}{2}}$$

$$= \log_5 (2^3 a^6)^{\frac{1}{2}}$$

2	8
2	4
2	2
	1

$8 = 2 \times 2 \times 2$

$8 = 2^3$

$$= \log_5 \left(2^{3 \times \frac{1}{2}} \; a^{6 \times \frac{1}{2}}\right)$$

$$= \log_5 \left(2^{\frac{3}{2}} \times a^3\right) \qquad \because \log(m \times n) = \log m + \log n$$

$$= \log_5 2^{\frac{3}{2}} + \log_5 a^3$$

$$= \boxed{\frac{3}{2}\log_5 2 + 3\log_5 a} \quad \text{Ans.}$$

(iii) $\ln\left(\frac{a^2 b}{c}\right)$

Sol. $\ln\left(\frac{a^2 b}{c}\right)$ $\quad \because \log\frac{m}{n} = \log m - \log n$

$= \ln(a^2 b) - \ln c$

$= \ln(a^2 \times b) - \ln c$ $\quad \because \log(m \times n) = \log m + \log n$

$= \ln a^2 + \ln b - \ln c$

$= \boxed{2\ln a + \ln b - \ln c}$ Ans.

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

Sol. $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

$= \frac{1}{9}\left[\log\left(\frac{xy}{z}\right)\right]$

$= \boxed{\frac{1}{9}\left[\log x + \log y - \log z\right]}$

Ans.

(v) $\ln \sqrt[3]{16x^3}$

Sol. $\ln \sqrt[3]{16x^3}$

$$= \ln (16x^3)^{\frac{1}{3}}$$

$$= \ln (2^4 x^3)^{\frac{1}{3}}$$

$$= \ln \left(2^{4\times\frac{1}{3}} \times x^{3\times\frac{1}{3}}\right)$$

$$= \ln \left(2^{\frac{4}{3}} \times x^1\right)$$

$$= \ln 2^{\frac{4}{3}} + \ln x^1$$

$$= \frac{4}{3}\ln 2 + 1 \ln x$$

$$= \boxed{\frac{4}{3}\ln 2 + \ln x}$$

Ans.

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$16 = 2\times2\times2\times2$

$16 = 2^4$

$\because \log(m\times n) = \log m + \log n$

$\because \log m^n = n \log m$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Sol.

$$\log_2\left(\frac{1-a}{b}\right)^5 \qquad \because \log m^n = n\log m$$

$$= 5\left[\log_2\left(\frac{1-a}{b}\right)\right]$$

$$= 5\left[\log_2\frac{(1-a)}{b}\right] \qquad \because \log\frac{m}{n} = \log m - \log n$$

$$= 5\left[\log_2(1-a) - \log_2 b\right] \quad \underline{\text{Ans.}}$$

EXERCISE # 2.4

Question # 4

Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$

Sol.

$$\log 2 + \log x = 1$$

$$\log(2 \times x) = 1 \qquad \because \log m + \log n = \log(m \times n)$$

$$\log_{10} 2x = 1 \qquad \because \log = \log_{10}$$

$$2x = 10^1$$

$$2x = 10$$

$$x = \frac{10}{2}$$

$$\boxed{x = 5} \quad \text{Ans}$$

Ex # 2.4 — {Question # 4(ii)} —

Find the value of "x".

$$\log_2 x + \log_2 8 = 5$$

Sol.

$$\log_2 x + \log_2 8 = 5$$

$\because \log m + \log n = \log(m \times n)$

$$\log_2 (x \times 8) = 5$$

$$\log_2 8x = 5$$

$$8x = 2^5$$

$$8x = 2 \times 2 \times 2 \times 2 \times 2$$

4 8 16 32

$$8x = 32$$

$$x = \frac{32}{8}$$

$$\boxed{x = 4}$$ Ans.

Ex # 2.4 — Question # 4(iii) —

Find the value of "x".

$$(81)^x = (243)^{x+2}$$

Sol.

$$(81)^x = (243)^{x+2}$$

3	81
3	27
3	9
3	3
	1

$81 = 3 \times 3 \times 3 \times 3$

$81 = 3^4$

$$(3^4)^x = (3^5)^{x+2}$$

$$3^{4x} = 3^{(5)(x+2)}$$

$$3^{4x} = 3^{5x+10}$$

3	243
3	81
3	27
3	9
3	3
	1

$243 = 3 \times 3 \times 3 \times 3 \times 3$

$243 = 3^5$

$$4x = 5x + 10$$

$$4x - 10 = 5x$$

$$-10 = 5x - 4x$$

$$-10 = 1x$$

$$\boxed{-10 = x}$$

Ans.

OR

$$\boxed{x = -10}$$

Ex # 2.4 — Question # 4(iv)

Find the value of "x".

$$\left(\frac{1}{27}\right)^{x-6} = 27$$

Sol.

$$\left(\frac{1}{27}\right)^{x-6} = 27$$

$$\left(27^{-1}\right)^{x-6} = (27)^{1}$$

$$(27)^{(-1)(x-6)} = (27)^{1}$$

$$(27)^{-x+6} = (27)^{1}$$

$$-x+6 = 1$$

$$+6-1 = x$$

$$+5 = x$$

$\boxed{5 = x}$ Ans.

Ex # 2.4 — Question # 4(v)

Find the value of "x"

$$\log(5x-10)=2$$

Sol.

$$\log(5x-10)=2$$

$$\log_{10}(5x-10)=2$$

$$5x-10=10^{2}$$

$$5x-10=10\times 10$$

$$5x-10=100$$

$$5x=100+10$$

$$5x=110$$

$$x=\frac{110}{5}$$

$$\begin{array}{r|l} & 22 \\ 5 & 110 \\ & -10 \\ & 10 \\ & -10 \\ & \times \end{array}$$

$$\boxed{x=22}$$ Ans.

Ex # 2.4 —{Question # 4(vi)}—

Find the value of "x".

$$\log_2(x+1) - \log_2(x-4) = 2$$

Sol.

$$\log_2(x+1) - \log_2(x-4) = 2$$

$$\log_2 \frac{(x+1)}{(x-4)} = 2 \qquad \because \log m - \log n = \log \frac{m}{n}$$

$$\frac{(x+1)}{(x-4)} = 2^2$$

$$\frac{(x+1)}{(x-4)} = 4$$

$$(x+1) = 4(x-4)$$

$$x+1 = 4x-16$$

$$+1+16 = 4x - x$$

$$17 = 3x$$

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ -15 \\ \hline 2 \end{array}$$

$$\boxed{\frac{17}{3} = x} \text{ Ans.} \quad \text{OR} \quad \boxed{x = 5\frac{2}{3}} \text{ Ans.}$$

EXERCISE #2.4 — Question # 5 (i)

Solve with the help of logarithm table:

(i) $\dfrac{3.68 \times 4.21}{5.234}$

Sol.

let

$$x = \frac{3.68 \times 4.21}{5.234}$$

Taking log on both side

$$\log x = \log\left(\frac{3.68 \times 4.21}{5.234}\right)$$

$\because \log \frac{m}{n} = \log m - \log n$

$$\log x = \log(3.68 \times 4.21) - \log(5.234)$$

$$\log x = \log 3.68 + \log 4.21 - \log(5.234)$$

$\because \log(m \times n) = \log m + \log n$

$$\log x = 0.5658 + 0.6243 - 0.7188$$

solve

$$\begin{array}{r} 7185 \\ +\quad 3 \\ \hline 7188 \end{array}$$

$$\log x = 1.1901 - 0.7188$$

$$\log x = 0.4713$$

$$x = \text{antilog}(0.4713)$$

solve

$$\begin{array}{r} 2958 \\ +\quad 2 \\ \hline 2960 \end{array}$$

$\boxed{x = 2.960}$ Ans.

EXERCISE #2.4 (Question #5(ii))

Solve with the help of logarithm table:

$$4.67 \times 2.11 \times 2.397$$

Sol.

let

$$x = 4.67 \times 2.11 \times 2.397$$

Taking log on both side

$$\log x = \log(4.67 \times 2.11 \times 2.397)$$

$$\log x = \log 4.67 + \log 2.11 + \log 2.397$$

Solve

$$\begin{array}{r} 3784 \\ +\ 13 \\ \hline 3797 \end{array}$$

$$\log x = 0.6693 + 0.3243 + 0.3797$$

$$\log x = 1.3733$$

Solve

$$x = \text{antilog } 1.3733$$

.37

$\boxed{3}$

$\boxed{3}$

$$\begin{array}{r} 2360 \\ +\ 2 \\ \hline 2362 \end{array}$$

$$x = 23.62$$

$$\boxed{x = 23.62}$$ Ans.

EXERCISE # 2.4 — (Question # 5(iii))

Solve with the help of logarithm table:

$$\frac{(20.46)^2 \times (2.4122)}{754.3}$$

Sol.

Let $x = \dfrac{(20.46)^2 \times (2.4122)}{754.3}$

Taking log on both side

$$\log x = \log\left[\frac{(20.46)^2 \times (2.4122)}{754.3}\right]$$

$$\log x = \log(20.46)^2 + \log(2.4122) - \log 754.3$$

$$\log x = 2\log 20.46 + \log 2.4122 - \log 754.3$$

$$\log x = 2(1.3109) + (0.3824) - (2.8776)$$

$$\log x = 2.6218 + 0.3824 - 2.8776$$

$$\log x = 3.0042 - 2.8776$$

$$\log x = 0.1266$$

$$x = \text{antilog } 0.1266$$

$$\boxed{x = 1.339} \text{ Ans}$$

$3096 + 13 = 3109$

$3820 + 4 = 3824$

$8774 + 2 = 8776$

$1337 + 2 = 1339$

EXERCISE # 4.2 — (Question # 5 (iv)) —

Solve with the help of logarithm table:

$$\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$$

Sol.

Let $x = \dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Taking log on both side $\quad \because \sqrt[3]{\ } = (\ )^{\frac{1}{3}}$

$$\log x = \log\left[\frac{(9.364)^{\frac{1}{3}} \times 21.64}{3.21}\right]$$

$$\log x = \log (9.364)^{\frac{1}{3}} + \log 21.64 - \log 3.21$$

$$\log x = \frac{1}{3}\log(9.364) + \log 21.64 - \log 3.21$$

$$\log x = \frac{1}{3}(0.9715) + (1.3353) - (0.5065)$$

$$\log x = 0.3238 + 1.3353 - 0.5065$$

$$\log x = 1.6591 - 0.5065$$

$$\log x = 1.1526$$

$$x = \text{antilog } 1.1526$$

$$x = 14.21$$

$$\boxed{x = 14.21} \text{ Ans.}$$

93: 9713 [6]; + 2; 9715 [4]

21: 3345 [6]; + 8; 3353 [4]

32: 5065 [1]

.15: 1419 [2]; + 2; 1421 [6]

EXERCISE # 2.4 (Question # 6)

The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_0}\right)$. If amplitude (A) is 10,000 and reference amplitude (A_0) is 10. What is the magnitude of the earthquake?

Sol.

Here, $A = 10000$

and $A_0 = 10$

$$M = \log_{10}\left(\frac{A}{A_0}\right)$$

$$M = \log_{10}\left(\frac{10000}{10}\right)$$

$$M = \log_{10}(1000)$$

$$M = \log_{10}(10^3)$$

$$M = 3\log_{10}10$$

$$M = 3(1)$$

$$\boxed{M = 3}$$

Therefore, the magnitude of earthquake is 3.

EXERCISE # 2.4 —(Question # 7)—

Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years in Rs y. This is modelled by an equation $y = 100,000 (1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

Sol.

Initial investment = Rs 100000

Interest Rate = 5% per annum

Total year after "t" years = y

Equation

$$y = 100000 (1.05)^t$$

We want to find years when the investment will be double i.e $y = 200000$

So,

$$200000 = 100000 (1.05)^t$$

$$\frac{200000}{100000} = (1.05)^t$$

$$2 = (1.05)^t$$

$$\log 2 = \log (1.05)^t$$

$$\log 2 = t \log(1.05)$$

$$\frac{\log 2}{\log(1.05)} = t$$

$$\frac{0.3010}{0.0212} = t$$

0212 $\boxed{110}$ 5

$$14.198 = t$$

$$\boxed{14.20 \approx t}$$

After 14.20 years the investment will be double.

EXERCISE # 2.4 — (Question # 8)

Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 meters gained in altitude. The initial temperature (T_i) at sea level is 20°c. Using the formula

$T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 metres.

Sol.

$$T = T_i \times (0.97)^{\frac{h}{100}}$$

$$T = 20 \times (0.97)^{\frac{500}{100}}$$

$$T = 20 \times (0.97)^5$$

$$\log T = \log\left(20 \times (0.97)^5\right)$$

$$\log T = \log 20 + \log(0.97)^5$$

$$\log T = \log 20 + 5 \log 0.97$$

$$\log T = 1.3010 + 5(-0.0132)$$

$$\log T = 1.3010 - 0.066$$

$$\log T = 1.235$$

$$T = \text{antilog } 1.235$$

$$\boxed{T \approx 17.18°C}$$

$\boxed{20}$ 3010 0

$\boxed{97}$ 9868 0

$\bar{1}.9868$

-0.0132

.23 1718 5