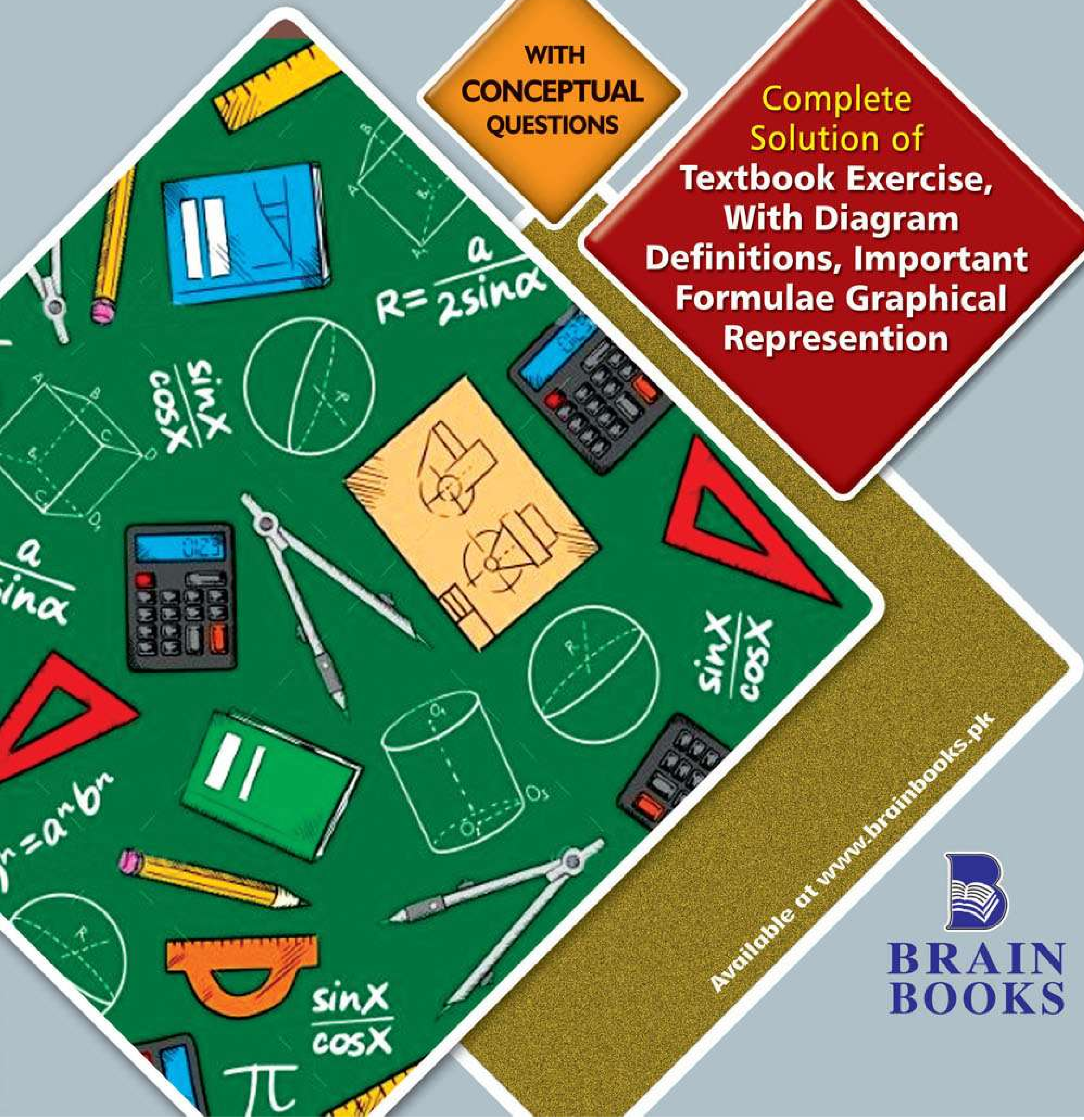
BRAIN SOLUTION

MATHEMATICS 9

WITH QUESTION BANK AND TUTOR

WITH CONCEPTUAL QUESTIONS

Complete Solution of **Textbook Exercise, With Diagram Definitions, Important Formulae Graphical Represention**



BRAIN BOOKS

BRAIN SOLUTION

Mathematics

with
CONCEPTUAL
Questions (CQs)

9

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CONTENTS

Unit - 1 REAL NUMBERS

1.1 Introduction to Real Numbers

The history of numbers comprises thousands of years, from ancient civilization to a modern Arabic system.

Sumerians

1. What is the range of years during which the Sumerians used the sexagesimal system?

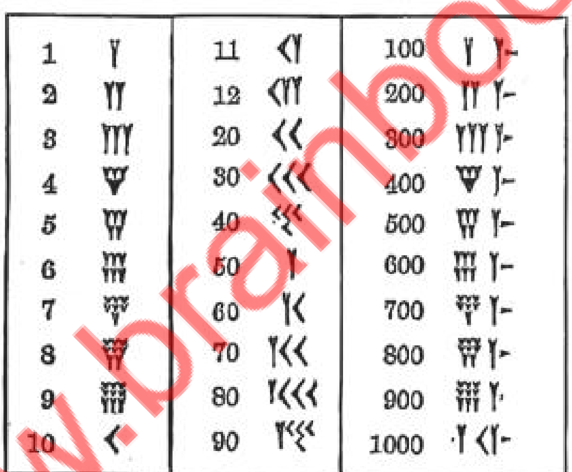
Ans: 4500 – 1900 BCE.

2. What numerical system did the Sumerians used for counting, and what was its base?

Ans: Sumerians used a sexagesimal (base 60) system for counting. They used a small cone, bead, large cone, large perforated cone, sphere and perforated sphere, corresponding to 1, 10, 60 (a large unit), 600.

3. Name the shapes or symbols used by the Sumerians for counting.

Ans:



Egyptians

4. What numerical system did the Egyptians use, and what was its base?

Ans: Egyptians used a decimal system for counting. Its base was 10.

5. How were Egyptian numbers written in terms of order and denominations?

Ans: The Egyptians usually wrote numbers left to right, starting with the highest denominator.

Example: 2525 would be written with 2000 first, then 500, 20 and 5.

Romans

6. Which numeral system was used by the Romans, and what letters did it use?

Ans: Roman numerals use 7 letters to represent different numbers. These are I, V, X, L, C, D, and M which represent the numbers 1, 5, 10, 50, 100, 500 and 1000 respectively.

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Indians

7. What is the contribution of ancient Indians to mathematics?

Ans: Ancient Indians (500 – 1200 CE) developed the concept of zero (0) and made a significant contribution to the decimal (base 10) system. The invention of zero is attributed to Indians, and this contribution outweighs all others made by any other nation since it is the basis of the decimal number system, without which no advancement in mathematics would have been possible.

Arabs

8. What are Indo – Arabic numerals and why are they called so?

Ans: The numbers system used today was invented by Indians, and it is still called Indo-Arabic numerals because Indians invented them and the Arab merchants took them to the Western world.

9. Who is credited with introducing algebra as a distinct field, and in which century?

Ans: Muhammad Ibn Musa-al-Khwarizmi introduced algebra as a distinct field in the 9^{th} century.

Modern era

10. Which modern number systems were developed in the modern era?

Ans: The modern era developed modern number system e.g. binary system (base–2) and hexadecimal system (base–16).

11. What is the basis for the modern decimal system?

Ans: The Arabic system is the basis for modern decimal system used globally today.

12. What set was adopted as the counting set in the modern era?

Ans: In the modern era, the set {1,2,3,…..} was adopted as the counting set.

13. What does the counting set represent?

Ans: The counting set represents the set of natural numbers.

14. What is set of numbers most frequently used in everyday life?

Ans: The set of real numbers which is used most frequently in everyday life.

1.1.1 Combination of Rational and Irrational Numbers

15. Define an irrational number.

Ans: Irrational Numbers: Irrational numbers are those numbers which cannot be put into the form $\frac{p}{q}$ where $p, q \in Z \wedge q \neq 0$. Set of irrational numbers is denoted by Q'.

Examples: $\sqrt{2}, \sqrt{3}, \frac{7}{\sqrt{5}}$ and $\sqrt{\frac{5}{16}}$ are irrational numbers.

16. Define rational number.

Ans: All the numbers of the form $\frac{p}{q}$ where p,q are integers and $q \neq 0$ are called rational numbers. It is denoted by "Q". The set of rational numbers is defined as:

$$Q=\left\{\frac{p}{q}; p,q \in Z \wedge q \neq 0, (p,q)=1\right\}$$

17. Define the set of Real Numbers.

Ans: The union of the set of rational numbers and irrational numbers is called set of real numbers. It is denoted by R. i.e $R = Q \cup Q'$ and $Q \cap Q' = \phi$.

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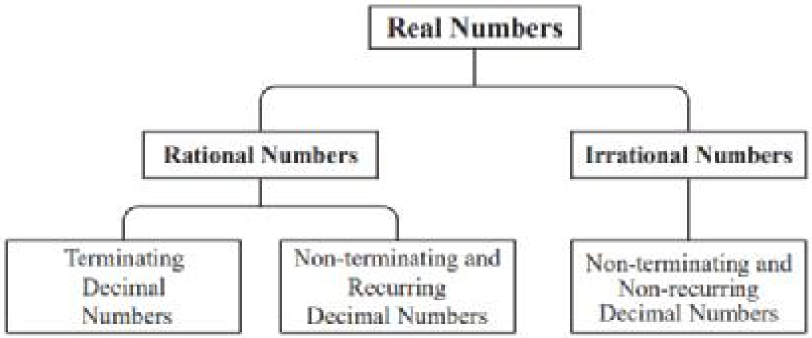
Note: Set of irrational numbers Q' contains those elements which cannot be expressed as quotient of integers.



1.1.2 Decimal Representation of Rational Numbers

(i) Terminating Decimal Numbers

18. What are terminating decimal numbers? Give examples.

Ans: A decimal numbers with a finite number of digits after the decimal point is called a terminating decimal numbers

Examples: $\frac{1}{4}=0.25, \frac{8}{25}=0.32, \frac{3}{8}=0.375, \frac{4}{5}=0.8$ are all terminating decimal numbers.

(ii) Non-Terminating and Recurring Decimal Numbers

19. Define Non-terminating and recurring decimal Number with an example.

Ans: The decimal numbers with an infinitely repeating pattern of digits after the decimal point are called non-terminating and recurring decimal numbers.

Examples:

(i) $\frac{1}{3}=0.333....=0.\overline{3}$ (3 repeats infinitely)

(ii) $\frac{1}{6}=0.1666.....=0.1\overline{6}$ (6 repeats infinitely)

(iii) $\frac{22}{7}=3.142857142857....=3.\overline{142857}$ (the pattern 142857 repeats infinitely)

(iv) $\frac{4}{9}=0.44444....=0.\overline{4}$ (03 repeats infinitely)

Note: Non-terminating and recurring decimal numbers are also rational numbers.

1.1.3 Decimal Representation of Irrational Numbers

20. Define irrational numbers.

Ans: **Irrational numbers:** Non-terminating and non-recurring decimal numbers are known as irrational numbers.

Decimal numbers that do not repeat pattern of digits after the decimal point continue indefinitely without terminating.

Examples: $\pi=3.1415926535897932....$

$e=2.71828182845904....$ and $\sqrt{2}=1.41421356237309....$

Remember!
e = 2.7182… is called Euler's

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1.1.4 Representation of Rational and Irrational Numbers on Number Line

Remember!

(i) Rational number + Irrational number = Irrational number

(ii) Rational number $(\neq 0)$ × Irrational number = Irrational number

Try Yourself

Q. What will be the product of two irrational numbers?

Ans: The product of two irrational numbers can either be rational or irrational, depending on the specific numbers involved.

(i) Irrational number × Irrational number = Irrational number

e.g., $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

(ii) Irrational number × Irrational number = Rational number

e.g., $\sqrt{2} \times \sqrt{2} = \left(\sqrt{2}\right)^2 = 2$ which is rational.

It is depends on the nature of the irrational numbers and how they interact when multiplied.

1.1.5 Properties of Real Numbers

All calculations involving addition, subtraction, multiplication, and divisions of real numbers are based on their properties.

Additive Properties

	Name of the property	$\forall a,b,c \in R$	Examples
(i)	Closure	$a+b \in R$	$2+3=5 \in R$
(ii)	Commutative	$a+b=b+a$	$2+5=5+2$ $7=7$
(iii)	Associative	$a+(b+c)=(a+b)+c$	$2+(3+5)=(2+3)+5$ $2+8=5+5$ $10=10$
(iv)	Identity	$a+0=a=0+a$	$5+0=5=0+5$
(v)	Inverse	$a+(-a)=-a+a=0$	$6+(-6)=(-6)+6=0$

Multiplicative Properties

	Name of the property	$\forall a,b,c \in R$	Examples
(i)	Closure	$ab \in R$	$2\times 5=10 \in R$
(ii)	Commutative	$ab=ba$	$2\times 3=3\times 2=6 \in R$
(iii)	Associative	$a(bc)=(ab)c$	$2\times(3\times 5)=(2\times 3)\times 5$ $2\times 15=6\times 5$ $30=30$
(iv)	Identity	$a\times 1=1\times a=a$	$5\times 1=1\times 5=5$
(v)	Inverse	$a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$7\times\frac{1}{7}=\frac{1}{7}\times 7=1$

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Distributive Properties

For all real numbers a,b,c

(i) $a(b + c) = ab + ac$ is called left distributive property of multiplication over addition.

(ii) $a(b - c) = ab - ac$ is called left distributive property of multiplication over subtraction.

(iii) $(a + b)c = ac + bc$ is called right distributive property of multiplication over addition.

(iv) $(a - b)c = ac - bc$ is called right distributive property of multiplication over subtraction.

Do you know?

0 and 1 are the additive and multiplicative identities of real numbers respectively.

Remember!

$0 \in R$ has no multiplicative inverse.

Properties of Equality of Real Numbers

i.	Reflexive property	$\forall a \in R,\ a = a$
ii.	Symmetric property	$\forall a,b \in R,\ a = b \Rightarrow b = a$
iii.	Transitive property	$\forall a,b,c \in R,\ a = b \wedge b = c \Rightarrow a = c$
iv.	Additive property	$\forall a,b,c \in R,\ a = b \Rightarrow a + c = b + c$
v.	Multiplicative property	$\forall a,b,c \in R,\ a = b \Rightarrow ac = bc$
vi.	Cancellation property w.r.t addition	$\forall a,b,c \in R,\ a + c = b + c \Rightarrow a = b$
vii.	Cancellation property w.r.t multiplication	$\forall a,b,c \in R$ and $c \neq 0, ac = bc \Rightarrow a = b$

Order Properties

i.	Trichotomy property	$\forall a,b \in R$, either $a = b$ or $a > b$ or $a < b$
ii.	Transitive property	$\forall a,b,c \in R$ • $a > b \wedge b > c \Rightarrow a > c$ • $a < b \wedge b < c \Rightarrow a < c$
iii.	Additive property	$\forall a,b,c \in R$ • $a > b \Rightarrow a + c > b + c$ • $a < b \Rightarrow a + c < b + c$
iv.	Multiplicative property	$\forall a,b,c \in R$ • $a > b \Rightarrow ac > bc$ if $c > 0$ • $a < b \Rightarrow ac < bc$ if $c > 0$ • $a > b \Rightarrow ac < bc$ if $c < 0$ • $a < b \Rightarrow ac > bc$ if $c < 0$ • $a > b \wedge c > d \Rightarrow ac > bd$ • $a < b \wedge c < d \Rightarrow ac < bd$
v.	Division property	$\forall a,b,c \in R$ • $a < b \Rightarrow \frac{a}{c} < \frac{b}{c}$ if $c > 0$ • $a < b \Rightarrow \frac{a}{c} > \frac{b}{c}$ if $c < 0$

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		• $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$ if $c > 0$ • $a > b \Rightarrow \frac{a}{c} < \frac{b}{c}$ if $c < 0$
vi.	Reciprocal property	$\forall a, b \in R$ and have same sign • $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$ • $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

21. What is additive identity?

Ans: There exists a unique real number 0 called additive identity such that $a + 0 = a = 0 + a \ \forall a \in \mathrm{R}$

22. Define the multiplicative identity.

Ans: There exists a unique real number 1, called the multiplicative identity, such that $\forall\ a \in \mathrm{R}, a.1 = a\mathrm{a} = 1.a$

SOLVED EXERCISE 1.1

1. Identify each of the following as a rational or irrational number:

(i) 2.353535

Sol: 2.353535 is a terminating decimal number, therefore it is a rational number.

(ii) $0.\overline{6}$

Sol: $0.\overline{6} = 0.666......$ is a non-terminating and recurring decimal number, therefore it is a rational number.

(iii) 2.236067………

Sol: 2.236067……… is a non-terminating and non-recurring decimal number, therefore it is a irrational number.

(iv) $\sqrt{7}$

Sol: $\sqrt{7} \simeq 2.64575131$ is a non-terminating and non-recurring decimal number, therefore it is an irrational number.

(v) e

Sol: $e \simeq 2.718281......$ is a non-terminating and non-recurring decimal number, therefore it is an irrational number.

(vi) π

Sol: $\pi \simeq 3.1415926...$ is a non-terminating and recurring decimal number, therefore it is an irrational number.

(vii) $5 + \sqrt{11}$

Sol: As sum of rational number and irrational number is an irrational number so $5 + \sqrt{11}$ is an irrational number.

(i) $\sqrt{3} + \sqrt{13}$

Sol: As $\sqrt{3}$ and $\sqrt{13}$ are both irrational numbers so sum of two irrational numbers is also an irrational number.

(i) $\frac{15}{4}$

Sol: $\frac{15}{4} = 3.75$ is a terminating decimal number, therefore it is a rational number.

(i) $\left(2 - \sqrt{2}\right)\left(2 + \sqrt{2}\right)$

Sol: As, $\left(2 - \sqrt{2}\right)$ and $\left(2 + \sqrt{2}\right)$ are conjugate surds of each other. The product of two conjugate surds is a rational number. Therefore,

$$\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right) = (2)^2 - \left(\sqrt{2}\right)^2$$

$$= 4 - 2 = 2$$

which is a rational number.

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2. Represent the following numbers on number line:

(i) $\sqrt{2}$

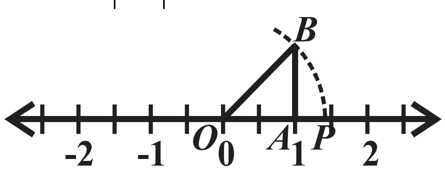
Sol: $\sqrt{2} \simeq 1.4142......$ which is near to 1. Take a side of length $m\overline{OA} = 1$ unit. By forming a right ΔOAB with side $m\overline{AB} = 1$ unit as shown in the figure. By Pythagoras theorem

$$\left(m\overline{OB}\right)^2 = \left(m\overline{OA}\right)^2 + \left(m\overline{AB}\right)^2$$

$$= (1)^2 + (1)^2 = 1 + 1 = 2$$

$\Rightarrow$ $m\overline{OB} = \sqrt{2}$

By drawing an arc with centre at O and radius $m\overline{OB} = \sqrt{2}$, we got point 'P' representing $\sqrt{2}$ on the number line. So $\left|\overline{OP}\right| = \sqrt{2}$



(ii) $\sqrt{3}$

Sol: $\sqrt{3} \simeq 1.73205......$

Draw a horizontal line and mark a point as **A**. Make another point at a unit distance from **A** and label it as 1 i.e. $m\overline{AB} = 1$ unit from the point **B** on the number line, draw a perpendicular line of length 1 unit $m\overline{BC} = 1$ unit. Connect this perpendicular end point **C** to **A** on the number line.

This forms a right triangle with a base of 1 unit and height of 1 unit.

By using Pythagoras theorem

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2$$

$$= (1)^2 + (1)^2 = 2$$

$m\overline{AC} = \sqrt{2}$

From point A, draw an arc of radius $m\overline{AC} = \sqrt{2}$ taking A as centre, we got point 'D" representing $\sqrt{2}$ on the number line. So, $\left|\overline{OD}\right| = \sqrt{2}$

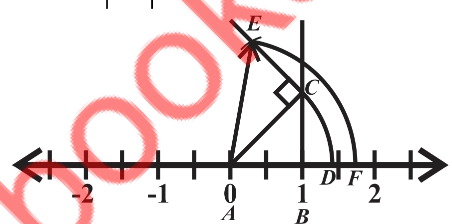
The new triangle with base $m\overline{AC} = \sqrt{2}$ and a height of 1 unit i.e. $m\overline{CE} = 1$. Again using Pythagoras theorem.

$$\left(m\overline{AE}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{CE}\right)^2$$

$$= \left(\sqrt{2}\right)^2 + (1)^2 = 2 + 1 = 3$$

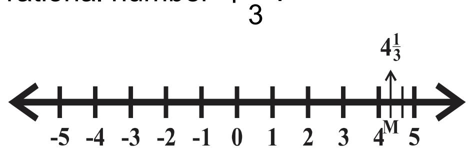
$m\overline{AE} = \sqrt{3}$

Draw an arc of radius $m\overline{AE} = \sqrt{3}$ taking A as centre, we got point 'F' representing $\sqrt{3}$ on the number line so $m\left|\overline{AF}\right| = \sqrt{3}$.



(iii) $4\frac{1}{3}$

Sol: For representing the rational number, $4\frac{1}{3}$ divide the unit length between 4 and 5 into three equal parts. Take the end of the first part from 4 to its right side. The point M in the following figure represents the rational number $4\frac{1}{3}$.



(iv) $-2\frac{1}{7}$

Sol: For representing the rational number, $-2\frac{1}{7}$ divide the unit length between -2 and -3 into seven equal parts. Take the end of the first part from -2 to its left side. The point M in the

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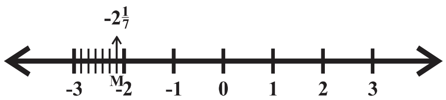
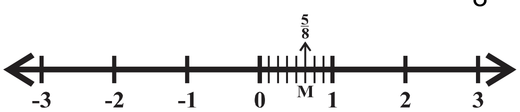
following figure represents the rational number $-2\frac{1}{7}$.



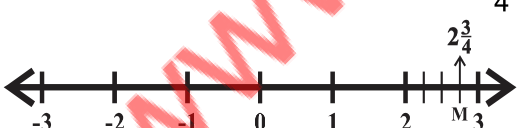
(v) $\frac{5}{8}$

Sol: For representing the rational number, $\frac{5}{8}$ on the number line, divide the unit length between 0 and 1 into eight equal parts and take the end of the fifth part from 0 to its right side.
The point M in the following figure represents the rational number $\frac{5}{8}$.



(vi) $2\frac{3}{4}$

Sol: For representing the rational number, $2\frac{3}{4}$ on the number line, divide the unit length between 2 and 3 into four equal parts and take the end of the three part from 2 to its right side.
The point M in the following figure represents the rational number $2\frac{3}{4}$.



3. Express the following as a rational number where $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i) $0.\overline{4}$

Sol: Let $x = 0.\overline{4}$

$x = 0.4444.......$ (i)

Multiply both sides by 10, we have

$10x = 10(0.4444......)$

$10x = 4.4444.......$ (ii)

Subtracting (i) from (ii)

$10x - x = (4.4444.....) - (0.4444....)$

$9x = 4$

$\Rightarrow \quad x = \frac{4}{9} \quad \Rightarrow \quad 0.\overline{4} = \frac{4}{9}$

Which shows the decimal number in the form of $\frac{p}{q}$

(ii) $0.\overline{37}$

Sol: Let $x = 0.\overline{37}$

$x = 0.373737.......$ (i)

Multiply both 100 on both sides

$100x = 100(0.373737.....)$

$100x = 37.373737.......$ (ii)

Subtracting (i) from (ii)

$100x - x = (37.373737.....) - (0.373737....)$

$99x = 37$

$\Rightarrow \quad x = \frac{37}{99} \quad \Rightarrow \quad 0.\overline{37} = \frac{37}{99}$

Which shows the decimal number in the form of $\frac{p}{q}$

(ii) $0.\overline{21}$

Sol: Let $x = 0.\overline{21}$

$x = 0.212121.......$ (i)

Multiply both 100 on both sides

$100x = 100(0.212121.....)$

$100x = 21.212121.......$ (ii)

Subtracting (i) from (ii)

$100x - x = (21.212121.....) - (0.212121....)$

$99x = 21$

$\Rightarrow \quad x = \frac{21}{99} \quad \Rightarrow \quad 0.\overline{21} = \frac{21}{99}$

Which shows the decimal number in the form of $\frac{p}{q}$

4. Name the property used in the following:

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(i) $(a+4)+b=a+(4+b)$

Ans: Association property over addition.

(ii) $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$

Ans: Commutative property over addition.

(iii) $x - x = 0$

Ans: Additive inverse.

(iv) **a (b + c) = ab + ac**

Ans: Left distributive property of multiplication over addition.

(v) **16 + 0 = 16**

Ans: Additive identity.

(vi) **100 × 1 = 100**

Ans: Multiplicative identity.

(vii) **4 × (5 × 8) = (4 × 5) × 8**

Ans: Associative property under multiplication.

(viii) **ab = ba**

Ans: Commutative property under multiplication.

5. Name the property used in the following:

(i) $-3 < -1$

Sol: $-3 < -1$

$\Rightarrow -3 + 3 < -1 + 3$

$\Rightarrow 0 < 2$

Additive property of inequality.

ii) **If a < b then** $\frac{1}{a} > \frac{1}{b}$

Sol: Reciprocal property of inequality.

(iii) **If a < b then a + c < b + c**

Sol: Additive property of inequality.

(iv) **If ac < bc and c > 0 then a < b**

Sol: Division property of inequality.

(v) **If ac < bc and c < 0 then a > b**

Sol: Division property of inequality.

(vi) **Either a > b or a = b or a < b**

Sol: Trichotomy property.

6. Insert two rational numbers between:

(i) $\frac{1}{3}$ **and** $\frac{1}{4}$

Sol: To find the rational number between $\frac{1}{3}$ and $\frac{1}{4}$, find the average of $\frac{1}{3}$ and $\frac{1}{4}$ as:

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)=\frac{1}{2}\left(\frac{4+3}{12}\right)=\frac{1}{2}\left(\frac{7}{12}\right)=\frac{7}{24}$$

So, $\frac{7}{24}$ is a rational number between $\frac{1}{3}$ and $\frac{1}{4}$.

To find another rational number between $\frac{1}{3}$ and $\frac{1}{4}$, we will again find average of $\frac{7}{24}$ and $\frac{1}{4}$ i.e.

$$\frac{1}{2}\left(\frac{7}{24}+\frac{1}{4}\right)=\frac{1}{2}\left(\frac{7+6}{24}\right)=\frac{1}{2}\left(\frac{13}{24}\right)=\frac{13}{48}$$

Hence, two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{13}{48}$

(ii) **3 and 4**

Sol: To find the rational number between 3 and 4, find the average of 3 and 4 as

$$\frac{1}{2}(3+4)=\frac{1}{2}(7)=\frac{7}{2}$$

So, $\frac{7}{2}$ is a rational number between 3 and 4.

To find another rational number between $\frac{7}{2}$ and 4 we will again find average of $\frac{7}{4}$ and 4.

i.e. $$\frac{1}{2}\left(\frac{7}{2}+4\right)=\frac{1}{2}\left(\frac{7+8}{2}\right)$$

$$=\frac{1}{2}\left(\frac{15}{2}\right)=\frac{15}{4}$$

Hence, two rational numbers between 3 and 4 are $\frac{7}{2}$ and $\frac{15}{4}$

(iii) $\frac{3}{5}$ **and** $\frac{4}{5}$

Sol: To find the rational number between

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$\frac{3}{5}$ and $\frac{4}{5}$, find the average of $\frac{3}{5}$ and $\frac{4}{5}$ as:

$$\frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{3+4}{5}\right)=\frac{1}{2}\left(\frac{7}{5}\right)=\frac{7}{10}$$

So $\frac{7}{10}$ is a rational number between $\frac{3}{5}$ and $\frac{4}{5}$.

To find another rational number between $\frac{3}{5}$ and $\frac{4}{5}$ we will again find average of $\frac{7}{10}$ and $\frac{4}{5}$.

i.e. $\frac{1}{2}\left(\frac{7}{10}+\frac{4}{5}\right)=\frac{1}{2}\left(\frac{7+8}{10}\right)$

$$=\frac{1}{2}\left(\frac{15}{10}\right)=\frac{15}{20}=\frac{3}{4}$$

Hence, two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{7}{10}$ and $\frac{3}{4}$

1.2 Radical Expressions

1. Define n^{th} root of "a".

Ans: If n is a positive integer greater than 1 and a is a real number x, then any real number such that $x=\sqrt[n]{a}$ is called n^{th} root of a.

Here, $\sqrt{\ }$ is called radical and n is the index of radical.

2. Define radicand with example.

Ans: Radicand: A real number under the radical sign is called a radicand.

Examples: $\sqrt[3]{5}, \sqrt[7]{7}$ are the examples of radical form.

3. Write the exponential form of $x=\sqrt[n]{a}$.

Ans: The exponential form of $x=\sqrt[n]{a}$ is $x=(a)^{\frac{1}{n}}$.

1.2.1 Laws of Radicals and Indices

Laws of Radical	Laws of Indices
(i) $\sqrt[n]{ab}=\sqrt[n]{a}.\sqrt[n]{b}$	(i) $a^m.a^n=a^{m+n}$
(ii) $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	(ii) $\left(a^m\right)^n=a^{mn}$
(iii) $\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$	(iii) $(ab)^n=a^nb^n$
(iv) $\left(\sqrt[n]{a}\right)^n=\left(a^{\frac{1}{n}}\right)^n=a$	(iv) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$
	(v) $\frac{a^m}{a^n}=a^{m-n}$
	(vi) $a^0=1$

1.2.2 Surds and their Applications

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4. Define surd with example.

Ans: An irrational radical with rational radicand is called a surd. If we take the n^{th} root of any rational number a then $\sqrt[n]{a}$ is a surd.

> **Remember!**
> Every surd is an irrational number but every irrational number is not a surd e.g., $\sqrt{\pi}$ is not surd.

Examples:

(i) $\sqrt{5}$ is a surd because the square root of 5 does not give a whole number but $\sqrt{9}$ is not a surd because it simplifies to a whole number 3 and our result is not an irrational number.

(ii) $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}$ are surds

(iii) $\sqrt{\pi}, \sqrt{e}$ are not surds.

5. Define monomial surd.

Ans: Monomial surd: A surd that contains a single term is called a monomial surd.
Example $\sqrt{5}, \sqrt{7}$ etc.

> **Remember!**
> **Q. What is the product of two conjugate surds?**
> **Ans.** The product of two conjugate surds is a rational number.

6. Define binomial surds.

Ans: Binomial surd: A surd that contains the sum of two monomial surds is called a binomial surd.
Examples: $\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{7}$ are the binomial surds.

7. Define conjugate surds.

Ans: $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

1.2.3 Rationalization of Denominator

8. How do you rationalize the denominator of a binomial surd?

Ans: To rationalize a denominator of the form $\sqrt{a}+b\sqrt{x}$ or $\sqrt{a}-b\sqrt{x}$, we multiply both the numerator and denominator by the conjugate factor.
Example: Multiply the numerator and denominator by the conjugate of $\sqrt{5}+\sqrt{2}$ which is $\sqrt{5}-\sqrt{2}$. i.e.,

$$\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}=\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}=\sqrt{5}-\sqrt{2}$$

SOLVED EXERCISE 1.2

1. Rationalize the denominator of following:

(i) $\dfrac{13}{4+\sqrt{3}}$

Sol: $$\frac{13}{4+\sqrt{3}}=\frac{13}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{(4)^2-\left(\sqrt{3}\right)^2}=\frac{13\left(4-\sqrt{3}\right)}{16-3}$$

$$=\frac{13\left(4-\sqrt{3}\right)}{13}=4-\sqrt{3}$$

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(ii) $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Sol: $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}=\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$=\dfrac{\sqrt{3}\left(\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{3}\right)^2}=\dfrac{\sqrt{6}+\sqrt{15}}{3}$

(iii) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

Sol: $\dfrac{\sqrt{2}-1}{\sqrt{5}}=\dfrac{\sqrt{2}-1}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{5}\left(\sqrt{2}-1\right)}{\left(\sqrt{5}\right)^2}$

$=\dfrac{\sqrt{10}-\sqrt{5}}{5}$

(iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Sol: $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}=\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}\times\dfrac{6-4\sqrt{2}}{6-4\sqrt{2}}$

$=\dfrac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$

$=\dfrac{(6)^2+\left(4\sqrt{2}\right)^2-2(6)\left(4\sqrt{2}\right)}{36-(16\times2)}$

$=\dfrac{36+(16\times2)-48\sqrt{2}}{36-(32)}$

$=\dfrac{36+32-48\sqrt{2}}{36-32}=\dfrac{68+48\sqrt{2}}{4}$

$=\dfrac{\cancel{4}\left(17-12\sqrt{2}\right)}{\cancel{4}}=17-12\sqrt{2}$

(v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Sol: $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$=\dfrac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$

$=\dfrac{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{3}\right)\left(\sqrt{2}\right)}{3-2}$

$=\dfrac{3+2-2\sqrt{6}}{3-2}=5-2\sqrt{6}$

(vi) $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Sol: $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}=\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}\times\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$=\dfrac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2}=\dfrac{4\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$

$=\dfrac{{}^{2}\cancel{4}\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)}{\cancel{2}}=2\sqrt{3}\left(\sqrt{7}-\sqrt{5}\right)$

2. Simplify the following:

(i) $\left(\dfrac{81}{16}\right)^{\frac{-3}{4}}$

Sol: $\left(\dfrac{81}{16}\right)^{\frac{-3}{4}}=\left(\dfrac{16}{81}\right)^{\frac{3}{4}}=\left(\dfrac{2^4}{3^4}\right)^{\frac{3}{4}}$

$=\left(\dfrac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}}\right)=\dfrac{2^3}{3^3}=\dfrac{8}{27}$

(ii) $\left(\dfrac{3}{4}\right)^{-2}\div\left(\dfrac{4}{9}\right)^3\times\dfrac{16}{27}$

Sol: $\left(\dfrac{3}{4}\right)^{-2}\div\left(\dfrac{4}{9}\right)^3\times\dfrac{16}{27}$

$=\left(\dfrac{4}{3}\right)^2\div\left(\dfrac{4}{9}\right)^3\times\dfrac{16}{27}$

$=\dfrac{16}{9}\div\dfrac{64}{729}\times\dfrac{16}{27}$

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$$=\frac{\cancel{16}^{\,1}}{\cancel{9}_{\,1}}\times\frac{\cancel{729}^{\,\cancel{81}\,\cancel{9}\,3}}{\cancel{64}_{\,1\,\cancel{4}}}\times\frac{\cancel{16}^{\,4}}{\cancel{27}_{\,\cancel{3}}}=12$$

(iii) $(0.027)^{-\frac{1}{3}}$

Sol: $(0.027)^{-\frac{1}{3}}=\left(\dfrac{27}{1000}\right)^{-\frac{1}{3}}$

$$=\left(\frac{1000}{27}\right)^{\frac{1}{3}}=\left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$$

$$=\left(\frac{10^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}\right)=\frac{10}{3}$$

(iv) $\sqrt[7]{\dfrac{x^{14}\times y^{20}\times z^{35}}{y^{14}\times z^{7}}}$

Sol: $\sqrt[7]{\dfrac{x^{14}\times y^{20}\times z^{35}}{y^{14}\times z^{7}}}$

$$=\sqrt[7]{x^{14}\times y^{20-14}\times z^{35-7}}$$

$$=\sqrt[7]{x^{14}\times y^{7}\times z^{28}}$$

$$=\left(x^{14}\times y^{7}\times z^{28}\right)^{\frac{1}{7}}$$

$$=x^{14\times\frac{1}{7}}\times y^{7\times\frac{1}{7}}\times z^{28\times\frac{1}{7}}$$

$$=x^2\times y\times z^4=x^2yz^4$$

(vi) $\dfrac{5.(25)^{n+1}-25.(5)^{2n}}{5.(5)^{2n+3}-(25)^{n+1}}$

Sol: $\dfrac{5.(25)^{n+1}-25.(5)^{2n}}{5.(5)^{2n+3}-(25)^{n+1}}$

$$=\frac{5.(5^2)^{n+1}-5^2.(5)^{2n}}{5.(5)^{2n+3}-(5^2)^{n+1}}$$

$$=\frac{5.5^{2n+2}-5^2.5^{2n}}{5.5^{2n+3}-5^{2n+2}}=\frac{5^{2n+2+1}-5^{2n+2}}{5^{2n+3+1}-5^{2n+2}}$$

$$=\frac{5^{2n+3}-5^{2n+2}}{5^{2n+4}-5^{2n+2}}=\frac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)}$$

$$=\frac{5-1}{5^2-1}=\frac{4}{24}=\frac{1}{6}$$

(vi) $\dfrac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$

Sol: $\dfrac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$

$$=\frac{(2^4)^{x+1}+20(2^2)^{2x}}{2^{x-3}\times(2^3)^{x+2}}$$

$$=\frac{2^{4x+4}+20.2^{4x}}{2^{x-3}\times 2^{3x+6}}=\frac{2^{4x+4}+20.2^{4x}}{2^{4x+3}}$$

$$=\frac{2^{4x}(2^4+20)}{2^{4x}(2^3)}=\frac{16+20}{8}$$

$$=\frac{36}{8}=\frac{9}{2}$$

(vii) $(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}}$

Sol: $(64)^{-\frac{2}{3}}\div(9)^{-\frac{3}{2}}=\left(4^3\right)^{-\frac{2}{3}}\div\left(3^3\right)^{-\frac{3}{2}}$

$$=4^{\cancel{3}x-\frac{2}{\cancel{3}}}\div3^{\cancel{2}x-\frac{3}{\cancel{2}}}=4^{-2}\div3^{-3}$$

$$=\frac{4^{-2}}{3^{-3}}=\frac{3^3}{4^2}=\frac{27}{16}$$

(viii) $\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

Sol: $\dfrac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}=\dfrac{3^n\times(3^2)^{n+1}}{3^{n-1}\times(3^2)^{n-1}}$

$$=\frac{3^n\times3^{2n+2}}{3^{n-1}\times3^{2n-2}}=\frac{3^{2n+2+n}}{3^{2n-2+n-1}}=\frac{3^{3n+2}}{3^{3n-3}}$$

$$=\frac{3^{3n}.3^2}{3^{3n}.3^{-3}}=3^2.3^3=3^5=243$$

(ix) $\dfrac{5^{n+3}-6.5^{n+1}}{9\times5^n-4\times5^n}$

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Sol: $\dfrac{5^{n+3}-6.5^{n+1}}{9\times 5^n-4\times 5^n}=\dfrac{5^n\left(5^3-6.5^1\right)}{5^n(9-4)}$

$=\dfrac{125-30}{9-4}=\dfrac{95}{5}=19$

3. **If $x=3+\sqrt{8}$ then find the value of:**

(i) $x+\dfrac{1}{x}$

Sol: $x=3+\sqrt{8}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}=\dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$=\dfrac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}=\dfrac{3-\sqrt{8}}{9-8}=\dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x}=3-\sqrt{8}$

Now, $x+\dfrac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)$

$=3+\cancel{\sqrt{8}}+3-\cancel{\sqrt{8}}$

$$\boxed{x+\frac{1}{x}=6}$$

(ii) $x-\dfrac{1}{x}$

Sol: $x=3+\sqrt{8}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}=\dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$=\dfrac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}=\dfrac{3-\sqrt{8}}{9-8}=\dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x}=3-\sqrt{8}$

Now,

$x-\dfrac{1}{x}=\left(3+\sqrt{8}\right)-\left(3-\sqrt{8}\right)$

$=\cancel{3}+\sqrt{8}-\cancel{3}+\sqrt{8}$

$$\boxed{x-\frac{1}{x}=2\sqrt{8}}$$

(iii) $x^2+\dfrac{1}{x^2}$

Sol: $x=3+\sqrt{8}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}=\dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$=\dfrac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}=\dfrac{3-\sqrt{8}}{9-8}=\dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x}=3-\sqrt{8}$

Now, $x+\dfrac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)$

$=3+\cancel{\sqrt{8}}+3-\cancel{\sqrt{8}}$

$x+\dfrac{1}{x}=6$

Taking square on both side

$\left(x+\dfrac{1}{x}\right)^2=(6)^2$

$x^2+\dfrac{1}{x^2}+2(x)\left(\dfrac{1}{x}\right)=36$

$x^2+\dfrac{1}{x^2}=36-2$

$$\boxed{x^2+\frac{1}{x^2}=34}$$

(iv) $x^2-\dfrac{1}{x^2}$

Sol: $x=3+\sqrt{8}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}=\dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$=\dfrac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}=\dfrac{3-\sqrt{8}}{9-8}=\dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x}=3-\sqrt{8}$

Now, $x+\dfrac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)$

$=3+\cancel{\sqrt{8}}+3-\cancel{\sqrt{8}}$

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$$x+\frac{1}{x}=6$$

and $x-\frac{1}{x}=(3+\sqrt{8})-(3-\sqrt{8})$

$$=\cancel{3}+\sqrt{8}-\cancel{3}+\sqrt{8}$$

$$x-\frac{1}{x}=2\sqrt{8}$$

$$\because a^2-b^2=(a+b)(a-b)$$

$$x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=(6)(2\sqrt{8})$$

$$\boxed{x^2-\frac{1}{x^2}=12\sqrt{8}}$$

(v) $x^4+\frac{1}{x^4}$

Sol: $x=3+\sqrt{8}$

$$\frac{1}{x}=\frac{1}{3+\sqrt{8}} \qquad =\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2} \qquad =\frac{3-\sqrt{8}}{9-8}=\frac{3-\sqrt{8}}{1}$$

$$\frac{1}{x}=3-\sqrt{8}$$

Now, $x+\frac{1}{x}=(3+\sqrt{8})+(3-\sqrt{8})$

$$=3+\cancel{\sqrt{8}}+3-\cancel{\sqrt{8}}$$

$$x+\frac{1}{x}=6$$

Taking square on both side

$$\left(x+\frac{1}{x}\right)^2=(6)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=36$$

$$x^2+\frac{1}{x^2}=36-2=34$$

Again squaring on both side

$$\left(x^2+\frac{1}{x^2}\right)^2=(34)^2$$

$$(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(x^2)\left(\frac{1}{x^2}\right)=1156$$

$$x^4+\frac{1}{x^4}+2=1156$$

$$x^4+\frac{1}{x^4}=1156-2$$

$$\boxed{x^4+\frac{1}{x^4}=1154}$$

(vi) $\left(x-\frac{1}{x}\right)^2$

Sol: $x=3+\sqrt{8}$

$$\frac{1}{x}=\frac{1}{3+\sqrt{8}}$$

$$=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2} \qquad =\frac{3-\sqrt{8}}{9-8}$$

$$=\frac{3-\sqrt{8}}{1}$$

$$\frac{1}{x}=3-\sqrt{8}$$

Now, $x-\frac{1}{x}=(3+\sqrt{8})-(3-\sqrt{8})$

$$=\cancel{3}+\sqrt{8}-\cancel{3}+\sqrt{8}$$

$$x-\frac{1}{x}=2\sqrt{8}$$

Taking square on both side

$$\left(x-\frac{1}{x}\right)^2=(2\sqrt{8})^2 \quad =(4\times 8)$$

$$\boxed{\left(x-\frac{1}{x}\right)^2=32}$$

4. Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$.

Sol:
$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}=p+q\sqrt{2}$$
$$\frac{8\text{-}3\sqrt{2}}{4+3\sqrt{2}}\times\frac{4\text{-}3\sqrt{2}}{4\text{-}3\sqrt{2}}=p+q\sqrt{2}$$
$$\frac{\left(8-3\sqrt{2}\right)\left(4-3\sqrt{2}\right)}{(4)^2-\left(3\sqrt{2}\right)^2}=p+q\sqrt{2}$$
$$\Rightarrow\quad \frac{32-24\sqrt{2}-12\sqrt{2}+\left(3\sqrt{2}\right)^2}{16-(9\times 2)}=p+q\sqrt{2}$$
$$\frac{32-36\sqrt{2}+(9\times 2)}{16-18}=p+q\sqrt{2}$$
$$\frac{32-36\sqrt{2}+18}{-2}=p+q\sqrt{2}$$
$$\frac{50-36\sqrt{2}}{-2}=p+q\sqrt{2}$$
$$\frac{-50}{2}+\frac{36\sqrt{2}}{2}=p+q\sqrt{2}$$
$$-25+18\sqrt{2}=p+q\sqrt{2}$$

By comparing of both side

$p=-25,\qquad q=18$

5. Simplify the following:

(i) $$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

Sol:
$$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}=\frac{\left(5^2\right)^{\frac{3}{2}}\times\left(3^5\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}}\times\left(2^3\right)^{\frac{4}{3}}}$$
$$=\frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}}=\frac{5^3\times 3^3}{2^5\times 2^4}$$
$$=\frac{125\times 27}{32\times 16}\quad=\frac{3375}{512}$$

(ii) $$\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216\left(3^{2x\text{-}1}\right)}$$

Sol:
$$\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216\left(3^{2x-1}\right)}$$
$$=\frac{\left(2\times 3^3\right)\times\sqrt[3]{\left(3^3\right)^{2x}}}{\left(3^2\right)^{x+1}+\left(2^3\times 3^3\right)\left(3^{2x-1}\right)}$$
$$=\frac{2\times 3^3\times\left(3^{6x}\right)^{\frac{1}{3}}}{3^{2x+2}+\left(2^3\times 3^3\right)\left(3^{2x-1}\right)}$$
$$=\frac{2\times 3^3\times 3^{2x}}{3^{2x+2}+2^3\times 3^{2x+2}}$$
$$=\frac{3^{2x}\left(2\times 3^3\right)}{3^{2x}\left(3^2+2^3\times 3^2\right)}$$
$$=\frac{2\times 3^3}{3^2+2^3\times 3^2}$$
$$=\frac{2\times 27}{9+(8\times 9)}$$
$$=\frac{54}{9+72}=\frac{54}{81}=\frac{2}{3}$$

2	54
3	27
3	9
3	3
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

(iii) $$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$

Sol:
$$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$
$$=\sqrt{\frac{\left(2^3\times 3^3\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{(25)^{\frac{-3}{2}}}}$$
$$=\sqrt{\frac{2^{3\times\frac{2}{3}}\times 3^{3\times\frac{2}{3}}\times 5^{2\times\frac{1}{2}}}{\left(5^{\cancel{2}}\right)^{\frac{-3}{\cancel{2}}}}}$$
$$=\sqrt{\frac{2^2\times 3^2\times 5^1}{(25)^{\frac{3}{2}}}}=\sqrt{\frac{2^2\times 3^2\times 5^1}{\left(5^{\cancel{2}}\right)^{\frac{3}{\cancel{2}}}}}$$
$$=\sqrt{\frac{2^2\times 3^2\times 5^1}{5^3}}=\sqrt{\frac{2^2\times 3^2}{5^{3-1}}}=\sqrt{\frac{2^2\times 3^2}{5^2}}$$

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$$=\sqrt{\left(\frac{2\times 3}{5}\right)^2} = \frac{2\times 3}{5} = \frac{6}{5}$$

(iv) $\left(\mathbf{a^{\frac{1}{3}}+b^{\frac{2}{3}}}\right)\times\left(\mathbf{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}}\right)$

Sol: $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

$$=a^{\frac{1}{3}+\frac{2}{3}}-a^{\frac{1}{3}+\frac{1}{3}}b^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{4}{3}}-a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}+\frac{2}{3}}+b^{\frac{2}{3}+\frac{4}{3}}$$

$$=a^1-\cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}}+\cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}}+\cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}}-\cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}}+b^2=a+b^2$$

Alternate:

$$\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$$

$$=\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left[\left(a^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right)+\left(b^{\frac{2}{3}}\right)^2\right]$$

$$\because (a+b)(a^2-ab+b^2)=a^3+b^3$$

$$=\left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^3=a+b^2$$

1.3 Applications of Real Numbers in Daily Life

1. Write applications of real numbers in daily life.

Ans: Real numbers are used in various fields including:

- **Science and engineering**: Physics, mechanical systems, electrical circuits
- Medicine and Health
- **Environmental science:** Climate modding, pollution monitoring etc.
- **Computer science**: Algorithm design, data compression, graphic rendering
- **Navigation and transportation:** GPS, flight planning
- Surveying and architecture
- Statistics and data

2. Define set of Integers.

Ans: The set of integers consist of positive integers, 0 and negative integers. It is denoted by "Z". i.e., Z = {.......-3, -2, -1, 0, 1, 2, 3, ...}

3. Define the set of Whole numbers.

Ans: If "0" is included in the set of natural numbers then set W={0,1,2,3,4,...} is called set of whole numbers. It is denoted by W.

4. Define Natural numbers and give example.

Ans: Numbers which are used to count different things, are called Natural numbers. Set N which include natural numbers can be written as: N = {1,2,3,4.........}

1.3.1 Temperature Conversions

5. Describe the conversion of temperature.

Ans: In the figure 1, three types of thermometers are shown. We can convert three temperature scales, Celsius, Fahrenheit, and Kelvin, with each other. Conversion formulae are given below:

(i) $K={}^{o}C+273$ (iii) ${}^{o}F=\frac{9\,{}^{o}C}{5}+32$ (ii) ${}^{o}C=\frac{5}{9}(F-32)^{o}$

Where K, °C and °F show the Kelvin, Celsius, and Fahrenheit scales respectively.

6. What is the normal human body temperature in Celsius?

Ans: The normal human body temperature in Celsius is 37 °C.

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7. What is the freezing point of water in Kelvin, Celsius and Fahrenheit?

Ans:

Scale	Freezing point
Kelvin	273 K
Celsius	0 °C
Fahrenheit	32 °F

8. What is the normal human body temperature in Kelvin?

Ans: The normal human body temperature in Kelvin is 310.15 K.

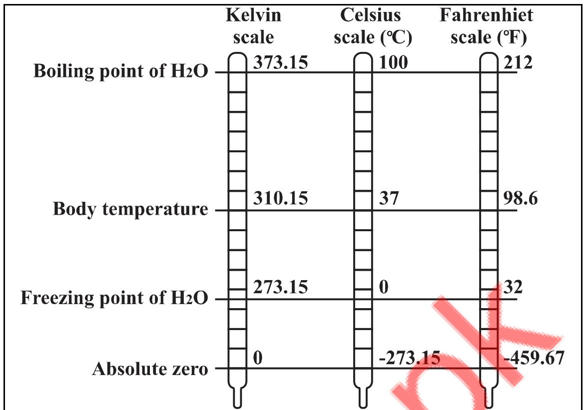

Figure 1

1.3.2 Profit and Loss

9. What is difference between profit and loss?

Ans:

Profit	Loss
When the selling price is more than cost price the profit will be occurs.	When the selling price is less than cost price the profit will be occurs.

10. What is the formula to calculate profit?

Ans: Profit = selling price – cost price

$P = SP - CP$

11. Write the formula to calculate profit percentage?

Ans: $Profit\% = \left(\frac{Profit}{CP} \times 100\right)\%$

12. What is the formula to calculate loss?

Ans: Loss = cost Price – selling price

Loss = $CP - SP$

13. Write the formula to calculate loss percentage?

Ans: $Loss\% = \left(\frac{Loss}{CP} \times 100\right)\%$

SOLVED EXERCISE 1.3

1. The sum of three consecutive integers is forty-two, find the three integers.

Sol: Let $x, x+1, x+2$ be the three consecutive integers,

By the given condition

$$x + (x+1) + (x+2) = 42$$

$$x + x + 1 + x + 2 = 42$$

$$3x + 3 = 42$$

$$3x = 42 - 3 = 39$$

$$3x = 39$$

$$x = \frac{39}{3} = 13$$

So, $x = 13$

$x + 1 = 13 + 1 = 14$

$x + 2 = 13 + 2 = 15$

Hence the three consecutive integers are 13, 14, 15.

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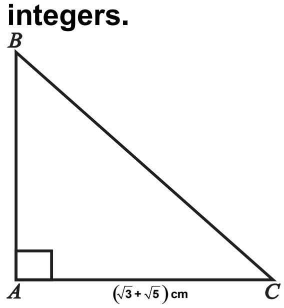
2. The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $\left(\sqrt{3}+\sqrt{5}\right)$cm. The area of ΔABC is $\left(1+\sqrt{15}\right)\text{cm}^2$. Find the length $\overline{AB}$ in the form $\left(a\sqrt{3}+b\sqrt{5}\right)$cm, where a and b are integers.



Sol: In a right angled triangle ABC

$$m\overline{AC}=\left(\sqrt{3}+\sqrt{5}\right)\text{cm}$$

Area of $\Delta ABC=\left(1+\sqrt{15}\right)\text{cm}^2$

$$m\overline{AB}=?$$

As we know that

$$\text{Area of } \Delta ABC=\frac{1}{2}\left(m\overline{AC}\right)\left(m\overline{AB}\right)$$

$$1+\sqrt{15}=\frac{1}{2}\left(\sqrt{3}+\sqrt{5}\right)\left(m\overline{AB}\right)$$

$$m\overline{AB}=\frac{2\left(1+\sqrt{15}\right)}{\sqrt{3}+\sqrt{5}}$$

$$m\overline{AB}=\frac{2\left(1+\sqrt{15}\right)}{\sqrt{3}+\sqrt{5}}\times\frac{\left(\sqrt{3}-\sqrt{5}\right)}{\sqrt{3}-\sqrt{5}}$$

$$=\frac{2\left(\sqrt{3}-\sqrt{5}+\sqrt{45}-\sqrt{75}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{2\left(\sqrt{3}-\sqrt{5}+\sqrt{9\times5}-\sqrt{25\times3}\right)}{3-5}$$

$$=\frac{\cancel{2}\left(\sqrt{3}-\sqrt{5}+3\sqrt{5}-5\sqrt{3}\right)}{-\cancel{2}}$$

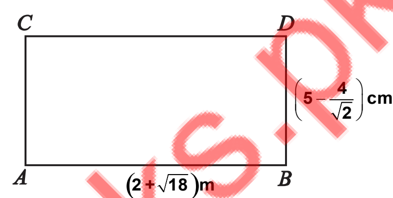
$$=-1\left(-4\sqrt{3}+2\sqrt{5}\right)$$

$$m\overline{AB}=4\sqrt{3}-2\sqrt{5}$$

Hence the length of $\overline{AB}=\left(4\sqrt{3}-2\sqrt{5}\right)$cm.

3. A rectangle has sides of length $\left(2+\sqrt{18}\right)$m and $\left(5-\frac{4}{\sqrt{2}}\right)$m. Express the area of the rectangle in the form $a+b\sqrt{2}$, where a and b are integers.

Sol:



Length of rectangle $=\ell=\left(2+\sqrt{18}\right)$m

Width of rectangle $=w=\left(5-\frac{4}{\sqrt{2}}\right)$m

Area of rectangle $=\ell\times w$

$$=\left(2+\sqrt{18}\right)\left(5-\frac{4}{\sqrt{2}}\right)$$

$$=10-\frac{8}{\sqrt{2}}+5\sqrt{18}-\frac{4\sqrt{18}}{\sqrt{2}}$$

$$=10-\frac{8}{\sqrt{2}}+5\left(\sqrt{9\times2}\right)-\frac{4\sqrt{9\times2}}{\sqrt{2}}$$

$$=10-\frac{8}{\sqrt{2}}+5\left(3\sqrt{2}\right)-\frac{4\left(3\sqrt{2}\right)}{\sqrt{2}}$$

$$=10-\frac{8}{\sqrt{2}}+15\sqrt{2}-12$$

$$=10-12+15\sqrt{2}-\frac{8}{\sqrt{2}}$$

$$=-2+15\sqrt{2}-\left(\frac{8}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$=-2+15\sqrt{2}-\frac{8\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$=-2+15\sqrt{2}-\frac{8\sqrt{2}}{2}$$

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$= -2 + 15\sqrt{2} - 4\sqrt{2}$
$= -2 + (15-4)\sqrt{2} = -2 + 11\sqrt{2}$
Area of rectangle $= \left(11\sqrt{2} - 2\right)m^2$

4. Find two numbers whose sum is 68 and difference is 22.

Sol: Let a and b are two numbers, then
$a + b = 68$ ....(1)
$a - b = 22$ ....(2)
Adding (1) and (2)
$2a = 90 \quad \Rightarrow \quad a = \frac{90}{2} = 45$
Put in (2)
$45 - b = 22 \quad \Rightarrow -b = 22 - 45$
$\Rightarrow \quad -b = -23 \quad \Rightarrow \quad b = 23$
So 45 and 23 are the required numbers.

5. The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperature as high as 48°C. By using the formula $\left({}^{\circ}F = \frac{9}{5}{}^{\circ}C + 32\right)$ find the temperature as Fahrenheit scale.

Sol: Given that ${}^{\circ}C = 48$
Using formula ${}^{\circ}F = \frac{9}{5}{}^{\circ}C + 32$
Putting value
${}^{\circ}F = \frac{9}{5}(48) + 32 = \frac{432}{5} + 32$
${}^{\circ}F = 86.4 + 32 = 118.4{}^{\circ}F$

6. The sum of the ages of the father son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?

Sol: Let Age of father = x years and
Age of Son = y years
By the given condition
$x + y = 72$ ....(1)
Six years ago
Age of father $= (x-6)$ years
Age of son $= (y-6)$ years
By the given condition
$(x-6) = 2(y-6)$
$x - 6 = 2y - 12$
$x = 2y - 12 + 6$
$x = 2y - 6$ ....(2)
put (2) in (1), we have
$2y - 6 + y = 72$
$3y - 6 = 72$
$y = \frac{78}{3} = 26$
putting value of in eq. (2)
$x = 2(26) - 6$
$x = 52 - 6 = 46$
so, Age of son six years ago
$= (y-6) = 26 - 6 = 20$ years

7. Mirha bought a toy for Rs.1500 and sold for Rs.1520. What was her profit percentage?

Sol: Cost price of toy = CP = Rs.1500
Sale price of toy = SP = Rs.1520
Profit = SP – CP = 1520–1500=20
Profit % $= \left(\frac{\text{Profit}}{\text{CP}} \times 100\right)\%$
$= \left(\frac{20}{1500} \times 100\right)\%$
$= \frac{4}{3}\% = 1.33\%$

8. The annual income of Tayyab is Rs.9,60,000, while the exempted amount is Rs.1,30,000. How much tax would he have to pay at the rate of 0.75%?

Sol: Annual income = Rs.9,60,000
Exempted amount = Rs.1,30,000
Taxable income
= 9,60,000 – 1,30,000
= Rs.8,30,000
Rate of tax = 0.75%
Amount of tax = 8,30,000 × 0.75%
$= 8{,}30{,}000 \times \frac{0.75}{100} = 8{,}300 \times \frac{75}{100}$

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$=83\times75=6225$

Hence amount of tax = Rs.6225

9. Find the compound markup on Rs.3,75,000 for one year at the rate of 14% compounded annually.

Sol: Principal amount =P= Rs.3,75,000

Rate $=r=14\%=\frac{14}{100}=0.14$

Time $=t=1$ year

Compound markup = ?

As, Compound markup$=P\left[(1+r)^t-1\right]$

$=3,75,000\left[(1+0.14)^1-1\right]$

$=3,75,000[1.14-1]$

$=3,75,000\times0.14=\text{Rs.}5,200$

Alternate

Principal amount = P = Rs.3,75,000

Rate $=R=14\%=0.14$

Time $=t=1$ year

Markup = PRT $=3,75,000\times0.14\times1$

$=\text{Rs.}52,500$

REVIEW EXERCISE 1

1. Four options are given against each statement. Choose the correct option.

(i) $\sqrt{7}$ **is:**

(a) Integer (b) rational number
(c) irrational number ✓ (d) natural number

(ii) π **and** e **are:**

(a) natural numbers (b) Integers
(c) rational numbers (d) irrational numbers ✓

(iii) If n **is not a perfect square, then** $\sqrt{n}$ **is:**

(a) rational number (b) natural number
(c) Integer (d) irrational number ✓

(iv) $\sqrt{3}+\sqrt{5}$ **is:**

(a) whole number (b) Integer
(c) rational number (d) irrational number ✓

(v) For all $x\in R,\ x=x$ **is called:**

(a) reflexive property ✓ (b) transitive number
(c) symmetric property (d) trichotomy property

(vi) Let $a,b,c\in R$**,** $a>b$ **and** $b>c\Rightarrow a>c$ **is called _____ property.**

(a) trichotomy (b) transitive ✓ (c) additive (d) multiplicative

(vii) $2^x\times8^x=64$ **then** $x=$**:**

(a) $\frac{3}{2}$ ✓ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(viii) Let $a,b\in R,$ **then** $a=b$ **and** $b=a$ **is called _______ property.**

(a) reflexive ✓ (b) symmetric (c) transitive (d) additive

(ix) $\sqrt{75}+\sqrt{27}=$ **?**

(a) $\sqrt{102}$ (b) $9\sqrt{3}$ (c) $5\sqrt{3}$ (d) $8\sqrt{3}$ ✓

(x) The product of $(3+\sqrt{5})(3-\sqrt{5})$ **is:**

(a) prime number (b) odd number
(c) irrational number (d) rational number ✓

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2. If $a=\frac{3}{2}, b=\frac{5}{3}$ and $c=\frac{7}{5}$, then verify that:

(i) Left Distributive property of multiplication over addition $a(b+c)=ab+ac$:

Sol: LHS: $=a(b+c)=\frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right)$

$=\frac{3}{2}\left(\frac{25+21}{15}\right)$

$=\frac{\cancel{3}}{\cancel{2}}\left(\frac{\cancel{46}_{23}}{\cancel{15}_{5}}\right)=\frac{23}{5}$

RHS $=ab+ac$

$=\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{2}\right)\left(\frac{7}{5}\right)$

$=\frac{5}{2}+\frac{21}{10}=\frac{25+21}{10}$

$=\frac{46}{10}=\frac{23}{5}$

L.H.S = R.H.S

Hence it is verified that $a(b+c)=ab+ac$

(ii) Right Distributive property of multiplication over addition $a(b+c)=ab+ac$.

Sol: L.H.S $=(a+b)c=\left(\frac{3}{2}+\frac{5}{3}\right)\frac{7}{5}$

$=\left(\frac{9+10}{6}\right)\frac{7}{5}=\left(\frac{19}{6}\right)\frac{7}{5}=\frac{133}{30}$

R.H.S $=ac+bc$

$=\left(\frac{3}{2}\right)\left(\frac{7}{5}\right)+\left(\frac{5}{3}\right)\left(\frac{7}{5}\right) \quad =\frac{21}{10}+\frac{7}{3}$

$=\frac{63+70}{30}=\frac{133}{30}$

L.H.S=R.H.S

Hence it is verified that $(a+b)c=ac+bc$.

3. If $a=\frac{4}{3}, b=\frac{5}{2}, c=\frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

Sol:

a) Associative property of real numbers w.r.t addition

$a(b+c)=(a+b)+c$

LHS $=a(b+c)=\frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right)$

$=\frac{4}{3}+\left(\frac{10+7}{4}\right)=\frac{4}{3}+\frac{17}{4}$

$=\frac{16+51}{12}=\frac{67}{12}$

RHS $=(a+b)+c$

$=\left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4}=\left(\frac{8+15}{6}\right)+\frac{7}{4}$

$=\frac{23}{6}+\frac{7}{4}=\frac{46+21}{12}=\frac{67}{12}$

L.H.S=R.H.S

Hence, it is verified that $a(b+c)=(a+b)+c$.

b) Associative property of real numbers w.r.t multiplication

$a(bc)=(ab)c$

LHS $=a(bc)=\frac{4}{3}\left(\frac{5}{2}\times\frac{7}{4}\right)$

$=\frac{\cancel{4}}{3}\left(\frac{35}{\cancel{8}_{2}}\right)=\frac{35}{6}$

RHS $=(ab)c=\left(\frac{{}^{2}\cancel{4}}{3}\times\frac{5}{\cancel{2}}\right)\frac{7}{4}$

$=\left(\frac{10}{3}\right)\frac{7}{4}=\frac{35}{6}$

L.H.S=R.H.S

Hence it is verified that $a(bc)=(ab)c$.

4. Is 0 a rational number? Explain.

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Sol. Yes, 0 is a rational number.

Explanation: A rational number is a member that can be expressed in the form $\frac{p}{q}$ where $p,q \in z, q \neq 0$.

'0' satisfied this definition because it can be written as $\frac{0}{q},(q \neq 0)$ such as $\frac{0}{1},\frac{0}{2}$ or $\frac{0}{-3}$.

5. State trichotomy property of real numbers.

Sol. Trichotomy property of real numbers $\forall a,b \in R$

either $a > b$ or $a = b$ or $a < b$.

6. Find two rational numbers between 4 and 5:

Sol. To find the rational number between 4 and 5

Rational number between 4 and 5

$= \frac{4+5}{2} = \frac{9}{2}$

So, $\frac{9}{2}$ is a rational member between 4 and 5.

To find another rational member between 4 and 5 we will again find average of $\frac{9}{2}$ and 5.

i.e. $\frac{\frac{9}{2}+5}{2} = \frac{1}{2}\left(\frac{9}{2}+5\right) = \frac{1}{2}\left(\frac{9+10}{2}\right)$

$= \frac{1}{2}\left(\frac{19}{2}\right) = \frac{19}{4}$

Hence, two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$

Sol. $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}}$

$= \left(\frac{x^{\frac{15}{5}}y^{\frac{35}{5}}}{z^{\frac{20}{5}}}\right) = \frac{x^3y^7}{z^4}$

(ii) $\sqrt[3]{(27)^{2x}}$

Sol. $\sqrt[3]{(27)^{2x}} = \left[\left(3^3\right)^{2x}\right]^{\frac{1}{3}} = \left(3^{6x}\right)^{\frac{1}{3}}$

$= 3^{6x \times \frac{1}{3}} = 3^{2x}$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

Sol. $\frac{6(3)^{n+2}}{3^{n+1}-3^n} = \frac{6.3^n.3^2}{3^n.3^1-3^n} = \frac{3^n(6.3^2)}{3^n(3-1)}$

$= \frac{6.3^2}{3-1} = \frac{{}^{3}\cancel{6} \times 9}{\cancel{2}} = 3 \times 9 = 27$

8. The sum of three consecutive odd integers is 51. Find the three integers.

Sol: Let $x,\ x+2,\ x+4$ be the three consecutive odd integers.

By the give condition.

$$x+(x+2)+(x+4) = 51$$
$$x+x+2+x+4 = 51$$
$$3x+6 = 51$$
$$3x = 51-6 = 45$$
$$3x = 45$$
$$x = \frac{45}{3} = 15$$
$$x = 15$$

So, $x+2 = 15+2 = 17$

$x+4 = 15+4 = 19$

Hence the three consecutive odd integers are 15, 17 and 19.

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9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Sol: Let the number of balls in the first bucket be x then the number of balls in the second bucket will be $(x+28)$

Total number of balls $= 96$

Therefore $x+(x+28)=96$

$$x+x+28=96$$
$$2x+28=96$$
$$2x=96-28$$
$$x=\frac{68}{2}=34$$

Hence, Number of balls in first bucket $=x=34$

Number of balls in second bucket

$$=x+28=34+28=62$$

10. Salma invested Rs.3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

Sol: Principal amount =P= Rs.3,50,000

Rate=R_1=$7\frac{1}{4}$%=7.25%per annum

Time = T_1= 2 years

$$\text{Profit}=\frac{PR_1T_1}{100}=\frac{3,50,000\times 7.25\times 2}{100}$$
$$=\text{Rs.}50,750$$

After 2 years:

P = Rs.3,50,000

R_2 = 8% per annum

T_2 = 5 years

$$\text{Profit }=\frac{PR_2T_2}{100}=\frac{100\times 8\times 5}{100}$$
$$=\text{Rs.}1,40,000$$

$$\text{Total profit }=\text{Rs.}50,750+1,40,00$$
$$=\text{Rs.}1,90,750$$

Amount at the end of 7 years

= principal amount + Total profit

$=3,50,000+1,90,750$

$=\text{Rs.}5,40,750$

MULTIPLE CHOICE QUESTIONS (MCQs)

1.1 Introduction to Real Numbers

1. The base of the Sumerian numerical system?
(A) 10 (B) 12 (C) 60 ✓ (D) 100

2. Which shape corresponds to 600 in the Sumerian numerical system?
(A) small cone (B) sphere (C) perforated sphere ✓ (D) large cone

3. Which shape corresponds to 1 in the Sumerian system?
(A) small circle ✓ (B) sphere (C) perforated sphere (D) large cone

4. Which shape corresponds to 10 in the Sumerian numeral system?
(A) small cone (B) sphere (C) large cone (D) perforated sphere

5. The Egyptians used a ____ system for counting.
(A) base 2 (B) base 10 ✓ (C) base 16 (D) base 60

6. How would the number 25 25 to written in Egyptian numerals?
(A) 1000, 1000, 500, 20, 5 (B) 500, 2000, 20, 5
(C) 2000, 500, 20, 5 ✓ (D) 500, 1000,1000, 5

7. Which of the following represents the Egyptian symbol for 10?
(A) A rope ✓ (B) A lotus flower (C) A heel bone (D) A spiral

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8. The Egyptian numeral system was used between ______:

(A) 4500 – 1900 BCE (B) 2000 – 1500 BCE
(C) 3000 – 2000 BCE ✓ (D) 1000 – 500 BCE

9. The Sumerians numeral system was used between _______.

(A) 4500 – 1900 BCE ✓ (B) 2000 – 1500 BCE
(C) 3000 – 2000 BCE (D) 1000 – 500 BCE

10. What numeral system did the Romans use?

(A) Decimal system (B) Sexagesimal system
(C) Roman numeral system ✓ (D) Indo–Arabic numeral system

11. How many letters are used in the Roman numeral system?

(A) 5 (B) 6 (C) 7 ✓ (D) 8

12. The invention of zero is attributed to which civilization?

(A) Romans (B) Arabs (C) Indians ✓ (D) Greeks

13. What number system is the basis of modern mathematics?

(A) Roman numeral system (B) Decimal system
(C) Binary system (D) Hexadecimal system

14. Who played a significant role in transforming mathematics in the Islamic world?

(A) Muhammad ibn Musa al-Khwarizmi (B) Al-Karaji (C) Euclid (D) Archimedes

15. What is the reason the numeral system used today is called Indo-Arabic numerals?

(A) It was invented by Indians and spread by Arab merchants ✓
(B) It was invented by Arabs and spread by Indians.
(C) It was invented by Europeans and improved by Arabs
(D) It was invented by Greeks and adopted by Arabs

16. Arab introduce Arabic numerals _____ to Europe.

(A) 0 – 1 (B) 0 – 5 (C) 0 – 8 (D) 0 – 9 ✓

17. When did the modern era of mathematics begin?

(A) 1500 CE (B) 1600 CE (C) 1700 CE✓ (D) 1800 CE

18. In _____ developed modern number system.

(A) Old era (B) India era (C) Modern era✓ (D) Roman era

19. _____ number system has a base of 2?

(A) Binary ✓ (B) Decimal (C) Hexadecimal (D) Roman

20. The hexadecimal system has a base of _______:

(A) 2 (B) 10 (C) 16 ✓ (D) 60

21. Which set was adopted as the counting set in the modern er?

(A) $\{1,2,3,.......\}$ ✓ (B) $\{0,1,2,3,.......\}$ (C) $\{0,1,10,100,.......\}$ (D) $\{2,4,6,8,.......\}$

22. The counting set represents the set of _____ numbers.

(A) Natural ✓ (B) Real (C) Integers (D) Complex

23. ______ number set is most frequently used in everyday life?

(A) Natural (B) Whole (C) Real ✓ (D) Complex

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1.1.1 Combination of Rational and Irrational Numbers

24. If n is a prime number then $\sqrt{n}$ is equals to:
(A) Rational number (B) Whole number (C) Natural (D) Irrational number ✓

25. The number $\sqrt{13}$ is known as:
(A) Rational Number (B) Prime number
(C) Irrational Number ✓ (D) Imaginary number

26. Which of the following is an irrational number?
(A) $\sqrt{\frac{68}{17}}$ (B) $\frac{\sqrt{16}}{7}$ (C) $\frac{4}{\sqrt{2}}$ ✓ (D) $\sqrt{\frac{3}{27}}$

27. "0" is:
(A) Rational number ✓ (B) Positive integer
(C) Singleton set (D) Binary set

28. 0.142857142857…… is number:
(A) Rational ✓ (B) Irrational (C) Whole (D) Imaginary

29. If Q and Q' are rational and irrational numbers then:
(A) $Q \cap Q' = R$ (B) $Q \cup Q' = N$ (C) $Q \cap Q' = Z$ (D) $Q \cup Q' = R$ ✓

30. Set of rational numbers is defined as:
(A) $Q = \left\{\frac{p}{q}; p,q \in Z \wedge q \neq 0\right\}$ ✓ (B) $Q = \left\{\frac{p}{q}; p,q \in P\right\}$
(C) $Q = \left\{\frac{p}{q}; p,q \in Q'\right\}$ (D) $Q = \left\{\frac{p}{q}; p,q \in R \wedge q \neq 0\right\}$

1.1.2 Decimal Representation of Rational numbers

31. Non-terminating and recurring decimal numbers are also ______ numbers.
(A) Real (B) Rational ✓ (C) Irrational (D) Whole

1.1.3 Decimal Representation of Irrational Numbers

32. Every non-recurring, non-terminating decimals represents.
(A) Rational number (B) Irrational number ✓
(C) Natural Number (D) Whole numbers

33. _______ is called Euler's Number.
(A) π (B) e ✓ (C) ϕ (D) $\sqrt{2}$

34. Number $\frac{1}{\pi}$:
(A) Rational (B) Irrational ✓ (C) Prime (D) Whole

35. π is a:
(A) Whole number (B) Natural number
(C) Rational number (D) Irrational number ✓

36. The numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$, π and e are called:
(A) Irrational Numbers ✓ (B) Rational Numbers
(C) Natural Numbers (D) Whole Numbers

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1.1.4 Representation of Rational and Irrational numbers on Number Line

37. Rational number + Irrational number = ________ :
(A) Rational number ✓ (B) Irrational number
(C) Real number (D) Both (A) and (B)

38. Rational number (non-zero) × Irrational number = _______
(A) Rational number (B) Irrational number ✓
(C) Real number (D) All of these

38. $0.\bar{5}$ = _ _ _ _ _
(A) 55 (B) $\frac{5}{10}$ (C) $\frac{5}{9}$ ✓ (D) 0.55

1.1.5 Properties of Real numbers

39. ______ has no multiplication inverse:
(A) –1 (B) 1 (C) 0 ✓ (D) 0 and 1

40. The multiplicative identity of real numbers is:
(A) 0 (B) 1✓ (C) 2 (D) 3

41. The property $\forall\, a,b \in R,\ a=b \Rightarrow b=a$ is called:
(A) Commutative (B) Transitive (C) Symmetric ✓ (D) Reflexive

42. The property $\forall\, a \in R$, a = a is called:
(A) Reflexive ✓ (B) Symmetric (C) Transitive (D) Commutative

43. The property used in $\forall\, a,b \in R\ a=b \wedge b=c \Rightarrow c=a$:
(A) Reflexive (B) Symmetric (C) Transitive✓ (D) Trichotomy

44. Trichotomy is the property of:
(A) Inequality ✓ (B) Equality (C) Division (D) Subtraction

45. Symbol "for all" is _______:
(A) A (B) $\forall$ ✓ (C) < (D) >

46. The property of real numbers used in $7 \times \frac{1}{7} = 1$ is:
(A) Additive Inverse (B) Additive Identity
(C) Multiplicative Inverse ✓ (D) Additive Property

47. The value of i^9 is:
(A) 1 (B) -1 (C) i ✓ (D) $-i$

48. Which of them is closure property under addition?
(A) $a+b \in R$ ✓ (B) $a-b \in R$ (C) $a \times b = b \times a$ (D) $a \div b \in R$

49. Which of them is commutative property under addition?
(A) $a+b=b+a$ ✓ (B) $a-b \in R$ (C) $a \times b = b \times a$ (D) $a \div b \in R$

50. $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$ is known as:
(A) Reciprocal property ✓ (B) Additive property
(C) Division property (D) Multiplicative property

1.2 Radical Expressions

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51. **In $\sqrt[3]{45}$ the radicand is _____:**

(A) 3 (B) $\frac{1}{3}$ (C) 45 ✓ (D) $\frac{45}{3}$

52. **$4^{2/5}$ with radical sign is:**

(A) $\sqrt[5]{4^2}$ ✓ (B) $\sqrt{4^5}$ (C) $\sqrt[2]{4^5}$ (D) $\sqrt{4^{10}}$

53. **Write $\sqrt[5]{2}$ in exponential form.**

(A) 2 (B) 2^5 (C) $2^{\frac{1}{5}}$ ✓ (D) $2^{\frac{5}{2}}$

54. **Write $\sqrt[7]{x^2}$ in exponential form.**

(A) x^2 (B) x^7 (C) $x^{\frac{7}{2}}$ (D) $x^{\frac{2}{7}}$ ✓

55. **$\sqrt{4x^0}$ = _____:**

(A) $4x$ (B) 2 ✓ (C) $2x$ (D) 4

56. **$\left(\frac{49}{36}\right)^{-1/2}$ = _____:**

(A) $\frac{7}{6}$ (B) $\frac{6}{7}$ ✓ (C) $-\frac{7}{6}$ (D) $-\frac{6}{7}$

57. **$\left(\frac{27}{64}\right)^{-1/3}$ = _____:**

(A) $\frac{3}{4}$ (B) $\frac{9}{4}$ (C) $\frac{9}{8}$ (D) $\frac{4}{3}$ ✓

58. **Write $\sqrt[7]{x}$ in exponential form.**

(A) x (B) x^7 (C) $x^{\frac{1}{7}}$ ✓ (D) $x^{\frac{7}{2}}$

59. **Write $4^{2/3}$ with radical sign.**

(A) $\sqrt[3]{4^2}$ ✓ (B) $\sqrt[2]{4^3}$ (C) $\sqrt{4^6}$ (D) $\sqrt{8}$

60. **If $\sqrt[3]{35}$ the radicand is:**

(A) 3 (B) $\frac{1}{3}$ (C) 35 ✓ (D) None

61. **$\left(\frac{25}{16}\right)^{-\frac{1}{2}}$ = _____:**

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ ✓ (C) $\frac{7}{4}$ (D) $-\frac{4}{5}$

62. **$\left(27x^{-1}\right)^{-\frac{2}{3}}$ = _____:**

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(A) $\frac{\sqrt[3]{x^2}}{9}$ ✓ (B) $\frac{\sqrt{x^2}}{9}$ (C) $\frac{\sqrt[3]{x^2}}{8}$ (D) $\frac{\sqrt{x^3}}{8}$

1.2.1 Laws of Radicals and Indices

63. $\sqrt[n]{ab} =$ _____ :

(A) $\sqrt[b]{a}$ (B) $\sqrt[a]{b}$ (C) $\sqrt[n]{a}.\sqrt[n]{b}$ ✓ (D) $\sqrt{ab} \times n$

64. $\frac{a^m}{a^n} =$ _____ :

(A) $\sqrt[m]{a}$ (B) $\sqrt[n]{a}$ (C) $\left(\sqrt[n]{a}\right)^m$ ✓ (D) $\left(\sqrt[m]{a}\right)^n$

65. $(64)^{-\frac{4}{3}} =$ _____ :

(A) 256 ✓ (B) $(64)^{\frac{1}{3}}$ (C) $\frac{1}{256}$ (D) 512

1.2.2 Surd and their Application

66. Every surd is an _____ number:

(A) Rational (B) Irrational ✓ (C) Real (D) Complex

67. Every _______ number is not a surd.

(A) Real (B) Rational (C) Irrational ✓ (D) Complex

68. Types of surds are:

(A) 2 (B) 3 ✓ (C) 4 (D) 5

69. The product of two conjugate surds is a ________ number:

(A) Real (B) Complex (C) Irrational (D) Rational ✓

70. Every surd is an irrational number but ever irrational number is ______.

(A) a surd (B) not a surd ✓
(C) may or may not be surd (D) must be a surd

1.2.3 Rationalization of Denominator

71. For rationalize a denominator, we ______ both the numerator and denominator by conjugate factor.

(A) Multiply ✓ (B) Division (C) Subtract (D) Add

72. Conjugate of $\sqrt{a} + \sqrt{b}$ is:

(A) $\sqrt{ab}$ (B) $\sqrt{a+b}$ (C) $\sqrt{a} - \sqrt{b}$ ✓ (D) $\sqrt{a-b}$

1.3 Application of Real numbers in Daily Life

73. The boiling point of water Kelvin is:

(A) 310.15 (B) 373.15 ✓ (C) 273.15 (D) 212

74. What is the absolute zero temperature in Celsius?

(A) 0 (B) –273.15 ✓ (C) 273.15 (D) –459.67

75. Formula used to convert Fahrenheit to Celsius is:

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(A) °F= °C + 273.15 (B) K= °C + 273.15 (C) $°C=\frac{5}{9}\left(°F-32\right)$ ✓ (D) $°F=\frac{9}{5}°C+32$

76. Body temperature in Fahrenheit is approximately:

(A) 100 °F (B) 32 °F (C) 98.6 °F ✓ (D) 37 °F

77. Formula used to convert Celsius to Kelvin is ______.

(A) K = F +273 (B) K = C +273 (C) K = C +273.15 ✓ (D) K = C –273

1.3.2 Profit and Loss

78. What is the formula to calculate profit?

(A) Profit = SP + CP (B) Profit = CP – SP

(C) Profit = SP – CP ✓ (D) $\text{Profit}=\frac{SP}{CP}$

79. What is the profit percentage if the cost price is Rs.1000 and Profit is Rs.200?

(A) 10% (B) 20% ✓ (C) 30% (D) 40%

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Unit - 2 LOGARITHMS

Introduction

1. Define logarithms and mention two fields where they are commonly used?

Ans: Logarithms are mathematical tools used to simplify complex calculations, especially involving exponential growth or decay.

Uses: Logarithms are commonly used in:

i) Banking and engineering, and information technology.

ii) Chemistry: The pH scale, which measures the acidity or alkalinity of a solution, is based on logarithms. They help in transforming non-linear data into linear form for analysis, solving exponential equations.

iii) Managing calculations involving very large or small numbers effectively.

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2.1 Scientific Notation

2. Define scientific notation.

Ans: A method used to express very large or very small numbers in a more manageable form is known as scientific notation. It is commonly used in science, engineering and mathematics to simplify complex calculations.

3. Write the general form of a number in scientific notation.

Ans: A number in scientific notation is written as:

$a\times10^{n}$, where $1\leq a\leq10$ and $n\in Z$.

Here "a" is called coefficient or base number.

Remember!

- If the number is greater than 1 then n is positive.
- If the number is less than 1 then n is negative.

2.1.1 Conversion of Number Form

4. How do you convert a number to scientific notation?

Ans: **Step 1:** Move the decimal to get a number between 1 and 10.

Step 2: Count the number of places you moved.

Step 3: Determine the sign of exponential base on the direction.

Try Yourself!

Convert the following into scientific notation.

(i) 29,000,000 **(ii) 0.000006**

Sol:

(i) 29,000,000

Step 1: Move the decimal to get a number between 1 and 10: 2.9

Step 2: Count the number of places you moved the decimal: 7 places left

Step 3: Write in scientific notation: $29,000,000 = 2.9\times10^{7}$

(ii) 0.000006

Step 1: Move the decimal to get a number between 1 and 10: 6.0

Step 2: Count the number of places you moved the decimal: 6 places right

Step 3: Write in scientific notation: $0.000006 = 6.0\times10^{-6}$

2.1.2Conversion of Numbers from Scientific Notation to Ordinary Notation

Remember!

Q. Explain the role of exponent in scientific notation.

Ans: The exponent indicates how many places the decimal point has been moved.

- If the exponential is positive then the decimal point will move to right.
- If the exponential is negative then the decimal point will move to left.

Try Yourself!

Convert the following into scientific notation.

(i) $\mathbf{5.63\times10^{3}}$ **(ii)** $\mathbf{6.6\times10^{-5}}$

Sol:

(i) $\mathbf{5.63\times10^{3}}$

Step 1: Identify the parts:

Coefficient: 5.63 , Exponent: 10^{3}

Step 2: Since, the exponents is positive 3, move the decimal point three places to the right.

$5.63\times10^{3} = 5630$

(ii) $\mathbf{6.6\times10^{-5}}$

Step 1: Identify the parts:

Coefficient: 6.6 , Exponent: 10^{-5}

Step 2: Since, the exponents is negative 5, move the decimal point five places to the left.

$6.6\times10^{-5} = 0.000066$

SOLVED EXERCISE 2.1

1. Express the following numbers in scientific notation:

(i) 2000000

Sol: **Step-1:** Move the decimal to get a number between 1 and 10: 2

Step-2: Count the number of places that moved the decimal: 6 places left

Step-3: Write in scientific notation:

$2000000 = 2\times10^{6}$

(ii) 48900

Sol: **Step-1:** Move the decimal to get a number between 1 and 10: 4.89

Step-2: Count the number of places that moved the decimal: 4 places left

Step-3: Write in scientific notation:

$48900 = 4.89\times10^{4}$

(iii) 0.0042

Sol: **Step-1:** Move the decimal to get a number between 1 and 10: 4.2

Step-2: Count the number of places that moved the decimal: 3 places right

Step-3: Write in scientific notation:

$0.0042 = 4.2\times10^{-3}$

(iv) 0.0000009

Sol: **Step-1:** Move the decimal to get a number between 1 and 10: 9

Step-2:

Count the number of places that moved the decimal: 7 places right

Step-3: Write in scientific notation:

$0.0000009 = 9\times10^{-7}$

(v) $\mathbf{73\times10^{3}}$

Sol: $73\times10^{3} = 73\times1000 = 73000$

Step-1: Move the decimal to get a number between 1 and 10: 7.3

Step-2: Count the number of places that moved the decimal: 4 places left

Step-3: Write in scientific notation:

$73000 = 7.3\times10^{4}$

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Alternate 73 × 10^5

Step-1:
Move the decimal to get a number between 1 and 10: 7.3
Step-2: Count the number of places that moved the decimal: 1 place left
Step-3: Write in scientific notation:
$73 \times 10^3 = 7.3 \times 10^1 \times 10^3$
$= 7.3 \times 10^4$

(vi) 0.65 × 10^2
Sol: Step-1: Move the decimal to get a number between 1 and 10: 6.5
Step-2: Count the number of places that moved the decimal: 1 place right
Step-3: Write in scientific notation:
$0.65 \times 10^2 = 6.5 \times 10^{-1} \times 10^2$
$= 6.5 \times 10^1$

Q.2: Express the following numbers in ordinary notation:

(i) 8.04 × 10^2
Sol: Step-1: Identify the parts:
Coefficient: 8.04
Exponent: 10^2
Step-2: Since the exponent is positive 2, move the decimal point 2 places to the right.
$8.04 \times 10^2 = 804$

(ii) 3 × 10^5
Sol: Step-1: Identify the parts:
Coefficient: 3
Exponent: 10^5
Step-2: Since the exponent is positive 5, move the decimal point 5 places to the right.
$3 \times 10^5 = 300000$

(iii) 1.5 × 10^{-2}
Sol: Step-1: Identify the parts:
Coefficient: 1.5
Exponent: 10^{-2}
Step-2: Since the exponent is negative 2, move the decimal point 2 places to the left.
$1.5 \times 10^{-2} = 0.015$

(iv) 1.77 × 10^7
Sol: Step-1: Identify the parts:
Coefficient: 1.77
Exponent: 10^7
Step-2: Since the exponent is positive 7, move the decimal point 7 places to the right.
$1.77 \times 10^7 = 17700000$

(v) 5.5 × 10^{-6}
Sol: Step-1: Identify the parts:
Coefficient: 5.5
Exponent: 10^{-6}
Step-2: Since the exponent is negative 6, move the decimal point 6 places to the left.
$5.5 \times 10^{-6} = 0.0000055$

(vi) 4 × 10^{-5}
Sol: Step-1: Identify the parts:
Coefficient: 4
Exponent: 10^{-5}
Step-2: Since the exponent is negative 5, move the decimal point 5 places to the left.
$4 \times 10^{-5} = 0.00004$

3. The speed of light is approximately 3 × 10^8 metres per second. Express it in standard form.
Sol: Speed of light = 3×10^8 m/s
For standard form,
Step-1: Identify the parts:
Coefficient: 3
Exponent: 10^8
Step-2: Since the exponent is positive 8, move the decimal point 8 places to the right.
$3 \times 10^8 = 3{,}00{,}000{,}000$ m/s
Hence
speed of light = 3,00,000,000 m/s

4. The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.
Sol: Circumference of Earth = 40075000 m
To convert into scientific notation.
Step-1: Move the decimal to get a number between 1 and 10. 4.0075
Step-2: Count the number of places that moved the decimal: 7 places left
Step-3: Write in scientific notation:

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40075000 m = 4.0075×10^7 m.
Hence, circumference of Earth is 4.0075×10^7 m

5. The diameter of Mars is 6.779×10^3 km. Express this number in standard form.

Sol: Diameter of Mars = 6.779×10^3 km.
To convert it into stand form.
Step-1: Identify the parts:
Coefficient: 6.779
Exponent: 10^3
Step-2: Since the exponent is positive 3, move the decimal point 3 places to the right.
$6.779 \times 10^3 = 6779$
Hence, diameter of Mars is 6779 km

6. The diameter of Earth is 1.2756×10^4 km. Express this number in standard form.

Sol: Diameter of Earth = 1.2756×10^4 km
To convert it into stand form.
Step-1: Identify the parts:
Coefficient: 1.2756
Exponent: 10^4
Step-2: Since the exponent is positive 4, move the decimal point 4 places to the right.
$1.2756 \times 10^4 = 12756$
Hence, diameter of Earth is 12756km.

2.2 Logarithm

1. What is meant by logarithm?
Ans: A logarithm is based on two Greek words: "logos" and "arithmos" which means ratio or proportion.

2. Who introduced the word "logarithm"?
Ans: The word "logarithm" was introduced by a Scottish mathematician, John Napier.

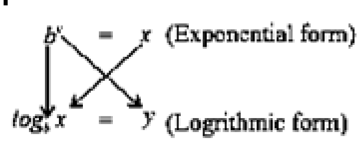
3. Why we use logarithm?
Ans: Logarithm is a way to simplify complex calculations, especially those involving multiplication and division of large numbers. Today, logarithm remain fundamental in mathematics, with applications in science, finance and technology.

2.2.1 Logarithm

4. What is the general form of algorithm?
Ans: The general form of a logarithm is: $log_b(x) = y$
Where b is the base, x is the **result** or the number whose algorithm is being taken.
x is the **exponent** or the logarithm of the x to the base b.
This means: $b^y = x$

5. What does $log_b(x) = y$ mean in words?
Ans: In words, "**the logarithm of the x to the base b is y.**"
This means when b is raised to the power y, it equals x.

6. What is the relationship between logarithmic and exponential forms?
Ans: The relation between logarithm form and exponential form is:
$log_b(x) = y \;\Leftrightarrow\; b^y = x$ where $b>0$, $x>0$ and $b \neq 1$



SOLVED EXERCISE 2.2

1. Express each of the following in logarithmic form:
(i) $10^3 = 1000$

Sol: $10^3 = 1000$
Its logarithmic form is
$\log_{10} 1000 = 3$

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(ii) $2^8 = 256$

Sol: $2^8 = 256$

Its logarithmic form is:

$\log_2 256 = 8$

(iii) $3^{-3} = \frac{1}{27}$

Sol: $3^{-3} = \frac{1}{27}$

Its logarithmic form is:

$\log_3\left(\frac{1}{27}\right) = -3$

(iv) $(20)^2 = 400$

Sol: $(20)^2 = 400$

Its logarithmic form is:

$\log_{20} 400 = 2$

(v) $(16)^{-\frac{1}{4}} = \frac{1}{2}$

Sol: $(16)^{-\frac{1}{4}} = \frac{1}{2}$

Its logarithmic form is:

$\log_{16}\left(\frac{1}{2}\right) = -\frac{1}{4}$

(vi) $(11)^2 = 121$

Sol: $(11)^2 = 121$

Its logarithmic form is:

$\log_{11} 121 = 2$

(vii) $p = q^r$

Sol: $p = q^r \quad \Rightarrow \quad q^r = p$

Its logarithmic form is:

$\log_q p = r$

(viii) $(32)^{-\frac{1}{5}} = \frac{1}{2}$

Sol: $(32)^{-\frac{1}{5}} = \frac{1}{2}$

Its logarithmic form is:

$\log_{32}\left(\frac{1}{2}\right) = -\frac{1}{5}$

2. Express each of the following in exponential form:

(i) $\log_5 125 = 3$

Sol: $\log_5 125 = 3$

Its exponential form is

$5^3 = 125$

(ii) $\log_2 16 = 4$

Sol: $\log_2 16 = 4$

Its exponential form is

$2^4 = 16$

(iii) $\log_{23} 1 = 0$

Sol: $\log_{23} 1 = 0$

Its exponential form is

$23^0 = 1$

(iv) $\log_5 5 = 1$

Sol: $\log_5 5 = 1$

Its exponential form is

$5^1 = 5$

(v) $\log_2\left(\frac{1}{8}\right) = -3$

Sol: $\log_2\left(\frac{1}{8}\right) = -3$

Its exponential form is

$2^{-3} = \frac{1}{8}$

(vi) $\frac{1}{2} = \log_9 3$

Sol: $\log_9 3 = \frac{1}{2}$

Its exponential form is

$9^{\frac{1}{2}} = 3$

(vii) $5 = \log_{10} 100000$

Sol: $\log_{10} 100000 = 5$

Its exponential form is

$10^5 = 100000$

(viii) $\log_4\left(\frac{1}{16}\right) = -2$

Sol: $\log_4\left(\frac{1}{16}\right) = -2$

Its exponential form is

$4^{-2} = \frac{1}{16}$

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3. Find the value of x in each of the following:

(i) $\log_x 64 = 3$

Sol: $\log_x 64 = 3$

Its exponential form is:

$x^3 = 64$

$\Rightarrow x^3 = 4^3 \Rightarrow x = 4$

(ii) $\log_5 1 = x$

Sol: $\log_5 1 = x$

Its exponential form is

$5^{x\ x} = 1$

$\Rightarrow 5^x = 5^0 \Rightarrow x = 0$

(iii) $\log_x 8 = 1$

Sol: $\log_x 8 = 1$

Its exponential form is

$x^1 = 8 \Rightarrow x = 8$

(iv) $\log_{10} x = -3$

Sol: $\log_{10} x = -3$

Its exponential form is

$x = (10)^{-3}$

$\Rightarrow x = \dfrac{1}{10^3} = \dfrac{1}{1000} \Rightarrow x = \dfrac{1}{1000}$

(v) $\log_4 x = \dfrac{3}{2}$

Sol: $\log_4 x = \dfrac{3}{2}$

Its exponential form is

$(4)^{\frac{3}{2}} = x$

$\Rightarrow (2^2)^{\frac{3}{2}} = x \Rightarrow 2^3 = x \Rightarrow x = 8$

(vi) $\log_2 1024 = x$

Sol: $\log_2 1024 = x$

Its exponential form is

$2^x = 1024$

$\Rightarrow 2^x = 2^{10} \Rightarrow x = 10$

2.3 Common Logarithm

1. Define common logarithm.

Ans: The **common logarithm** is the logarithm with a base of 10. It is written as log_{10} or simply as log.

Examples:

$10^1 = 10 \Leftrightarrow \log 10 = 1$

$10^2 = 100 \Leftrightarrow \log 100 = 2$

$10^3 = 1000 \Leftrightarrow \log 1000 = 3$

$10^{-1} = \dfrac{1}{10} \Leftrightarrow \log 0.1 = -1$

$10^{-2} = \dfrac{1}{100} \Leftrightarrow \log 0.01 = -2$

$10^{-3} = \dfrac{1}{1000} \Leftrightarrow \log 0.001 = -3$

Note: When no base is mentioned, it is usually assumed to be base 10.

History

English Mathematician Henry Briggs extended Napier's work and developed the common logarithm. He also introduced logarithmic table.

2.3.1 Characteristics and Mantissa of Logarithms

2. How many parts of logarithm? Write their names.

Ans: Logarithm of a number consists of two parts:

(i) The characteristic (ii) The mantissa

(a) Characteristic

3. Define characteristic.

Ans: The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

4. Write the rules for finding the characteristic of logarithm.

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Ans: Rules for finding the characteristic:

(i) For a number greater than 1:

Characteristic = number of digits to the left of the decimal point –1.

For example, log 567 characteristic = 3–1 = 2

(ii) For a number less than 1:

Characteristic = – (number of zeros between the decimal point and the first non-zero digit +1).

For example,

log 0.0123 the characteristic = -(-1+1) = -2 or $\bar{2}$.

Remember!

Q. When we write a number with bar?

Ans: When the characteristics is negative, we write it be bar.

Characteristic of logarithm

Characteristic of a logarithm of numbers can be find by expressing them in scientific notation.

Number	Scientific Notation	Characteristics of the logarithm
725	7.25×10^{2}	2
9.87	9.87×10^{0}	0
0.00045	4.5×10^{-4}	-4
0.54	5.4×10^{-1}	-1

(a) Mantissa

5. Define Mantissa.

Ans: The mantissa is the decimal part of the logarithm. It represents the “fractional” component and is always positive.

Example: In log 5000 = 3.698, the mantissa is 0.698.

2.3.2 Finding Common Logarithm of a Number

Do you know!

log (0) = undefined	log (1) = 0	log_a (a) = 1

Remember!

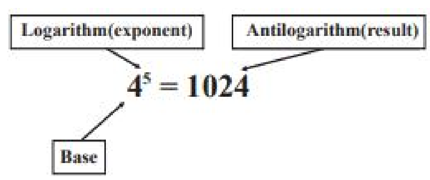
log (Number) = Characteristic + Mantissa

2.3.3 Concept of Anti-Logarithm

6. Define antilogarithm.

Ans: An antilogarithm is the inverse operation of a logarithm. An antilogarithm helps to find a number whose logarithmic value is given.

If $log_b(x) = y \Leftrightarrow b^y = x$ then the process of finding x is called anti-logarithm of y.



7. Write the steps to find the anti-logarithm of a number.

Ans: Finding anti-logarithm of a number using tables:

Step 1: Separate the characteristic and mantissa parts:

Step 2: Find corresponding value of mantissa from anti-logarithm table.

Step 3: Find the mean difference.

Step 4: Add the number found in step 2 and 3.

Step 5: Insert the decimal point.

Remember!

The word antilogarithm is another word for the number or result.

For example, in 4^3 = 64, the result 64 is antilogarithm.

Remember!

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Q. **What is reference position?**
Ans: The place between the first non-zero digit from left and its next digit is called reference position.
Example: In 1332, the reference point is between 1 and 3.

2.3.4 Natural Logarithm

8. Define natural logarithm.
Ans: The natural logarithm is the logarithm is the logarithm with base e, where e is a mathematical constant approximately equal to 2.71828. It is denoted by ln.
Example: $lne^2 = 2$ i.e., the logarithm of e^2 to the base e is 2.

9. Write the uses of natural logarithm.
Ans: The natural logarithm is commonly used in mathematical particularly in calculus, to describe exponential growth, decay and many other natural phenomenon.

Remember!
1. Who introduced the base of natural logarithm?
Ans: Swiss mathematician and physicist Leonhard Euler introduced 'e' for the base of natural logarithm.

10. What is the value of lne?
Ans: The value of $lne = 1$.

11. Write the differences between common and natural logarithms.
Ans:

Common Logarithms	Natural Logarithms
Base: The base of a common logarithm is 10.	**Base:** The base of a common logarithm is e.
Representation: It is written as $log_{10}(x)$ or simply $log(x)$. when no base is specified.	**Representation:** It is written as $ln(x)$.
Uses: Common logarithms are widely used in everyday calculations, especially in scientific engineering applications.	**Uses:** Natural logarithms are commonly used in higher level mathematics particularly calculus and applications involving growth/decay processes.

SOLVED EXERCISE 2.3

1. Find characteristic of the following numbers:
Sol:

Sr.#	Number	Scientific Notation	Characteristic
i)	5287	5.287×10^3	3
ii)	59.28	5.928×10^3	1
iii)	0.0567	5.67×10^{-2}	-2
iv)	234.7	$2.347\times10^{+2}$	2
v)	0.000049	4.9×10^{-5}	-5
vi)	145000	1.45×10^5	5

2. Find logarithm of the following numbers:

(i) 43
Sol: log 43
Characteristic = 2 – 1 = 1
Mantissa = 0.6335
So, log (43) = 1 + 0.6335
= 1.6335

(ii) 579
Sol: log (579)
Characteristic = 3 – 1 = 2
Mantissa = (0.7627)
So, log (579) = 2 + 0.7627
= 2.7627

(iii) 1.982
Sol: log (1.982)
Characteristic = 1 – 1 = 0
Mantissa = (0.2967 + 4)
= 0.2971
So, log (1.982) = 0 + 0.2971
= 0.2971

(iv) 0.0876
Sol: log (0.0876)

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Characteristic = –(1 + 1) = –2
Mantissa = 0.9425
So, log (0.0876) = –2 + 0.9425
= –1.0575

(v) 0.047

Sol: log (0.047)
Characteristic = –(1 + 1) = –2
Mantissa = 0.6721
So, log (0.047) = -2 + 0.6721
= -1.3279

(vi) 0.000354

Sol: log (0.000354)
Characteristic = –(3 + 1) = –4
Mantissa = 0.5490
So, log (0.047) = -4 + 0.5490
= –3.4510

3. If log 3.177 = 0.5019, then find:

(i) log 3177

Sol: log 3177
Characteristic = 4 – 1 = 3
Mantissa = 0.5019
So, log (3177) = 3 + 0.5019
= 3.5019

(ii) log (31.77)

Sol: log 31.77
Characteristic = 2 – 1 = 1
Mantissa = 0.5019
So, log (31.77) = 1 + 0.5019
= 1.5019

(iii) log 0.03177

Sol: log 0.03177
Characteristic = –(1 + 1) = -2
Mantissa = 0.5019
So, log (0.03177) = -2 + 0.5019
= -1.4981

4. Find the value of x.

(i) log x = 0.0065

Sol: log x = 0.0065
Characteristic = 0
Mantissa = 0.0065
Table value of 0.0065 = (1014+1)
= 1015
So, x = antilog (0.0065)
x = 1.015

(ii) log x = 1.192

Sol: log x = 1.192
Characteristic = 1
Mantissa = 0.192
Table value of 0.192 = 1556
So, x = antilog (1.192)
x = 15.56

(iii) log x = –3.434

Sol: log x = –3.434
Since mantissa is negative, so we make it positive by adding and subtracting 4
log x = –4 + 4 – 3.434
$= \bar{4} + 0.566 = \bar{4}.566$
Here, characteristic = $\bar{4}$
Mantissa = 0.566
Table value of 0.566 = 3681
So, x = antilog ($\bar{4}.566$)
x = 0.0003681

(iv) log x = –1.5726

Sol: log x = –1.5726
Since mantissa is negative, so we make it positive by adding and subtracting 2.
log x = –2 + 2 – 1.5726
$= \bar{2} + 0.4274 = \bar{2}.4274$
Here, Characteristic = $\bar{2}$
Mantissa = 0.4274
Table value of 0.4274 = (2673+2)
= 2675
So, x = antilog ($\bar{2}.4274$)
x = 0.002675

(v) log x = 4.3561

Sol: log x = 4.3561
Characteristic = 4
Mantissa = 0.3561
Table value of 0.3561 = (2270+1)
= 2271
So, x = antilog (4.3561)
x = 22710

(vi) log x = –2 .0184

Sol: log x = –2.0184
Since mantissa is negative, so we make it positive by adding and subtracting 3
log x = –3 + 3 – 2.0184

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$= \bar{3} + 0.9816 = \bar{3}.9816$

Here, characteristic $= \bar{3}$

Mantissa $= 0.9816$

Table value of 0.9816 = (9572+13)

$= 9585$

So, x = antilog ($\bar{3}.9816$)

x = 0.009585

2.4 Laws of Logarithm

Laws of logarithm are also known as rules or properties of logarithm. These laws help to simplify logarithmic expressions and solve logarithmic equations.

1. Product Law

1. State the product law of logarithm.

Ans: The logarithm of the product is the sum of the logarithms of the factors. i.e.,

$$\log_b xy = \log_b x + \log_b y$$

2. Prove that $\log_b xy = \log_b x + \log_b y$.

Proof: Let $m = \log_b x$ (i) and $n = \log_b y$ (ii)

Express (i) and (ii) in exponential form:

$$x = b^m \quad \text{and} \quad y = b^n$$

Multiply x and y, we get

$$x.y = b^m.b^n = b^{m+n} \Rightarrow b^{m+n} = xy$$

It logarithmic form is:

$$\log_b xy = m + n$$

$$\log_b xy = \log_b x + \log_b y \quad \text{[From (i) and (ii)]}$$

2. Quotient Law

3. State the quotient law of logarithm.

Ans: The logarithm of the quotient is the difference of the logarithms of the numerator and denominator. i.e., $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

4. Prove that $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$.

Proof: Let $m = \log_b x$ (i) and $n = \log_b y$ (ii)

Express (i) and (ii) in exponential form:

$$x = b^m \quad \text{and} \quad y = b^n$$

Dividing x and y, we get

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n} \Rightarrow b^{m-n} = \frac{x}{y}$$

It logarithmic form is:

$$\log_b\left(\frac{x}{y}\right) = m - n$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \text{[From equation (i) and (ii)]}$$

Activity

- Divide the students into small groups.
- Distribute the logarithmic expression cards randomly among the groups.
- Each group will work together to identify which logarithmic law applies to each expression.
- After completing the task, each group will present its findings.

(Practical work)

3. Power Law

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5. State the power law of logarithm.

Ans: The logarithm of a number raised to a power is the product of the power and the logarithm of the base number. i.e., $\log_b x^n = m.\log_b y$

6. Prove that $\log_b x^n = n.\log_b x$.

Proof: Let $m = \log_b x$ (i)

Its exponential form is:

$$x = b^m$$

Raise both sides to power n

$$x^n = (b^m)^n = b^{mn} \Rightarrow b^{mn} = x^n$$

It logarithmic form is:

$$log_b x^n = nm$$

$$\log_b x^n = m.\log_b y \qquad \text{[From equation (i)]}$$

4. Change of Base Law

7. What is change of base law of logarithm?

Ans: Fourth law of algorithm allows to change the base of a logarithm from "b" to any other base "a". i.e.,

$$\log_b x = \frac{\log_a x}{\log_a b}$$

8. Prove that $\log_b x = \frac{\log_a x}{\log_a b}$.

Ans: Proof: Let $m = \log_b x$ (i)

Its exponential form is:

$$b^m = x$$

Taking log with base "a" on both sides, we get

$$log_a b^m = log_a x \Rightarrow m\, log_a b = log_a x$$

$$m = \frac{\log_a x}{\log_a b} \Rightarrow \log_b x = \frac{\log_a x}{\log_a b} \qquad \text{From equation (i)}$$

2.4.1 Applications of Logarithm

Logarithms have a wide range of applications in many fields.

Do you know!

ln (0) = undefined	ln (1) = 0	ln (e) = 1

SOLVED EXERCISE 2.4

Q.1 Without using calculator evaluate the following:

(i) $\log_2 18 - \log_2 9$

Sol: $\log_2 18 - \log_2 9 = \log_2\left(\frac{18}{9}\right)$ $\quad \because log_b x - log_b y = log_b\left(\frac{x}{y}\right)$

$= \log_2(2) = 1$ $\quad \because log_a a = 1$

(ii) $\log_2 64 + \log_2 2$

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Sol: $\log_2 64 + \log_2 2$ $= \log_2(64\times2)$ $\because \log_b x + \log_b y = \log_b xy$

$= \log_2(128) = \log_2\left(2^7\right) = 7\log_2 2$ $\because \log_b x^n = n\log_b x$

$= 7\,(1) = 7$ $\because \log_a a = 1$

(iii) $\left(\frac{1}{3}\right)\log_3 8 - \log_3 18$

Sol: $\frac{1}{3}\log_3 8 - \log_3 18 = \frac{1}{3}\log_3 2^3 - \log_3\left(2\times3^2\right)$

$= \frac{1}{3}\log_3 2^3 - \left(\log_3 2 + \log_3 3^2\right)$ $\because \log_b x + \log_b y = \log_b xy$

$= \frac{1}{3}\log_3 2^3 - \log_3 2 - \log_3 3^2$

$= \frac{3}{3}\log_3 2 - \log_3 2 - 2\log_3 3$ $\because \log_b x^n = n\log_b x$

$= \log_3 2 - \log_3 2 - 2\log_3 3$

$= -2(1) = -2$ $\because \log_a a = 1$

(iv) 2log 2+ log 25

Sol: $2\log 2 + \log 25 = \log 2^2 + \log 25$ $\because \log_b x^n = n\log_b x$

$= \log 4 + \log 25 = \log(4\times25)$ $\because \log_b x + \log_b y = \log_b xy$

$= \log(100) = \log 10^2$

$= 2\log 10$ $\because \log_b x^n = n\log_b x$

$= 2(1) = 2$ $\because \log_a a = 1$

(v) $\frac{1}{3}\log_4 64 + 2\log_5 25$

Sol: $\frac{1}{3}\log_4 64 + 2\log_5 25 = \frac{1}{3}\log_4 4^3 + 2\log_5 5^2$

$= \frac{3}{3}\log_4 4 + 2(2)\log_5 5$ $\because \log_b x^n = n\log_b x$

$= \log_4 4 + 4\log_5 5$

$= 1 + 4(1) = 1+5 = 5$ $\because \log_a a = 1$

(vi) $\log_3 12 + \log_3 0.25$

Sol: $\log_3 12 + \log_3 0.25 = \log_3(12\times0.25)$ $\because \log_b x + \log_b y = \log_b xy$

$= \log_3(3) = 1$ $\because \log_a a = 1$

2. Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 + 2\log 3$

Sol: $\frac{1}{2}\log 25 + 2\log 3 = \log(25)^{\frac{1}{2}} + \log 3^2$ $\because \log_b x^n = n\log_b x$

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$= \log\left(5^2\right)^{\frac{1}{2}} + \log 3^2$

$= \log 5 + \log 9 = \log(5\times 9)$ $\quad \because \log_b x + \log_b y = \log_b xy$

$= \log 45$

(ii) $\log 9 - \log\frac{1}{3}$

Sol: $\log 9 - \log\left(\frac{1}{3}\right) = \log\left(\frac{9}{\frac{1}{3}}\right)$ $\quad \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

$= \log(9\times 3) = \log 27$

(iii) $\log_5 b^2 . \log_a 5^3$

Sol: $\log_5 b^2 . \log_a 5^3 = \frac{\log_a b^2}{\log_a 5} . \log_a 5^3$ $\quad \because \log_b x = \frac{\log_a x}{\log_a b}$

$= \frac{\log_a b^2}{\log_a 5} . 3\log_a 5 = 3\log_a b^2$

$= 3\left(2\log_a b\right) = 6\log_a b$ $\quad \because \log_b x^n = n\log_b x$

(iv) $2\log_3 x + \log_3 y$

Sol: $2\log_3 x + \log_a y = \log_3 x^2 + \log_3 y$ $\quad \because \log_b x^n = n\log_b x$

$= \log_a\left(x^2 y\right)$ $\quad \because \log_b x + \log_b y = \log_b xy$

(v) $4\log_5 x - \log_5 y + \log_5 z$

Sol: $4\log_5 x - \log_5 y + \log_5 z = \log_5 x^4 - \log_5 y + \log_5 z$ $\quad \because \log_b x^n = n\log_b x$

$= \log_5 x^4 + \log_5 z - \log_5 y$

$= \log_5\left(\frac{x^4 z}{y}\right)$ $\quad \left(\begin{array}{l}\because \log_b x + \log_b y = \log_b xy \\ \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)\end{array}\right)$

(iv) $2\ \ell n\, a + 3\ \ell n\, b - 4\ \ell n\, c$

Sol: $2\ \ell n\, a + 3\ \ell n\, b - 4\ \ell n\, c = \ell n\, a^2 + \ell n\, b^3 - \ell n\, c^4$ $\quad \because \log_b x^n = n\log_b x$

$= \ell n\left(\frac{a^2 b^3}{c^4}\right)$ $\quad \left(\begin{array}{l}\because \log_b x + \log_b y = \log_b xy \\ \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)\end{array}\right)$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$

Sol: $\log\left(\frac{11}{5}\right) = \log 11 - \log 5$ $\quad \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

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(ii) $\log_5 \sqrt{8a^6}$

Sol: $\log_5 \sqrt{8a^6} = \log_5 (8a^6)^{\frac{1}{2}}$ $\quad \because \log_b x^n = n \log_b x$

$= \frac{1}{2}\log_5 (8a^6)$ $\quad \because \log_b x + \log_b y = \log_b xy$

$= \frac{1}{2}\left[\log_5 8 + \log_5 a^6\right] \quad = \frac{1}{2}\left[\log_5 2^3 + \log_5 a^6\right]$

$= \frac{1}{2}\left[3\log_5 2 + 6\log_5 a\right] \quad = \frac{3}{2}\log_5 2 + \frac{6}{2}\log_5 a$ $\quad \because \log_b x^n = n \log_b x$

$= \frac{3}{2}\log_5 2 + 3\log_5 a$

(iii) $\ell n\left(\frac{a^2 b}{c}\right)$

Sol: $\ell n\left(\frac{a^2 b}{c}\right) = \ell n(a^2 b) - \ell n c$ $\quad \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

$= \ell n a^2 + \ell n b - \ell n c = 2\ell n a + \ell n b - \ell n c$ $\quad \because \log_b x^n = n \log_b x$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

Sol: $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}} = \frac{1}{9}\log\left(\frac{xy}{z}\right)$ $\quad \because \log_b x^n = n \log_b x$

$= \frac{1}{9}\left[\log(xy) - \log z\right]$ $\quad \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

$= \frac{1}{9}\left[\log x + \log y - \log z\right]$ $\quad \because \log_b x + \log_b y = \log_b xy$

(v) $\ell n\sqrt[3]{16x^3}$

Sol: $\ell n\sqrt[3]{16x^3} = \ell n(16x^3)^{\frac{1}{3}}$

$= \frac{1}{3}\ell n(16x^3)$ $\quad \because \log_b x^n = n \log_b x$

$= \frac{1}{3}\left[\ell n(2^4 x^3)\right] = \frac{1}{3}\left[\ell n\, 2^4 + \ell n\, x^3\right]$ $\quad \because \log_b x + \log_b y = \log_b xy$

$= \frac{1}{3}\left[4\ell n\, 2 + 3\ell n\, x\right]$ $\quad \because \log_b x^n = n \log_b x$

$= \frac{4}{3}\ell n\, 2 + \frac{3}{3}\ell n\, x = \frac{4}{3}\ell n\, 2 + \ell n\, x$

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(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Sol: $\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2\left(\frac{1-a}{b}\right)$ $\qquad \because \log_b x^n = n\log_b x$

$=5\left[\log_2(1-a)-\log_2 b\right]$ $\qquad \because \log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$

Sol: $\log 2 + \log x = 1 \Rightarrow \log 2 + \log x = \log 10$ $\qquad \because \log 10 = 1$

$\log(2x) = \log 10$ $\qquad \because \log_b x + \log_b y = \log_b xy$

$\Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = \frac{10}{2} = 5$

(ii) $\log_2 x + \log_2 8 = 5$

Sol: $\log_2 x + \log_2 8 = 5 \Rightarrow \log_2(8x) = 5$ $\qquad \because \log_b x + \log_b y = \log_b xy$

In exponential form is

$2^5 = 8x \quad \Rightarrow \quad 32 = 8x$

$x = \frac{32}{8} = 4 \quad \Rightarrow \quad \boxed{x=4}$

(iii) $(81)^x = (243)^{x+2}$

Sol: $(81)^x = (243)^{x+2} \Rightarrow (3^4)^x = (3^5)^{x+2} \Rightarrow 3^{4x} = 3^{5x+10}$

Equating exponents

$4x = 5x + 10 \Rightarrow 4x - 5x = 10 \Rightarrow -x = 10 \Rightarrow$

$\boxed{x=-10}$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

Sol: $\left(\frac{1}{27}\right)^{x-6} = 27 \Rightarrow \left[(27)^{-1}\right]^{x-6} = 27 \Rightarrow (27)^{-x+6} = (27)^1$

Equating exponents

$-x + 6 = 1 \Rightarrow -x = 1 - 6 \Rightarrow -x = -5 \Rightarrow \boxed{x=5}$

(v) $\log(5x-10) = 2$

Sol: $\log(5x-10) = 2$

In exponential form is

$10^2 = 5x - 10 \Rightarrow 100 = 5x - 10$

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$$5x = 100+10=110 \Rightarrow \quad x=\frac{110}{5}=22$$

(vi) $\log_2(x+1)-\log_2(x-4)=2$

Sol: $\log_2(x+1)-\log_2(x-4)=2$

$$\log_2\left(\frac{x+1}{x-4}\right)=2 \qquad \because \log_b x-\log_b y=\log_b\left(\frac{x}{y}\right)$$

In exponential form is

$$\left(\frac{x+1}{x-4}\right)=2^2 \quad \Rightarrow \quad \frac{x+1}{x-4}=4$$

$$4(x-4)=x+1 \quad \Rightarrow \quad 4x-16=x+1$$

$$4x-x=16+1$$

$$17=3x$$

$$x=\frac{17}{3}$$

$$\boxed{x=5\frac{2}{3}}$$

$$\begin{array}{r|r} & 5 \\ \hline 3 & 17 \\ & -15 \\ \hline & 2 \end{array}$$

5. Find the values of the following with the help of logarithm table:

(i) $\dfrac{3.68\times 4.21}{5.234}$

Sol: Let $x=\dfrac{3.68\times 4.21}{5.234}$

Taking log on both sides

$$\log x=\log\left(\frac{3.68\times 4.21}{5.234}\right)$$

Using the laws of logarithm

$$\log x=\log 3.68+\log 4.21-\log 5.234$$

$$\log x=(0+0.5658)+(0+0.6243)-(0+0.7188)=(0.5658)+(0.6243)-(0.7188)$$

$$\log x=0.4713$$

$$x=\text{antilog}(0.4713)$$

$$x=2.960$$

(ii) $4.67\times 2.11\times 2.397$

Sol: Let $x=4.67\times 2.11\times 2.397$

Taking log on both sides

$$\log x=\log(4.67\times 2.11\times 2.397)$$

Using the laws of logarithm

$$\log x=\log 4.67+\log 2.11+\log 2.397$$

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$\log x = (0+0.6693)+(0+0.3243)+(0+(0.3784+13))$

$= 0.6693+0.3243+0.3797$

$\log x = 1.3733$

0.3784
13
0.3797

$x = \text{antilog}(1.3733)$

$x = 23.62$

(iii) $\dfrac{(20.46)^2 \times (2.4122)}{754.3}$

Sol: Let $x = \dfrac{(20.46)^2 \times (2.4122)}{754.3}$

Taking log on both sides

$\log x = \log\left[\dfrac{(20.46)^2 \times 2.4122}{754.3}\right]$

Using the laws of logarithm

$\log x = \log(20.46)^2 + \log(2.4122) - \log(754.3)$

$= 2\log(20.46) + \log(2.4122) - \log(754.3)$

$= 2[1+(0.3096+13)] + [0+(0.3820+4)] - [2+(0.8774+2)]$

$= 2[(1+0.3109)+(0+0.3824)-(2+0.8776)]$

$= 2(1.3109)+(0.3824)-(2.8776) \quad = 2.6218+0.3824-2.8776$

$\log x = 0.1266$

$x = \text{antilog}(0.1266) \quad \Rightarrow \quad x = 1.339$

(iv) $\dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Sol: Let $x = \dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Taking log on both sides

$\log x = \log\left[\dfrac{(9.364)^{\frac{1}{3}} \times 21.64}{3.21}\right]$

Using the laws of logarithm

$\log x = \log(9.364)^{\frac{1}{3}} + \log(21.64) - \log(3.21) = \frac{1}{3}\log(9.364) + \log(21.64) - \log(3.21)$

$= \frac{1}{3}[0+(0.9713+2)] + [1+(0.3345+8)] - [0+(0.5065)]$

$\log x = \frac{1}{3}[(0+0.9715)] + (1+0.3353) - (0+0.5065)$

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$\log x = \frac{1}{3}(0.9715) + 1.3353 - 0.5065 = 0.3238 + 1.3353 - 0.5065$

$\log x = 1.1526$

$x = \text{antilog}(1.1526) \quad \Rightarrow \quad x = 14.21$

6. The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_o}\right)$. If amplitude (A) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

Sol: Magnitude of earth quakes

$= M = \log_{10}\left(\frac{A}{A_o}\right)$

Putting A = 10,000 and A_o = 10

$M = \log_{10}\left(\frac{10000}{10}\right) = \log_{10}(1000)$

$M = \log_{10}(10^3)$

$M = 3\log_{10}10 \qquad (\because \log_b x^n = n\log_b x)$

$M = 3(1) \qquad (\because \log_a a = 1)$

Magnitude of Earth quake = $M = 3$

7. Abdullah invested Rs.100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs. y. This is modelled by an equation $y = 100{,}000(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

Sol: Invested amount = P = Rs.100,000

Rate = r = 5% Per annum , Time = t = years

Formula:

$y = P(1+r)^t$

$y = 100{,}000(1+0.05)^t$

$y = 100{,}000(1.05)^t$

We need to find the time when the investment doubles. For this put

$y = 200{,}000$

$200{,}000 = 100{,}000(1.05)^t \quad \Rightarrow \quad \frac{200{,}000}{100{,}000} = (1.05)^t$

$\Rightarrow \quad 2 = (1.05)^t \quad$ or $\quad (1.05)^t = 2$

Taking log on both side

$t\log(1.05) = \log 2 \qquad (\because \log_b x^n = n\log_b x)$

$t(0 + 0.0212) = 0.3010$

$$t\,(0.0212)=0.3010 \qquad \Rightarrow \qquad t=\frac{0.3010}{0.0212}=14.2$$

Hence, the investment will double in approximately 14-years.

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20ºC. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 metres.

Sol: Temperature = T = ?

Initial temperature = T_1 = 20ºC

Altitude = h = 500m

Temperature (T) decrease by 3% (or a factor of 0.97) for every 100 metres gained in altitude

Formula: $T = T_1 \times (0.97)^{\frac{h}{100}}$

Putting value of h and T_1 in it

$T = 20 \times (0.97)^{\frac{500}{100}} = 20 \times (0.97)^5$

Taking log on both sides

$$\log T = \log\left[20 \times (0.97)^5\right]$$

Using the laws of logarithm

$$\log T = \log 20 + \log(0.97)^5 \qquad \because \log_b x + \log_b y = \log_b xy$$

$$= \log 20 + 5\log(0.97) \qquad \because \log_b x^n = n\log_b x$$

$$=(1+0.3010)+5(-1+0.9868) = 1+3010+5(-0.0132) = 1.3010-0.066$$

$$\log T = 1.235$$

$$T = \text{antilog}(1.235) \qquad \Rightarrow \qquad T = 17.18$$

Hence, the temperature at an altitude of 500 meters is approximately 17.18ºC.

REVIEW EXERCISE 2

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) The standard form of 5.2×10^6 is:

(a) 52,000 (b) 520,000 (c) 5,200,000 ✓ (d) 52,000,000

(ii) Scientific notation of 0.00034 is:

(a) 3.4×10^3 (b) 3.4×10^{-4} ✓ (c) 3.4×10^4 (d) 3.4×10^{-3}

(iii) The base of common logarithm is:

(a) 2 (b) 10 ✓ (c) 5 (d) e

(iv) $\log_2 2^3$ = __________ :

(a) 1 (b) 2 (c) 5 (d) 3 ✓

(v) log 100 = _________:

(a) 2 ✓ (b) 3 (c) 10 (d) 1

(vi) If log 2 = 0.3010, then log 200 is:

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(a) 1.3010 (b) 0.6010 (c) 2.3010 ✓ (d) 2.6010

(vii) log (0) = __________:

(a) positive (b) negative (c) zero (d) undefined ✓

(viii) log 10,000 =:

(a) 2 (b) 3 (c) 4 ✓ (d) 5

(ix) log 5 + log 3 = __________.

(a) log 0 (b) log 2 (c) $\log\left(\frac{5}{3}\right)$ (d) log 15 ✓

(x) $3^4 = 81$ in logarithmic form is:

(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$ (c) $\log_3 81 = 4$ ✓ (d) $\log_4 81 = 3$

2.

Express the following numbers in scientific notation

(i) 0.000567

Sol: Step-1: Move the decimal to get a number between 1 and 10: 5.6

Step-2: Count the number of places that moved the decimal: 4 places right

Step-3: Write in scientific notation:

$0.000567 = 5.67 \times 10^{-4}$

(ii) 734

Sol: Step-1: Move the decimal to get a number between 1 and 10: 7.3

Step-2: Count the number of places that moved the decimal: 2 places left

Step-3: Write in scientific notation:

$734 = 7.34 \times 10^2$

(iii) 0.33×10^3

Sol: Step-1: Move the decimal to get a number between 1 and 10: 3.3

Step-2: Count the number of places that moved the decimal: 1 places right

Step-3: Write in scientific notation:

$0.33 \times 10^3 = 3.3\times10^{-1} \times 10^3$

$= 3.3\times10^{-1+3} = 3.3 \times 10^2$

3. Express the following numbers in ordinary notation:

(i) 2.6×10^3

Sol: Step-1: Identify the parts:

Coefficient: 2.6, **Exponent:** 10^3

Step-2: Since the exponent is positive 3 move the decimal point 3 places to the right.

Step-3: $2.6 \times 10^3 = 2600$

(ii) 8.794×10^{-4}

Sol: Step-1: Identify the parts:

Coefficient: 8.794, **Exponent:** 10^{-4}

Step-2: Since the exponent is negative 4 move the decimal point 4 places to the left.

Step-3: $8.794 \times 10^{-4} = 0.0008794$

(iii) 6×10^{-6}

Sol: Step-1: Identify the parts:

Coefficient: 6 , **Exponent:** 10^{-6}

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Step-2: Since the exponent is negative 6 move the decimal point 6 places to the left.

Step-3: $6 \times 10^{-6} = 0.000006$

4. Express each of the following logarithmic form:

(i) $3^7 = 2187$

Sol: Its logarithm form is: $\log_3 2187 = 7$

(ii) $a^b = c$

Sol: Its logarithm form is: $\log_a c = b$

(iii) $(12)^2 = 144$

Sol: Its logarithm form is: $\log_{12} 144 = 2$

5. Express each of the following exponential form:

(i) $\log_4 8 = x$

Sol: Its exponential form is: $4^x = 8$

(ii) $\log_9 729 = 3$

Sol: Its exponential form is: $9^3 = 729$

(iii) $\log_4 1024 = 5$

Sol: Its exponential form is: $4^5 = 1024$

6. Find value of x in the following:

(i) $\log_9 x = 0.5$

Sol: $\log_9 x = 0.5$

Its exponential form is:

$$(9)^{0.5} = x \quad \Rightarrow \quad x = \sqrt{9} = 3 \quad \Rightarrow \quad \boxed{x = 3}$$

(ii) $\left(\frac{1}{9}\right)^{3x} = 27$

Sol: $$\left(\frac{1}{9}\right)^{3x} = 27 \quad \Rightarrow \quad \left(\frac{1}{3^2}\right)^{3x} = 3^3$$

$$\Rightarrow (3^{-2})^{3x} = 3^3 \quad \Rightarrow \quad 3^{-6x} = 3^3$$

Equating exponent

$$\Rightarrow -6x = 3 \quad \Rightarrow x = \frac{-3}{6} = -\frac{1}{2}$$

$$\Rightarrow \quad \boxed{x = \frac{-1}{2}}$$

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$

Sol: $$\left(\frac{1}{32}\right)^{2x} = 64 \quad \Rightarrow \quad \left(\frac{1}{2^5}\right)^{2x} = 2^6 \quad \Rightarrow (2^{-5})^{2x} = 2^6$$

$$\Rightarrow \quad 2^{-10x} = 2^6$$

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Equating exponent

$\Rightarrow \quad -10x = 6 \qquad \Rightarrow \quad x = \frac{-6}{10} = \frac{-3}{5}$

$\boxed{x = \frac{-3}{5}}$

7. Write the following as a single logarithm:

(i) $7\log x - 3\log y^2$

Sol: $7\log x - 3\log\left(y^2\right)$

Using laws of logarithm

$= \log x^7 - \log\left(y^2\right)^3 = \log x^7 - \log y^6 = \log\left(\frac{x^7}{y^6}\right)$

(ii) $3\log 4 - \log 32$

Sol: $3\log 4 - \log 32$

Using laws of logarithm

$= 3\log\left(2^2\right) - \log 2^5 = 3(2\log 2) - 5\log 2$

$= 6\log 2 - 5\log 2 = (6-5)\log 2$

$= (1)\log 2 = \log 2$

(iii) $\frac{1}{3}\left(\log_5 8 + \log_5 27\right) - \log_5 3$

Sol: $\frac{1}{3}\left[\log_5 8 + \log_5 27\right] - \log_5 3$

Using laws of logarithm

$= \frac{1}{3}\left[\log_5 2^3 + \log_5 3^3\right] - \log_5 3 = \frac{1}{3}\left[3\log_5 2 + 3\log_5 3\right] - \log_5 3$

$= \log_5 2 + \log_5 3 - \log_5 3 = \log_5 2$

8. Expand the following using laws of logarithms:

(i) $\log(x\,y\,z^6)$

Sol: $\log(x\,y\,z^6)$ $\qquad \left(\because \log_b(xy) = \log_b x + \log_b y\right)$

$= \log x + \log y + \log z^6 = \log x + \log y + 6\log z$

(ii) $\log_3 \sqrt[6]{m^5 n^3}$

Sol: $\log_3 \sqrt[6]{m^5 n^3} = \log_3\left(m^5 n^3\right)^{\frac{1}{6}} = \frac{1}{6}\log_3\left(m^5 n^3\right)$ $\qquad \left(\because \log_b x^n = n.\log_b x\right)$

$= \frac{1}{6}\left[\log_3 m^5 + \log_3 n^3\right]$ $\qquad \left(\because \log_b(xy) = \log_b x + \log_b y\right)$

$= \frac{1}{6}\left[5\log_3 m + \log_3 n\right]$ $\qquad \left(\because \log_b x^n = n.\log_b x\right)$

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(iii) $\log\sqrt{8x^3}$

Sol: $\log\sqrt{8x^3} = \log\left(8x^3\right)^{\frac{1}{2}} = \frac{1}{2}\log\left(8x^3\right)$ $\left(\because \log_b x^n = n.\log_b x\right)$

$= \frac{1}{2}\log\left(2^3.x^3\right) = \frac{1}{2}\log(2x)^3 = \frac{3}{2}\log(2x)$

$\log\sqrt{8x^3} = \frac{3}{2}[\log 2 + \log x]$ $\left(\because \log_b(xy) = \log_b x + \log_b y\right)$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$

Sol: Let $x = \sqrt[3]{68.24} = (68.24)^{\frac{1}{3}}$

Taking log on both sides

$\log x = \log(68.24)^{\frac{1}{3}}$

$= \frac{1}{3}\log(68.24) = \frac{1}{3}\left[1 + (0.8338 + 3)\right]$ $\left(\because \log_b x^n = n.\log_b x\right)$

$= \frac{1}{3}[1 + 0.8341] \quad = \frac{1}{3}[1.8341]$

$\log x = 0.6114$

$x = \text{antilog}(0.6114)$

$x = 4.087$

(ii) $\mathbf{319.8 \times 3.543}$

Sol: Let $x = 319.8 \times 3.543$

Taking log on both sides

$\log x = \log(319.8 \times 3.543)$

$\log x = \log(319.8) + \log(3.543)$ $\left(\because \log_b(xy) = \log_b x + \log_b y\right)$

$= \left[(2 + 0.5038 + 11)\right] + \left[0 + (0.5494 + 4)\right] = (2 + 0.5049) + (0 + 0.5494)$

$= 2.5049 + 0.5049$

$\log x = 3.0543$

$x = \text{antilog}(3.0543)$

$x = 1133$

(iii) $\dfrac{\mathbf{36.12 \times 750.9}}{\mathbf{113.2 \times 9.98}}$

Sol: Let $x = \dfrac{36.12 \times 750.9}{113.2 \times 9.98}$

Taking log on both sides

$\log x = \log\left[\dfrac{36.12 \times 750.9}{113.2 \times 9.98}\right]$

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$\log x = \log(36.12) + \log(750.9) - \log(113.2) - \log(9.98) \quad (\because \log_b(xy) = \log_b x + \log_b y)$

$\log x = [1 + (0.5575 + 2)] + [2 + (0.8751 + 5)] - [2 + (0.0531 + 8)] - (0 + 0.9991)$

$\log x = (1 + 0.5577) + (2 + 0.8756) - (2 + 0.0539) - (0 + 0.9991)$

$\log x = 1.5577 + 2.8756 - 2.0539 - 0.9991$

$\log x = 1.3803$

$x = \text{antilog}(1.3803) \quad \Rightarrow \quad x = 24.01$

10. In the year 2016, the population of city was 22 millions and was growing at a rate of 2.5% per year. The function p(t) = 22(1.025)^t gives the population in millions, *t* years after 2016. Use the model to determine in which year the population will reach 35 millions. Round the answer to the nearest year.

Sol: Here $t_o = 2016$

Population = $P_o(t_o) = 22$ Millions

Rate = 2.5% Per year

Give function is

$P_1(t) = 22(1.025)t$ ___(1)

New population $P_1(t_1) = 35$ Millions

put in (1), we get

$$35 = 22(1.025)^{t_1}$$

$$\frac{35}{22} = (1.025)^{t_1}$$

$$1.591 = (1.025)^{t_1}$$

Taking log on both sides

$$\log(1.591) = \log(1.025)^{t_1}$$

$$\log(1.591) = t_1 \log(1.025)$$

$$[0 + (0.2014 + 3)] = t_1[0 + (0.0086 + 21)]$$

$$(0 + 0.2017) = t_1(0 + 0.0107)$$

$$(0.2017) = t_1(0.0107) \quad \Rightarrow \quad 0.2017 = t_1 0.0107$$

$$t_1 = \frac{0.2017}{0.0107}$$

$t_1 = 18.9 \approx 19$ years

Required year: $t_o = t_o + t_1 = 2016 + 19 = 2035$

So, the year when population will be 35 millions is 2035.

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MULTIPLE CHOICE QUESTIONS (MCQs)

Introduction

1. Logarithms are used in which of the following fields?
(A) Banking (B) Engineering (C) Chemistry (D) All of these ✓

2. Which of the following in Not a purpose of logarithms?
(A) Transforming non-linear calculation involving into linear form
(B) Managing calculations involving very large or small numbers
(C) Measuring distance in astronomy ✓ (D) Solving exponential equations

2.1 Scientific Notation

3. What is the correct range for the co-efficient "*a*" in scientific notation?
(A) $1 \le a < 10$ ✓ (B) $1 < a < 10$ (C) $1 \le a \le 100$ (D) $0 \le a < 1$

4. In scientific notation, if the number is greater than 1, the exponent is _____.
(A) Negative (B) Positive ✓ (C) Zero (D) None of these

5. In scientific notation, if the number is less than 1, the exponent is _____.
(A) Negative ✓ (B) Positive (C) Zero (D) None of these

6. What is the scientific notation for the number 78,000,000?
(A) 7.8×10^6 (B) 7.8×10^7 ✓ (C) 7.8×10^8 (D) 7.8×10^{-7}

7. What is the co-efficient in 3.47×10^6?
(A) 6 (B) 3.47 ✓ (C) 10 (D) 10^6

8. A number in scientific notation is written as:
(A) $a \times 10^n$ ✓ (B) $a \times 10^{-n}$ (C) $a - 10^{-n}$ (D) $a + 10^n$

2.1.1 Conversion of Number Form

9. Which of the following number is in proper scientific notation?
(A) 0.42×10^3 (B) 42×10^3 (C) 4.2×10^3 ✓ (D) 0.0042×10^3

2.1.2 Conversion of Numbers from Scientific Notation to Ordinary Notation

10. If the decimal point is moved to the right when converting to scientific notation, the exponent is _____.
(A) Positive ✓ (B) Negative (C) Zero (D) Constant

11. If the decimal point is moved to the left when converting to scientific notation, the exponent is _____.
(A) Positive (B) Negative ✓ (C) Zero (D) Constant

12. Convert the 0.00000623 to scientific notation.
(A) 6.23×10^4 (B) 6.23×10^{-4} (C) 6.23×10^{-6} ✓ (D) 6.23×10^6

13. What is the ordinary notation for 1.77×10^7?
(A) 1.77×10^7 (B) 0.0000000177 (C) 17,700,000 ✓ (D) 1,770,000

2.2 Logarithm

14. What does the word "logarithm" mean?
(A) Ratio or proportion ✓ (B) Power and base
(C) Exponent and division (D) Multiplication and division

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2.2.1 Logarithm of a Real Number

15. The exponent form of $log_3 27 = 3$?
(A) $3^2 = 27$ (B) $3^3 = 27$ ✓ (C) $27^2 = 3$ (D) $3\times3=27$

16. In the logarithm form $log_b x = y$, what does "b" represent?
(A) The result (B) The exponent (C) The base ✓ (D) The logarithm

17. If $log_2 x = 5$, what is the value of x?
(A) 2 (B) 25 (C) 32 ✓ (D) 64

18. Which condition is Not for the logarithmic form $log_b x=y$?
(A) $b>0$ (B) $x>0$ (C) $b=1$ ✓ (D) $b\neq1$

19. If $log_4 x=-2$, then $x=$ _____ :
(A) 8 (B) 16 (C) $\frac{1}{8}$ (D) $\frac{1}{16}$ ✓

20. The relation $x=log_a y$ implies _____:
(A) $a^y = x$ (B) $y^a = x$ (C) $y^x = a$ (D) $a^x = y$ ✓

21. If $a^x = n$ then:
(A) $a = log_x n$ (B) $x = log_n a$ (C) $x = log_a n$ ✓ (D) $x = log_n x$

22. If $y = log_z x$ then _____:
(A) $z^y = x$ ✓ (B) $x^y = z$ (C) $x^z = y$ (D) $y^z = x$

23. The general form of logarithm is:
(A) $log_b x = y$ ✓ (B) $log_b = xy$ (C) $log_{xy} = b$ (D) $log_b y = b$

24. If $3^4 = 81$, then logarithmic form is:
(A) $log_3 4 = 81$ (B) $log_4 3 = 81$ (C) $log_3 81 = 4$ ✓ (D) $log_{81} 3 = 4$

25. If $log_5 25 = x$, then:
(A) $x = 1$ (B) $x = 2$ ✓ (C) $x = 3$ (D) $x = 4$

2.3 Common Logarithm

26. _____ introduced logarithm table?
(A) John Napier (B) Henry Briggs ✓ (C) Euler (D) Khwarizmi

27. _____ of the logarithm of numbers can also be find by expression them in scientific notation.
(A) Characteristics ✓ (B) Mantissa (C) Base (D) Ordinary notation

28. If a number and base of its logarithm are same then answer will be:
(A) 0 (B) −1 (C) 1 ✓ (D) 10

29. For common logarithm the base is:
(A) 1 (B) e (C) 10 ✓ (D) 5

30. $log_a m^n =$ _____:
(A) $nlog_a m$ ✓ (B) $mlog_a n$ (C) $log_a m$ (D) $log_a n$

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2.3.1 Characteristics and Mantissa of Logarithms

31. The logarithm of a number consists of ________pairs:

(A) two ✓ (B) three (C) four (D) five

32. The decimal part of logarithm is:

(A) Characteristic (B) mantissa ✓ (C) real (D) imaginary

33. The integral part of logarithm is known as:

(A) real (B) natural (C) mantissa (D) characteristic ✓

34. In $log_b x = 725$, the characteristic is:

(A) 0 (B) 1 (C) 2 ✓ (D) 3

35. In $\log 0.00045$, the characteristic is:

(A) $\bar{1}$ (B) $\bar{2}$ (C) $\bar{3}$ (D) $\bar{4}$ ✓

2.3.2 Finding Common Logarithm of a Number

36. The logarithm of unity to any base is:

(A) 1 (B) 10 (C) e (D) 0 ✓

37. The logarithm of 345 is:

(A) 1.5378 (B) 2.5738 (C) 2.5738 (D) 3.5738 ✓

2.3.3 Concept of Anti-Logarithm

38. In $\log_x = 0.2568$, the value of x is:

(A) 1.806 (B) 2.806 (C) 3.806 (D) 4.806

39. In $\log_x = -2.1234$ the value of x is:

(A) 0.007526 ✓ (B) 0.07526 (C) 0.7526 (D) 7.526

2.3.4 Natural Logarithm

40. $\log e = ...$ where $e \cong 2.718$:

(A) 0 (B) 0.4343 ✓ (C) ∞ (D) 1

2.4 Laws of Logarithm

41. $\log_b a \times \log_c b$ can be written as:

(A) $\log_a c$ (B) $\log_c a$ ✓ (C) $\log_a b$ (D) $\log_b c$

42. $\log_y x$ will be equal to:

(A) $\dfrac{log_z x}{log_y z}$ (B) $\dfrac{log_x z}{log_y z}$ (C) $\dfrac{log_z x}{log_z y}$ ✓ (D) $\dfrac{log_z y}{log_z x}$

2.4.1 Applications of Logarithm

43. $\log_3 20 =$ ______ :

(A) $2\log_3 2 + \log_3 5$ ✓ (B) $2\log_3 2 + \log_3 2$

(C) $2\log_3 5 + \log_3 2$ (D) $2\log_3 4 + \log_3 5$

44. $2\log_3 10 - \log_3^4 =$ ______ :

(A) $\log_3 20$ (B) $\log_3 25$ ✓ (C) $\log_3 30$ (D) $\log_3 40$

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Unit - 3 Sets and Functions

3.1 Mathematics as the Study of Patterns, Structures and Relationship

Mathematics is the science of patterns, structures and relationships, comprising various branches that explore ad analyze our world's logical and quantitative aspects. The strength of mathematics is based upon relations that enhance the understanding between the patterns and structure and their generalizations.

1. Define Mathematical pattern with example.

Ans: A mathematical pattern is a predictable arrangement of numbers, shapes or symbols that follows a specific rule or relationship. Virtually, patterns are the key to learning structural knowledge involving numerical and geometrical relationships.

Example: Following numerical pattern of the numbers.

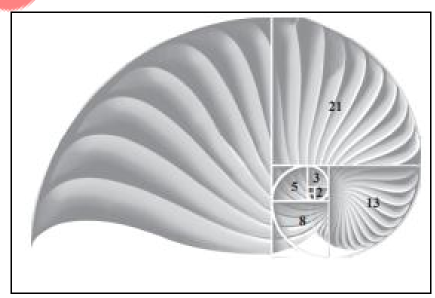
1st term	2nd term	3rd term	4th term	5th term
1	4	7	10	13

$$1 \xrightarrow{+3} 4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13$$

In the above pattern, every term is obtaining by adding 3 in the preceding term. This predictable rule or pattern extends continuously, making it a sequence where each term increases at a constant rate.

2. What is Fibonacci sequence?

Ans: The sequence 0, 1, 1, 2, 3, 5, 8, 13, 21,…., known as the Fibonacci sequence. This sequence starts with two terms, 0 and 1. Each term of the sequence is obtained by adding previous two terms.



3. What is the formula of Fibonacci sequence?

Ans: The formula for the Fibonacci sequence is $F_n = F_{n-1} + F_{n-2}$ where $F_0 = 0$ and $F_1 = 1$ are the first and the second terms respectively. This recursive pattern occurs more frequently in nature.

4. Define mathematical structure.

Ans: A mathematical structure is typically a rule of a numerical, geometric and logical relationship that holds consistency with the domain. A structure is a collection of items or objects, along with particular relationships defined among them.

3.1.1 Basic Definitions

5. How sets helps us?

Ans: The study of sets helps us in understanding the concept of relations, functions and especially in statistics. We use sets to understand probability and other important ideas.

6. Define set.

Ans: A set is described as a well-defined collection of distinct objects, numbers or elements, so that we may be able to decide whether the object belongs to the collection or not.

Q. Who was George Cantor? Write its contribution in mathematics.

Ans: George Cantor (1845-1918) was a German mathematician.

Contribution:

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7. How we represent the sets and elements of sets?

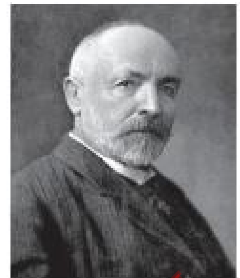
Ans: Capital letters A, B, C, X, Y, Z etc., are generally used as names of sets and small letters a, b, c, x, y, z etc., are used as members or elements of sets.

8. How many different ways of described a set? Write their names.

Ans: There are three different ways to describing a set.

(i) The Descriptive form
(ii) The Tabular form
(iii) Set-builder method

9. Differentiate between descriptive and tabular form.



i) He significantly contributed to the development of set theory, a key area in mathematics.
ii) He showed how to compare sets by matching their members one-to-one.

Q. What did George Cantor proved?

Ans: Cantor defined different types of infinite sets and proves that there are more real numbers than natural numbers. His proof revealed that there are many sizes of infinity. Additionally, he introduced the concepts of cardinal and ordinal numbers, along with their arithmetic operations.

Ans: The Descriptive form: A set may be described in words.

Example: The set of all vowels of English alphabets.

The Tabular form: A set may be described by listing its elements within brackets.

Example: It A is the mentioned above, then by we may write:

$A = \{a, e, i, o, u\}$

Note: The tabular form is also known as Roster form.

10. Define set-builder method.

Ans: Set builder notation is a mathematical way to describe a set by stating the properties of its elements.

Example: Let $A = \{a, e, i, o, u\}$

Its set builder notation is:

$A = \{x | x$ is a vowel of the English alphabets$\}$

This is read as A is the set of all x such that x is a vowel of the English alphabets.

Some Important Sets

N = The set of natural numbers $= \{1, 2, 3, ...\}$

W = The set of whole numbers $= \{0, 1, 2, ...\}$

Z = The set of integers $= \{0, \pm 1, \pm 2, ...\}$

O = The set of odd integers $= \{\pm 1, \pm 3, \pm 5, ...\}$

E = The set of even integers $= \{\pm 2, \pm 4, \pm 6, ...\}$

P = The set of prime numbers $= \{2, 3, 5, 7, 11, 13, 17 ....\}$

Q = The set of all rational numbers $= \left\{x \mid x = \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$

Q' = The set of all irrational numbers $= \left\{x \mid x \neq \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$

R = The set of all real numbers $= Q \cup Q'$

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11. Define singleton set.

Ans: A set with only one element is called singleton set.

Examples: {3}, {a} and {Saturday} are singleton sets.

Remember!

The set $\{0\}$ is a singleton set having zero as its only element, and not the empty set.

12. Define empty set.

Ans: The set with no elements (zero number of elements) is called an empty set, null set, or void set.

Symbolical Representation: The empty set is denoted by the symbol Φ or {}.

13. Differentiate between equal and equivalent sets.

Ans: **Equal sets:** Two sets A or B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A or B are equal, we write $A=B$.

Example: Set {1,2,3} and {2,1,3}

Equivalent sets: Two sets A or B are equivalent if they have the same number of elements.

Example: If $A=\{a,b,c,d,e\}$ and {1,2,3,4,5}, then A or B are equivalents sets. The symbol ~ is used to represent equivalent sets. Thus, we can write $A\sim B$.

14. Define subset.

Ans: **Subset:** If every element is a set of A is an element of set B, then A is a subset of B. Symbolically, this is written as $A \subseteq B$ (A is a subset of B). In such a case, B is a superset of A.

Remember!

The subset of a set can also be stated as follows:

$A \subseteq B$ iff $\forall x \in A \Rightarrow x \in B$

Symbolic Representation: Symbolically, this is written as: $B \supseteq A$ (B is a superset of A)

15. Define proper subset.

Ans: **Proper subset:** If A is a subset of B, and B contains at least one element that is not an element of A, then A is said to be a proper subset of set B. In such a case, we write:

$$A \subset B \qquad (A \text{ is a proper subset of } B)$$

16. Define Improper subset.

Ans: **Improper subset:** If A is a subset of B and $A=B$, then we say that A is an improper subset of B.

From, this definition, it also follows that every set A is a subset of itself and is called an improper subset.

Example: Let set $A=\{a,b,c\}$, $B=\{c,a,b\}$ and $C=\{a,b,c,d\}$, then clearly

$$A \subset C,\ B \subset C \text{ but } A=B$$

Remember!

When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that:

$N \subset W \subset Z \subset Q \subset R$

Note: Each of sets A and B is an improper subset of the other because $A=B$.

17. Define universal set.

Ans: **Universal subset:** The set that contains all objects or elements under consideration is called universal set or the universe of discourse. It is denoted by U.

18. Define power set.

Ans: **Power set:** The power set of a set S denoted by $P(S)$ is the set containing all the possible subsets of S.

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Examples:

(i) If $C = \{a,b,c,d\}$, then $P(C) = \left\{ \begin{array}{l} \phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\}, \\ \{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},\{a,b,c,d\} \end{array} \right\}$

(ii) If $D = \{a\}$, then $P(D) = \{\phi,\{a\}\}$

Note: If S is a finite set with $n(S) = m$ representing the number of elements of the set S, then $n\{P(S)\} = 2^m$ is the number of elements of the power set.

SOLVED EXERCISE 3.1

1. Write the following sets in set builder notation:

(i) $\{1,4,9,16,25,36,\ldots\ldots,484\}$

Sol: $\{x \mid x = n^2, n \in N \wedge 1 \le x < 500\}$

or $\{x \mid x = n^2, n \in N \wedge n \le 22\}$

(ii) $\{2,4,8,16,\ldots\ldots,256\}$

Sol: $\{x \mid x = 2^n, n \in N \wedge 1 \le x \le 256\}$

(iii) $\{0,\pm1,\pm2,\ldots,\pm1000\}$

Sol: $\{x \mid x \in Z \wedge -1000 \le x \le 1000\}$

(iv) $\{6,12,18,\ldots,120\}$

Sol: $\{x \mid x = 6n, n \in N \wedge 1 \le n \le 20\}$

(v) $\{100,102,104,\ldots,400\}$

Sol: $\{x \mid x = 100 + 2n, n \in W \wedge 0 \le n \le 150\}$

(vi) $\{1,3,9,27,81\ldots\}$

Sol: $\{x \mid x = 3^n, n \in W\}$

(vii) $\{1,2,4,5,10,20,25,50,100\}$

Sol: $\{x \mid x$ is a divisor of $100\}$

(viii) $\{5,10,15,\ldots,100\}$

Sol: $\{x \mid x = 5n, n \in N \wedge 1 \le n \le 20\}$

(ix) The set of all integers between – 100 and 1000

Sol: $\{x \mid x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in tabular form:

(i) $\{x \mid x$ ***is a multiple of*** $3 \wedge x < 35\}$

Sol: $\{3,6,9,12,15,18,21,24,27,30,33\}$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

Sol: $\left\{-\frac{1}{2}\right\}$

(iii) $\{x \mid x \in P \wedge x < 12\}$

Sol: $\{2,3,5,7,11\}$

(iv) $\{x \mid x$ ***is a divisor of*** $128\}$

Sol: $\{1,2,4,8,16,32,64,128\}$

(v) $\{x \mid x = 2^n, n \in N \wedge n < 8\}$

Sol: $\{2,4,8,16,32,64,128\}$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

Sol: $\{\ \}$

(vii) $\{x \mid x \in N \wedge x = x\}$

Sol: $\{1,2,3,4,5\ldots\}$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

Sol: $\{\ \}$

3. Write two proper subsets of each of the following sets:

(i) $\{a,b,c\}$

Sol: Let $A = \{a,b,c\}$

Two proper subsets are $\{a\},\{b,c\}$

(ii) $\{0,1\}$

Sol: Let $B = \{0,1\}$

Two proper subsets are $\{0\},\{1\}$

(iii) N

Sol: Let C= N= Set of natural number

$C = \{1,2,3,\ldots\}$

Two proper subsets are $\{1,2\},\{3,4\}$

(iv) Z

Sol: Let D = Z = set of integers

$D = \{0,\pm1,\pm2,\pm3,\ldots\}$

Two proper subsets are

$\{0\},\{\pm2\pm3\}$

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(v) Q

Sol: Let E=Q=Set of rational number.
Two proper subsets are:
Z = set of integers and
$\{x \mid x \in Q \wedge 0 < x < 1\}$

(vi) R

Sol: Let F = R = Set of real numbers.
Two proper subsets are
$$Q = \left\{x \mid x = \frac{p}{q}, \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$$
$$Q' = \left\{x \mid x \neq \frac{p}{q}, \text{where } p, q \in Z \text{ and } q \neq 0\right\}$$
i.e. $Q \subset R$ and $Q' \subset R$

(vii) $\{x \mid x \in Q \wedge 0 < x \le 2\}$

Sol: Let $G = \{x \mid x \in Q \wedge 0 < x \le 2\}$
Two proper subsets are
$\left\{1, \frac{1}{2}, \frac{1}{3}\right\}$ and $\{x \mid x \in Q \wedge 0 < x < 1\}$

Q4. Is there any set which has no proper subset? If so, name that set.

Sol: Yes, the set that has no proper subset is the empty set. i.e ϕ or { }

Q5. What is the difference between {*a*,*b*} and {{*a*,*b*}}?

Sol: {a,b} is a set containing two elements a and b while $\{\{a,b\}\}$ is a set containing one element $\{a,b\}$.

Q6. What is the number of elements of the power set of each of following sets?

(i) { }

Sol: Let A = { }
Number of elements = 2^0 =1 in P(A)

(ii) {0,1}

Sol: Let B = {0,1}
Number of elements in P(B) = 2^2 = 4

(iii) {1,2,3,4,5,6,7}

Sol: Let C = {1,2,3,4,5,6,7}
Number of elements in P(C) =2^7 = 128

(iv) {0,1,2,3,4,5,6,7}

Sol: Let D = {0,1,2,3,4,5,6,7}
Number of elements in P(D)=2^8 =256

(v) {a,{b,c}}

Sol: Let E = {a,{b,c}}
Number of elements in P(E) = 2^2 = 4

(v) {{a,b},{b,c},{d,e}}

Sol: Let F = $\{\{a,b\},\{b,c\},\{d,e\}\}$
Number of elements in P(F) =2^3 = 8

Q7. Write down the power set of each of the following sets:

(i) {9,11}

Sol: Let A = {9,11}
Number of elements in P(A) = 2^2 = 4
$P(A) = \{\phi, \{9\}, \{11\}, \{9,11\}\}$

(ii) {+, –, × , ÷}

Sol: Let B = {+, –, ×, ÷}
Number of elements in P(B)=2^4 = 16
$$P(B) = \left\{\begin{array}{l}\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+,-\}, \{+,\times\}, \\ \{+,\div\}, \{-,\times\}, \{-,\div\}, \{\times,\div\}, \{+,-,\times\}, \\ \{+,-,\div\}, \{+,\times,\div\}, \{-,\times,\div\}, \{+,-,\times,\div\}\end{array}\right\}$$

(iii) $\{\phi\}$

Sol: Let C = $\{\phi\}$
Number of elements in P(C) = 2^1 = 2
P(C) = $\{\phi, \{\phi\}\}$

(iv) {*a*, {*b*,*c*}}

Sol: Let D = {*a*, {*b*,*c*}}
Number of elements in P(D)=2^2 = 4
$P(D) = \{\phi, \{a\}, \{b,c\}, \{a, \{b,c\}\}\}$

3.2 Operations of Two Sets

Just as operations of addition, subtraction etc., are performed on numbers, the operations of union, intersection etc., are performed on sets. We are already familiar with them.

1. Define Union of two sets.

Ans: **Union of two sets:** The union of two sets A or B, denoted by $A \cup B$, is the set of all elements that belong to both A or B.

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Symbolical representation: $A \cup B = \{x \mid x \in A \vee x \in B\}$

Example: If $A = \{1,2,3\}$, $B = \{2,3,4,5\}$ then $A \cup B = \{1,2,3,4,5\}$.

2. Define intersection of two sets.

Ans: Intersection of two sets: The intersection of two sets A or B, denoted by $A \cap B$, is the set of all elements that belong to both A and B.

Remember!
The symbol $\vee$ means or.
The symbol $\wedge$ means and.

Symbolical Representation: $A \cap B = \{x \mid x \in A \wedge x \in B\}$

Example: If $A = \{1,2,3\}$, $B = \{2,3,4,5\}$ then $A \cap B = \{2,3\}$.

3. Define disjoint sets.

Ans: Disjoint sets: If the intersection of two sets is non-empty set, the sets are said to be disjoint.

Examples:

(i) S_1 = The set of odd natural numbers and S_2 = The set of even natural numbers, then S_1 and S_1 are disjoint sets.

(ii) The set of arts students and the set of science students of a college are disjoint sets.

4. Define overlapping of two sets.

Ans: Overlapping sets: If the intersection of two sets is non-empty but neither is a subset of the others, the sets are called overlapping set.

Example: $L = \{2,3,4,5,6\}$ and $M = \{5,6,7,8,9,10\}$, L and M are overlapping sets.

5. Define difference of two sets.

Ans: Difference of two sets: The difference between the sets A and B, denoted by A-B, consists of all the elements that belong to both A but not belong to B.

Symbolical Representation:

$A - B = \{x \mid x \in A \wedge x \notin B\}$ and $B - A = \{x \mid x \in B \wedge x \notin A\}$

Example: If $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8,9,10\}$ then

$A - B = \{1,2,3,4,5\} - \{4,5,6,7,8,9,10\} = \{1,2,3\}$

and $B - A = \{4,5,6,7,8,9,10\} - \{1,2,3,4,5\} = \{6,7,8,9,10\}$

6. Define Complement of a Set.

Ans: Complement of a Set: The complement of a set A, denoted by A' or A^c relative to the universal set U is the set of all elements U, which do not belong to A.

Remember!
In view of the definition of complement and difference sets it is evident that for any set A, $A' = U - A$

Symbolical Representation:

$A' = \{x \mid x \in U \wedge x \notin A\}$

Examples:

(i) If $U = X$ then $E' = O$ and $O' = E$

(ii) If U = Set of alphabets of English language
C = Set of consonants, W = Set of vowels
then $C' = W$ and $W' = C$

3.2.1 Identification of Sets Using Venn Diagram

7. What do you know about Venn diagram?

Ans: Venn diagrams are very useful in depicting visually the basic concepts of sets and relationships between sets. The diagrams were first used by an English logician and mathematician John Venn (1834 to 1883 AD).

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In the adjoining figures, the rectangle represents the universal set U and the shaded circular region represents a of set A and the remaining portion of the rectangle represents the A' or U-A.

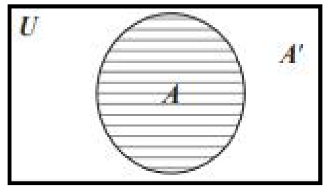
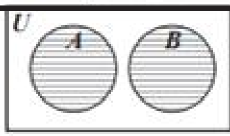
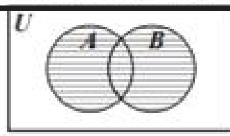
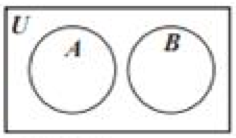
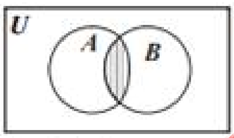
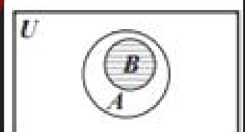
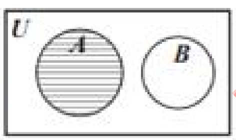
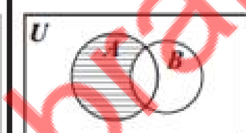
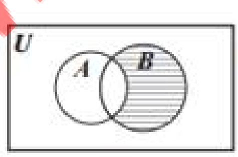
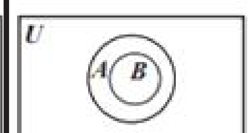


Illustration of Basic operations of Sets:

Below are given some more diagram **illustrating basic operations on two sets** in different cases (the lined region represents the result of the relevant operation in each case show below):

	Disjoint sets	Overlapping sets	$A \subseteq B$	$B \subseteq A$
$A \cup B$	 • $A \cap B = \phi$ • $n(A \cup B) = n(A) + n(B)$	 • $A \cap B \neq \phi$ • $n(A \cup B) = n(A) + n(B) - n(A \cap B)$	 • $A \cup B = B$ • $n(A \cup B) = n(B)$	 • $A \cup B = A$ • $n(A \cup B) = n(A)$
$A \cap B$	 • $A \cap B = \phi$ • $n(A \cap B) = 0$	 • $A \cap B \neq \phi$	 • $A \cap B = A$ • $n(A \cap B) = n(A)$	 • $A \cap B = B$ • $n(A \cap B) = n(B)$
$A - B$	 • $A - B = A$ • $n(A - B) = n(A)$	 • $n(A - B) = n(A) - n(A \cap B)$	 • $A - B = \phi$ • $n(A - B) = 0$	 • $A - B \neq \phi$ • $n(A - B) = n(A) - n(B)$
$B - A$	 • $B - A = B$ • $n(B - A) = n(B)$	 • $n(B - A) = n(B) - n(A \cap B)$	 • $B - A \neq \phi$ • $n(B - A) = n(B) - n(A)$	 • $B - A = \phi$ • $n(B - A) = 0$

3.2.2 Operations on Three Sets

If A, B and C are three given sets, operations of union and intersection can be performed on them in the following ways:

(i)	$A \cup (B \cup C)$	(ii)	$(A \cup B) \cup C$	(iii)	$A \cap (B \cup C)$
(iv)	$(A \cap B) \cap C$	(v)	$A \cup (B \cap C)$	(vi)	$(A \cap C) \cup (B \cap C)$
(vii)	$(A \cup B) \cap C$	(viii)	$A \cap (B \cup C)$	(ix)	$(A \cup C) \cap (B \cup C)$

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3.2.2.1 Properties of union and intersection

(i)	$A \cup B = B \cup A$	(Commutative property of Union)
(ii)	$A \cap B = B \cap A$	Commutative property of Intersection
(iii)	$A \cup (B \cup C) = (A \cup B) \cup C$	Associative property of Union
(iv)	$(A \cap B) \cap C = (A \cap B) \cap C$	Associative property of Intersection
(v)	$A \cup (B \cap C) = (A \cup C) \cap (B \cup C)$	Distributivity of Union over Intersection
(vi)	$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$	Distributivity of Intersection over Union
(vii)	$(A \cup B)' = A' \cap B'$	De Morgan's law
(viii)	$(A \cap B)' = A' \cup B'$	

3.2.3 Real World Applications

8. Define cardinality of a set.

Ans: The cardinality of a set is defined as the total number of elements of a set. The cardinality of a set is basically the size of the set. For a non-empty set A, the cardinality of a set is denoted by $n(A)$.

Example: If $A = \{1, 3, 5, 7, 9, 11\}$ then $n(A) = 6$.

9. What is principle of inclusion and exclusion for two sets.

Ans: To find the cardinality of a set, we use the following rule called the inclusion-exclusion principle for two or three sets.

Principle of Inclusion and Exclusion for Two Sets:

Let A and B be finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

and $A \cup B$ and $A \cap B$ are also finite.

Principle of Inclusion and Exclusion for Three Sets:

If A, B and C are finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

and $A \cup B \cup C$, $A \cap B$, $A \cap C$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$ are also finite.

SOLVED EXERCISE 3.2

Q1. Consider the universal set U = {x: x is multiple of 2 and 0 < x $\leq$ 30}, A = {x: x is multiple of 6} and B = {x: x is multiple of 8}. Find:

(i) List all elements of sets A and B in tabular form (ii) $A \cap B$

(iii) Draw a Venn diagram

Sol: U = {x:x is multiple of 2 and 0 < $x \leq$ 30} = {2, 4, 6, 8, 10, ..., 30}

A = {x:x is multiple of 6} = {6, 12, 18, 24, 30}

B = {x:x is multiple of 8} = {8, 16, 24}

(i) List all elements of sets A and B in tabular form.

A={6,12,18,24,30} and B = {8,16,24}

(ii) Find $A \cap B$

$A \cap B$ = {6,12,18,24,30} $\cap$ {8,16,24}

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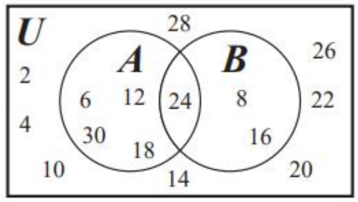
$A \cap B = \{24\}$

(iii) Draw a Venn diagram



Q2. Let, U = {x:x is an integer and $0 < x \leq 150$}, G={x:x =2^m for integer m and $0 \leq m \leq 12$} and H = {x:x is a square}. Find (i) List all elements of sets G and H in tabular form (ii) $G \cup H$ (iii) $G \cap H$.

Sol: Let, U = {x:x is an integer and $0 < x \leq 150$} = {1,2,3,…,150}

G = {x:x =2^m for inter m and $0 \leq m \leq 7$} = {1,2,4,8,16,3,64,128}

H= { x:x is a square} = $\{1,4,9,16,25,36,49,64,81,100,121,144\}$

(i) List all elements of sets G and H in tabular form.

$G = \{1,2,4,8,16,32,64,128\}$

$H = \{1,4,9,16,25,32,36,49,64,81,100,121,128,144\}$

(ii) $G \cup H$

$G \cup H = \{1,2,4,8,16,32,64,128\} \cup \{1,4,9,16,25,32,36,49,64,81,100,121,128,144\}$

$G \cup H = \{1,2,4,8,9,16,25,36,49,64,81,100,121,128,144\}$

(iii) $G \cap H$

$G \cap H = \{1,2,4,8,16,32,64,128\} \cap \{1,4,9,16,25,36,49,64,81,100,121,144\}$

$G \cap H = \{1,4,16,64\}$

Q3: Consider the sets P= {x:x is a prime number and $0 < x \leq 20$} and Q = {x:x is a divisor of 210 and $0 < x \leq 20$}. Find $P \cap Q$ (ii) $P \cup Q$.

Sol: P = { x:x is a prime number and $0 < x \leq 20$}$= \{2,3,5,7,11,13,17,19\}$

Q = {x:x is a divisor of 210 and $0 < x \leq 20$} $= \{1,2,3,5,6,7,10,14,15\}$

(i) $P \cap Q$

$P \cap Q = \{2,3,5,7,11,13,17,19\} \cap \{1,2,3,5,6,7,10,14,15\}$

$P \cap Q = \{2,3,5,7\}$

(ii) $P \cup Q$

$P \cup Q = \{2,3,5,7,11,13,17,19\} \cup \{1,2,3,5,6,7,10,14,15\}$

$P \cup Q = \{1,2,3,5,6,7,10,11,13,14,15,17,19\}$

Q4. Verify the commutative properties of union and intersection for the following pairs of sets:

(i) A = {1,2,3,4,5}, B = {4,6,8,10} (ii) N,Z (iii) A = {$x \mid x \in$ R $\wedge x \geq 0$},B = R.

Sol:

(i) A = {1,2,3,4,5}, B = {4,6,8,10}

a) Commutative property of union:

$A \cup B = B \cup A$

$LHS = A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\} = \{1,2,3,4,5,6,8,10\}$ (i)

$RHS = B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,8,10\}$ (ii)

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From (i) and (ii)

LHS = RHS

$A \cup B = B \cup A$

b) Commutative property of intersection:

$A \cap B = B \cap A$

$LHS = A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\} = \{4\}$ (i)

$RHS = B \cap A = \{4,6,8,10\} \cap \{1,2,3,4,5\} = \{4\}$ (ii)

From (i) and (ii) LHS = RHS

$A \cap B = B \cap A$

(ii) N,Z

Sol: Let, $A = N = \{1,2,3,....\}$ and $B = Z = \{0,\pm 1,\pm 2,\pm 3,....\}$

a) Commutative property of union:

$A \cup B = B \cup A$

$LHS = A \cup B = \{1,2,3...\} \cup \{0,\pm,1,\pm 2,\pm,3...,\} = \{0,\pm,1,\pm 2,\pm,3,...\} = Z$ (i)

$RHS = B \cup A = \{0,\pm 1,\pm 2,\pm 3,....\} \cup \{1,2,3,....\} = \{0,\pm 1,\pm 2,\pm 3,....\} = Z$ (ii)

Form (i) and (ii)

LHS = RHS

$A \cup B = B \cup A$

b) Commutative property of intersection:

$A \cap B = B \cap A$

$LHS = A \cap B = \{1,2,3,....\} \cup \{0,\pm 1,\pm 2,\pm 3,....\} = \{1,2,3,....\} = N$ (i)

$RHS = B \cap A = \{0,\pm 1,\pm 2,\pm 3,....\} \cup \{1,2,3,....\} = \{1,2,3,....\} = N$ (ii)

Form (i) and (ii) LHS = RHS

$A \cap B = B \cap A$

(iii) $\mathbf{A} = \{x \mid x \in \mathbf{R} \wedge x \geq 0\}, \mathbf{B} = \mathbf{R}.$

Sol: A = $\mathbb{R}^+$ = set of all positive real numbers and

B = $\mathbb{R}$ = set of all real numbers

Here $\mathbb{R}^+ \subset \mathbb{R}$

a) Commutative property of union:

$A \cup B = B \cup A$

$LHS = A \cup B = \mathbb{R}^+ \cup \mathbb{R} = \mathbb{R}$ = set of all real numbers (i)

$RHS = B \cup A = \mathbb{R} \cup \mathbb{R}^+ = \mathbb{R}$ = set of all real numbers (ii)

$\therefore$ LHS=RHS

$A \cup B = B \cup A$

b) Commutative property of intersection.

$A \cap B = B \cap A$

$LHS = A \cap B = \mathbb{R}^+ \cap \mathbb{R} = \mathbb{R}^+$ = Set of all positive real numbers ($\because \mathbb{R}^+ \subset \mathbb{R}$)

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$RHS = B \cap A = \mathbb{R}^+ \cap \mathbb{R} = \mathbb{R}^+$ = Set of all positive real numbers $(\because \mathbb{R}^+ \subset \mathbb{R})$

$\therefore$ LHR = RHS

$A \cap B = B \cap A$

Q5: Let U = {a,b,c,d,e,f,g,h,i,j}, A = {a,b,c,d,g,h}, B={c,d,e,f,j}
Verify De Morgan's Laws for these sets. Draw Veen diagram.

Sol: Let U = $\{a,b,c,d,e,f,g,h,i,j\}$, A = $\{a,b,c,d,g,h\}$, B = $\{c,d,e,f,j\}$

<u>De Morgan's Laws:</u>

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(i) **$(A \cup B)' = A' \cap B'$**

LHS = $(A \cup B)'$

$(A \cup B) = \{a,b,c,d,g,h\} \cup \{c,d,e,f,j\} = \{a,b,c,d,e,f,g,h,j\}$

$(A \cup B)' = U - (A \cup B) = \{a,b,c,d,e,f,g,h,i,j\} - \{a,b,c,d,e,f,g,h,j\} = \{\ \}$ ...(i)

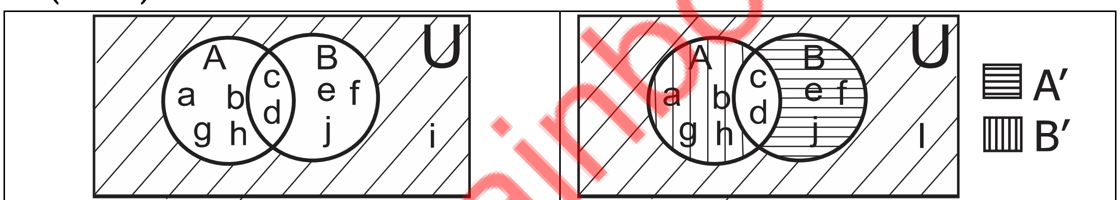
RHS = $A' \cap B'$

$A' = U - A = \{a,b,c,d,e,f,g,h,i,j\} - \{a,b,c,d,g,h\} = \{e,f,i,j\}$

$A' \cap B' = \{e,f,i,j\} \cap \{a,b,g,h,i\} = \{i\}$ (ii)

From (i) and (ii)

$(A \cup B)' = A' \cap B'$



$(A \cup B)'$ $A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Sol: LHS = $(A \cap B)'$

$(A \cap B) = \{a,b,c,d,g,h\} \cap \{c,d,e,f,j\} = \{c,d\}$

$(A \cap B)' = U - (A \cap B) = \{a,b,c,d,e,f,g,h,i,j\} - \{c,d\} = \{a,b,e,f,g,h,i,j\}$ (i)

RHS $= A' \cup B'$

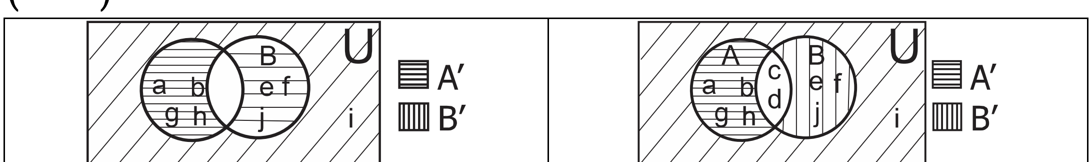
$A' = U - A = \{a,b,e,f,g,h,i,j\} - \{a,b,c,d,g,h\} = \{e,f,i,j\}$

$B' = U - B = \{a,b,c,d,e,f,g,h,i,j\} - \{c,d,e,f,j\} = \{a,b,g,h,i\}$

$A' \cup B' = \{e,f,i,j\} \cup \{a,b,g,h,i\} = \{a,b,e,f,g,h,i,j\}$ (ii)

From (i) and (ii)

$(A \cap B)' = A' \cup B'$



$(A \cap B)'$ $A' \cup B'$

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Q6: If U = {1,2,3,….20} and A = {1,3,5,…, 19}, verify the following:

Sol: U = {1,2,3,….20}, A ={1,3,5,….19}

(i) AUA′ = U

LHS = AUA′

A′ = U – A = {1,2,3….,20} – {1,3,5,….19} = {2,4,6,8,10,12,14,16,18,20}

AUA = {1,3,5,….,19} U {2,4,6,8,10,12,14,16,18,20} = {1,2,3,4,5,6,….20}

= U = RHS

ii) $A \cap U = A$

Sol: L.H.S = $A \cap U$ = {1,3,5,7,9,11,13,15,17,19} $\cap$ {1,2,3,4,...20}

={ 1,3,5,7,9,11,13,15,17,19} = A = RHS

iii) $A \cap A' = \phi$

Sol: L.H.S = $A \cap A'$

A′ = U – A = {1,2,3,….,20} – {1,3,5,….,19} = {2,4,6,8,10,12,14,16,18,20}

$A \cap A'$ {1,3,5,….,19} $\cap$ {2,4,6,8,……20} = ϕ = R.H.S

Q7: In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

Sol: Let U = {Total student in a class}

A = {Students who like to play cricket}

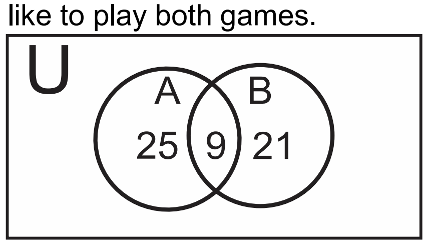
B = {Students who like to play hockey}

From the statement of problems we have

$n(U) = n(A \cup B) = 55$, $n(A)=34$, $n(B) = 30$

We want to find the total number of students who like to play both games.

$n(A \cap B) = ?$



Using the principles of inclusion and exclusion for two sets:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 34 + 30 - 55 = 9$

Thus, 9 Students like to play both games.

Q8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

Sol: Let U = {Total number of employees in a group}

U = {Employees who speak Urdu} , E = {Employees who speak English}

P = {Employees who speak Punjabi}

From the statement of problems, we have

$\Rightarrow$ $n(U) = 500$, $n(u) = 250$, $n(E) = 150$, $n(P) = 50$, $n(U \cap E) = 40$, $n(E \cap P) = 30$

$n(U \cap P) = 10$

We want to find the total numbers of employees who speak all the three languages.

$n(U \cap E \cap P) = ?$

Using the principle of inclusion and exclusion for three sets.

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$n(U \cup E \cup P) = n(U) + n(E) + n(P) - n(U \cap E) - n(E \cap P) - n(U \cap P)$

$$\Rightarrow \quad n(U \cap E \cap P) = n(U \cup E \cup P) - n(U) - n(E) - n(P) + n(U \cap E) + n(E \cap P) + n(U \cap P)$$

$= 500 - 250 - 150 - 150 + 40 + 30 + 10 = 500 - 450 + 80 = 580 - 450 = 130$

Therefore 130 Employees speak all the three languages.

Q9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap a and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Sol: Let U = {Total numbers of people is sport event}, B = {people who wear blue shirts}
G = {People who wear green shirts}, C = {People who wear caps}

From the statement of problems, we have

$n(U) = n(B \cup G \cup C) = 34$, $n(B) = 19$, $n(G) = 15$,

$n(B \cap G) = 3$, $n(C \cap B) = 4$, $n(C \cap G) = 2$, $n(B \cap G \cap C) = 0$

We want to find the number of people who wearing caps i.e n(C) = ?

Using the principle of inclusion and exclusion for three sets:

$n(B \cup G \cup C) = n(B) + n(G) + n(C) - n(B \cap G) - n(G \cap C) - n(B \cap C) + n(B \cap G \cap C)$

$34 = 19 + 15 + n(C) - 3 - 2 - 4 + 0$

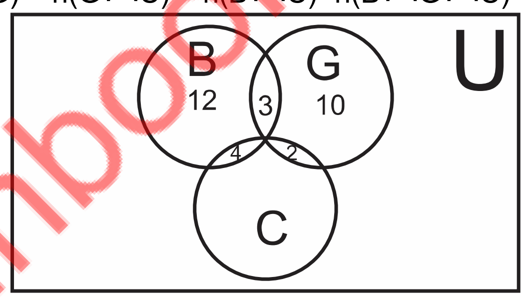
$34 = 34 + n(C) - 9$

$34 = 25 + n(C)$

$34 - 25 = n(C)$

$\Rightarrow \quad n(C) = 9$

Thus, 9 people are wearing caps.



Q10. In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Sol: Let

U = {Total number of participants in a training session}

L = {Participants having laptops},

T = {Participants having tablets}

B = {Participants having books}

From the statement of problems, we have

$n(U) = n(L \cup T \cup B) = 35$, $n(L) = 17$, $n(T) = 11$, $n(L \cap T) = 9$

$n(T \cap B) = 4$, $n(L \cap B) = 6$, $n(L \cap T \cap B) = 8$

We want to find the total number of participants who have books. We are to find n(B) using the principle of inclusion and exclusion for three sets.

$n(L \cup T \cup B) = n(L) + n(T) + n(B) - n(L \cap T) - n(T \cap B) - n(L \cap B) + n(L \cap T \cap B)$

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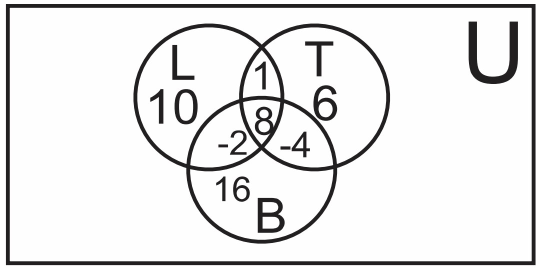
$35 = 17 + 11 + n(B) - 9 - 4 - 6 + 8$
$35 = 36 - 19 + n(B)$
$35 = 17 + n(B)$
$35 - 17 = n(B)$
$\Rightarrow \quad n(B) = 18$

Thus, 18 participants have books.



Q11. A shopping mall has 150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:
Set A: 40 employees with a salary range of 30 k–45 k, labelled from 50 to 89.
Set B: employees with a salary range of 50 k–80 k, labelled from 101 to a 150.
Set C: 60 employees with a salary range of 100k–150k, labelled from 1 to 49 and 90 to 100.

(a) Find $(A' \cup B') \cup C$ (b) Find $n\{A \cap (B' \cap C')\}$

Sol: Let

$U = \{1,2,3,\ldots,150\}$ $\Rightarrow n(U)=150$,
$A = \{50,51,\ldots,89\}$ $\Rightarrow n(A)=40$
$B = \{101,102,\ldots,150\}$ $\Rightarrow n(B)=50$,
$C = \{1,2,3,\ldots,49,90,91,\ldots,100\}$ $\Rightarrow n(C)=60$

a) $A' = U - A = \{1,2,3,\ldots150\} - \{50,51,\ldots89\} = \{1,2,3,\ldots,49,90,91,\ldots,150\}$

$B' = U - B = \{1,2,3,\ldots,150\} - \{101,102,\ldots,150\} = \{1,2,3,\ldots,100\}$

$(A' \cup B') \cap C$

$= (\{1,2,3,\ldots,49,90,91,150\} \cup \{1,2,3,\ldots,100\}) \cap \{1,2,3,\ldots,49,90,91\ldots,100\}$
$= \{1,2,3,\ldots,100,101,\ldots,150\} \cap \{1,2,3,\ldots,49,90,91,\ldots,100\}$
$= \{1,2,3,\ldots,49,90,91,\ldots,100\}$

b) $B' = U - B = \{1,2,3,\ldots,150\} - \{101,102,\ldots,150\} = \{1,2,3,\ldots,100\}$

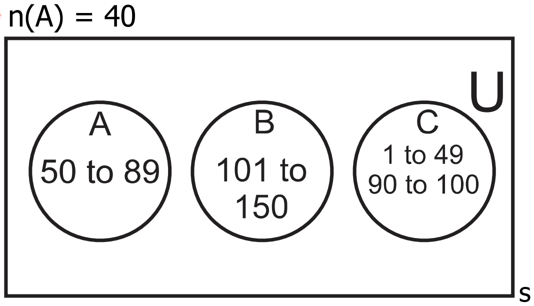
$C' = U - C = \{1,2,3,\ldots,150\} - \{1,2,3,4\ldots,49,90,91,\ldots100\}$

$C' = \{50,51,\ldots89,101,102,\ldots150\}$

$B' \cap C' = \{1,2,3,\ldots,100\} \cap \{50,51,\ldots,89,101,102,\ldots,150\} = \{50,51,\ldots,89\}$

Now $A' \cap (B' \cap C') = \{50,51,\ldots,89\} \cap \{50,51,\ldots,89\} = \{50,51,\ldots,89\} = A$

$n\{A \cap (B' \cap C')\} = n(A) = 40$



Since, A,B and C are disjoint sets so $B' \cap C' = (B \cup C)' = A$

$A \cap (B' \cap C') = A \cap (B \cup C)' = A \cap A = A$

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Q12. In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.
60 Students play cricket, 70 Students play football.
40 Students play hockey, 25 Students play both cricket and football.
15 Students play both football and hockey, 10 play both cricket and hockey.

a) How many students play all three sports?

b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Sol: Let U = {Total number of students participate in sports}
C = {Students who play cricket}
F = {Students who play football}
H = {Students who play hockey}

Form the statement of problems, we have
$n(U) = n(C \cup F \cup H) = 125$, $n(C) = 60$, $n(F) = 70$
$n(H) = 40$, $n(C \cap F) = 25$, $n(F \cap H) = 15$, $n(C \cap H) = 10$

a) We went to find the total number of students who play all the three sports.
We are to find $n(C \cap F \cap H)$
Using the principle of inclusion and exclusion for three sets.

$n(C \cup F \cup H) = n(C)+n(F)+n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$

$125 = 60+70+40 - 25 - 15 - 10 + n(C \cap F \cap H)$

$125 = 170 - 50 + n(C \cap F \cap H)$

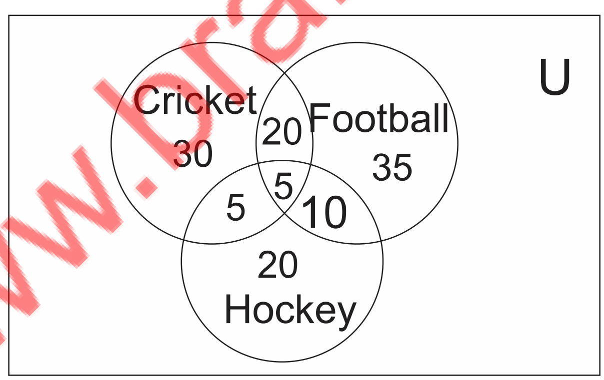
$125 = 120 + n(C \cap F \cap H)$

$125 - 120 = n(C \cap F \cap H)$

$\Rightarrow$ $n(C \cap F \cap H) = 5$

Therefore 5 students play all three sports.

b)



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Q13. A survey was conducted in which 130 people were asked about their favourite foods. They survey results showed the following information:
40 people said they liked nihari. 65 people said they liked biryani.
50 people said they liked korma. 20 people said they liked nihari and biryani.
35 people said they liked biryani and korma.
27 people said they liked nihari and korma
12 people said they liked all three foods nihari, biryani, and korma.

a) At least how many people like nihari, biryani or korma?

b) How many people did not like nihari, biryani, or korma?

c) How many people like only one of the following foods: nihari, biryani, or korma?

d) Draw a Venn diagram.

Sol: Let U = {Total number of people in survey}, N = {People who like nihari}
B = {People who like biryani} , K = { People who like korma}

From the statement of problems, we have

n(U) =130

n(N) = 40, n(B) = 65, n(K) = 50

n(N $\cap$B) =20, n(B $\cap$K)=35, n(N $\cap$K)=27, n(N $\cap$B $\cap$K) =12

a) We want to find the total number of people like at least one of the foods: nihari, biryani or korma.
Using the principle of inclusion and exclusion for three sets. i.e n(N $\cup$B $\cup$K)

n(N $\cup$B $\cup$K) = n(N)+n(B)+n(K) – n(N $\cap$B) – n(B $\cap$K) – n(N $\cap$K)+ n(N $\cap$B $\cap$K)
= 40+65+50 – 20 – 35 – 27+12 = 85

b) We want to find the number of people who did not like nihari, biryani or korma.
i.e n(N $\cup$B $\cup$K)'
n(N $\cup$B $\cup$K)' =n (U) – n(N $\cup$B $\cup$K) =130-85 = 45

c) We want to find the number of people who like only nihari, biryani or korma.

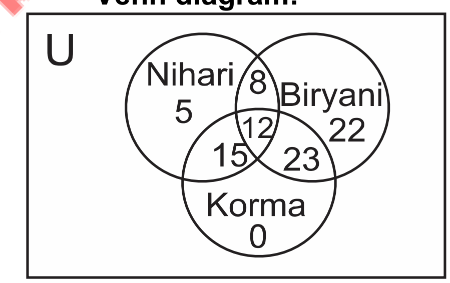
People who like only nihari = n(N) – n(N $\cap$B) – n(N $\cap$K)+n(N $\cap$B $\cap$K)
= 40 – 20 – 27+12 = 5

People who like only biryani
= n(B) – n(N $\cap$B) – n(B $\cap$K) + n(N $\cap$B $\cap$K)
= 65 – 20 – 35 + 12 = 22

People who like only korma
= n(K) – n(B $\cap$K) – n(N $\cap$K)+n(N $\cap$B $\cap$K)
= 50 – 35 – 27+12 = 0

(d) Therefore the total number of people only like nihari, biryani or Korma=5+22+0=27

Venn diagram:



3.3 Binary Relations

1. What is meant by Cartesian product?

Ans: Let A and B be two non-empty sets, then the Cartesian product is the set of all ordered pairs (x,y) such that $x \in A$ and $y \in B$ and is denoted by $A \times B$. Symbolically, we can write it as $A \times B = \{(x,y) \mid x \in A \, and \, y \in B\}$.

2. Define binary relation and give an example.

Ans: Any subset $A \times B$ is called a binary relation, or simply relation, from A to B. Ordinarily, a relation will be denoted by the letter r.

Example: Let c_1, c_2, c_3 be the children and m_1 , m_2 be a such that the father of both c_1, c_2 is m_1 and father c_3 is m_2. Find the relation {(child, father)}.

C = Set of children = $\{c_1, c_2, c_3\}$ and F = Set of Father = $\{m_1, m_2\}$

The Cartesian product C and F

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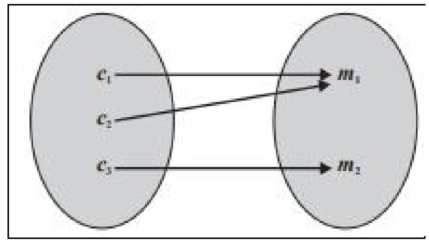
$C \times F = \{(c_1,m_1),(c_1,m_2),(c_2,m_1),(c_2,m_2),(c_3,m_1),(c_3,m_2)\}$

r = set of ordered pairs (child, father)

$r = \{(c_1,m_1), (c_2,m_1), (c_3,m_2)\}$

Domain $r = \{c_1, c_2, c_3\}$, Range $r = \{m_1, m_2\}$

The relation is shown diagrammatically in adjacent figure.



3. Define Domain and Range of a function.

Ans: Domain: The set of the first elements of the ordered pairs forming a relation is called its domain. The domain of any relation r is denoted by Dom r.

Range: The set of the second elements of the ordered pairs forming a relation is called its range. The range of any relation r is denoted by Ran r.

Note: If A is a non-empty set, any subsets of $A \times A$ is called relation in A.

3.3.1 Relation as Table, Ordered Pair and Graph

4. Define abscissa and ordinate of ordered pair.

Ans: Each ordered pair consists of two coordinates, x and y. The x coordinates is called abscissa and y is ordinate, often representing an input and an output.

5. Define ordered pairs.

Ans: Any two numbers can x and y can be written in the form (x,y) is called order pair. A relation can be represented by a set of ordered pairs.

Example: Consider a water tank that starts with 1 litre of water already inside. Each minute, 1 additional litre of water is added to the tank. The situation can be represented by the relation r = $\{(x,y) | y = x+1\}$. where x is the number of minutes (time) that have passed since the filling started and y is the total amount of water (in litres) in the tank. When $x = 0$, $y = 1$ and $x = 1$, $y = 2$ In order pair this relation is represented as: {(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} The above relation in table form can be represented as given below:

Table

y (time in minutes)	$y = x + 1$ (water in litres)
0	y = 0+1 = 1
1	y = 1+1 = 2
2	y = 2+1 = 3
3	y = 3+1 = 4
4	y = 4+1 = 5
5	y = 5+1 = 6

6. Define graph with example.

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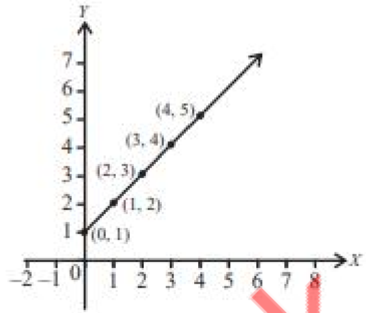
Ans: We can represent the relation visually by drawing a graph. To draw a graph, we use ordered pairs. Each ordered pair (x,y) is plotted as a point in the coordinate plane, where x is the first element and y is the second element.

Example: The relation is represented graphically the line passing through the points, $\{(0,1),(1,2),(2,3),(3,4),(4,5),(5,6)\}$ as shown in the adjacent figure.



3.3.2 Function and its Domain and Range

7. Define a function Give an example.

Ans: Let A and B be two non-empty sets such that:

i) f is a relation from A to B, that is, f is a subset of $A\times B$.

ii) Domain $f = A$

iii) First elements of no two pairs of f are equal, then f is said to be a function from A and B.

The function f is also written as: $f : A \to B$

8. Define domain set and range set.

Ans: The set of all first elements of each ordered pair represents the domain f, and all the second elements represent the range of f.

Here, the domain of f is A, and range of f is B.

Note: If (x, y) is an element of f when regarded as a set of ordered pairs.

We write $y = f(x)$. y is called the value of f for x or the image of x under f.

Types of functions

9. How many types of functions?

Ans: There are four types of functions.

(i) Into function

(ii) One – one function (or Injective function)

(iii) Surjective / Onto function

(iv) Onoe – one and onto / Bijective function

10. What is meant by into function?

Ans: **Into function:** If a function $f : A \to B$ is such that Range $f \subset B$ i.e., Range $f \neq B$, then f is said to be a function from A into B. In figure (i), f is clearly a function. But Range $f \neq B$. Therefore, f is a function from A into B.

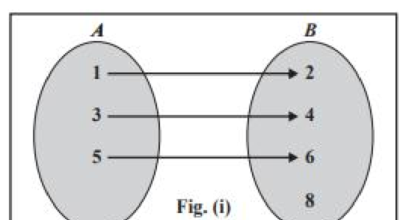


Fig. (i)

$f = \{(1, 2), (3, 4), (5, 6)\}$

11. Define one-one function and give an example.

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Ans: One – one function or Injective function:

If a function f from A into B is such that second elements of no two of its ordered pairs are same, then it is called injective function; the function shown in figure (ii) one - one a function.

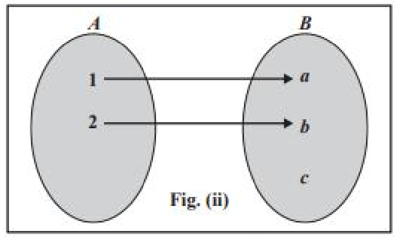

Fig. (ii)

$f = \{(1, a), (2, b)\}$

12. Define Surjective / Onto function and give one example.

Ans: Onto Function: If a function $f : A \to B$ is such that Range $f = B$ i.e., every element of B is the image of some element A, then f is called onto function or a surjective function.

Example:

If $A = \{c_1, c_2, c_3\}, B = \{m_1, m_2\}$ and $f : A \to B$ such that $f = \{(c_1, m_1), (c_1, m_2), (c_3, m_2)\}$.

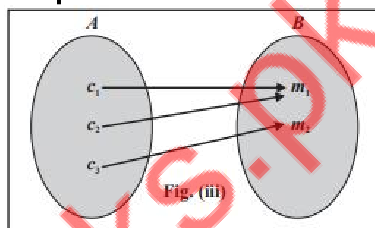

Fig. (iii)

$f = \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

Here, Range $f = \{m_1, m_2\} = B$. So, f is a surjective or onto function.

13. Define bijective function and give an example.

Ans: One – One and onto function or Bijective function:

A function f from A into B is said to be a bijective function if it is both one-one and onto. Such a function is also called (1-1) correspondence between the sets A and B.

Example:

If $A = \{a, b, c\}, B = \{x, y, x\}$ and $f : A \to B$ such that $f = \{(a, z), (b, x), (c, y)\}$.

Which is a bijective function or (1-1) corresponding between the sets A and B.

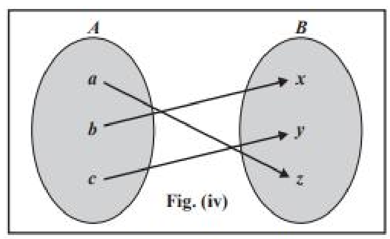

Fig. (iv)

$f = \{(a, z), (b, x), (c, y)\}$

3.3.3 Notation of Function

Set-builder notation is more than suitable for infinite sets. So, is the case with respect to a function comprising an infinite number of ordered pairs.

Example: Consider the function

$f = \{(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), ...\}$

Dom $f = \{-1, 0, 1, 2, 3, 4, ...\}$ and

Range $f = \{0, 1, 4, 9, 16, ...\}$

This function may be written as:

Dom $f = \{(x, y) \mid y = x^2, x \in N\}$

The mapping diagram for the function is shown in the Fig.

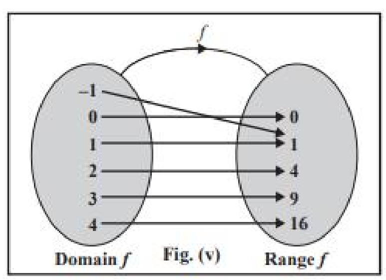

Fig. (v)

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3.3.4 Linear and Quadratic Functions

14. What is the difference between linear and quadratic functions?

Ans: **Linear Function:** The function $\{(x,y) \mid y = mx + c\}$ is called linear function because its graph (geometric representation) is a straight line.

Quadratic Function: The function $\{(x,y) \mid y = ax^2 + bx + c\}$ is called a quadratic function.

Note: We know that an equation of the form $y = mx + c$ represents a straight line.

SOLVED EXERCISE 3.3

Q1. For A={1,2,3,4}, find the following relations in A. State the domain and range of each relation.

i) $\{(x,y) \mid y = x\}$ ii) $\{(x,y) \mid y + x = 5\}$

iii) $R_3 = \{(x,y) \mid x + y < 5\}$ iv) $R_4 = \{(x,y) \mid x + y > 5\}$

Sol: A={1,2,3,4}

$$A \times A = \{1,2,3,4\} \times \{1,2,3,4\} = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4) \\ (3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4) \end{Bmatrix}$$

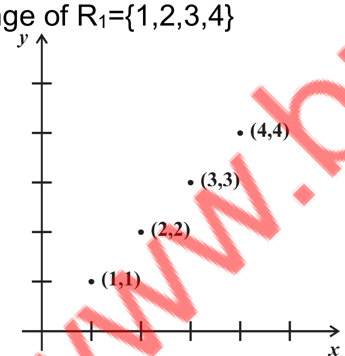
i) $\{(x,y) \mid y = x\}$

$R_1 = \{(x,y) \mid y = x\}$

$R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$

Domain of R_1, ={1,2,3,4}

Range of R_1={1,2,3,4}



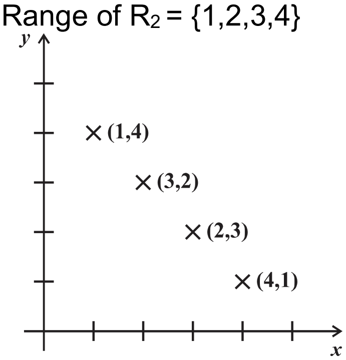
ii) $\{(x,y) \mid y + x = 5\}$

$R_2 = \{(x,y) \mid y + x = 5\}$

$R_2 = \{(1,4),(2,3),(3,2),(4,1)\}$

Domain of R_2 = {1,2,3,4}

Range of R_2 = {1,2,3,4}



iii) $R_3 = \{(x,y) \mid x + y < 5\}$

$R_3 = \{(x,y) \mid y + x < 5\}$

R_3={(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)}

Domain of R_3 ={1,2,3}

Range of R_3 ={1,2,3}

iv) $R_4 = \{(x,y) \mid x + y > 5\}$

$R_4 = \{(x,y) \mid y + x > 5\}$

R_4={(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)

Domain of R_4 = {2,3,4}

Range of R_4 = {2,3,4}

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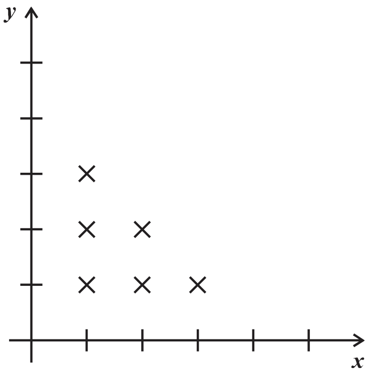

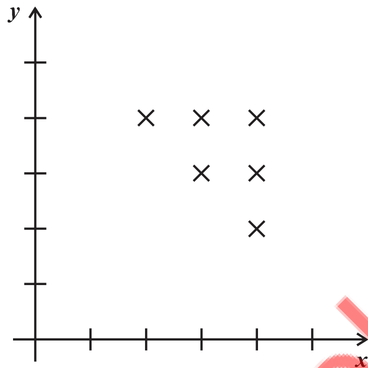

Q2. Which of the following diagrams represent functions and of which type?

Sol:

(i) **Fig(1)** Does not represent a function because first element of two pairs are equal i.e, $(1,a)$, $(1.\ b)$

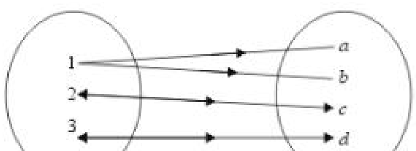

Fig (1)

(ii) **Fig(2)** Represents a function it is one –one and onto so it is a objective function.

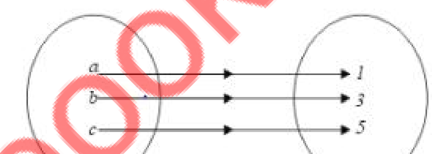

Fig (2)

(iii) **Fig(3)** Represents a function it is one-one and onto so it is a bijective function.

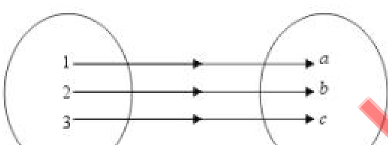

Fig (3)

(iv) **Fig(4)** Represents a function As range is {x,z} $\subset$ (x,y,z} so it is into function.


Fig (4)

Q3. If g(x) =3 x +2 and $h(x)=x^2+1$, then find:

ii) g(–3) ii) g(–3) iii) $g\left(\frac{2}{3}\right)$ iv) h(1) v) h(-4) vi) $h\left(-\frac{1}{2}\right)$

Sol: g(x) =3 x +2 and h(x) = x^2+1

i) g(0) =3 (0) + 2 = 0 + 2 = 2

ii) g(–3) = 3(–3) + 2 = –9 + 2 = –7

iii) $g\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)+2=2+2=4$

iv) h(1) =$(1)^2$+1=1+1=2

v) h(-4) =$(-4)^2$+1=16+1=17

vi) $h\left(-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)^2+1=\frac{1}{4}+1=\frac{5}{4}$

Q4. Given that f (x) = ax + b + 1, where a and b are constant numbers. If $f(3)$=8 and f (6)=14, then find the values of a and b.

Sol:

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Given $f(x) = ax + b + 1$

$f(3) = 8$ and $f(6) = 14$

$f(x) = ax + b + 1$

Putting $x = 3$

$f(3) = a(3) + b + 1 = 3a + b + 1$

$\Rightarrow \quad 8 = 3a + b + 1$

$8 - 1 = 3a + b$

$\Rightarrow 3a + b = 7$ …. (i)

and

$f(x) = ax + b + 1$

Putting $x = 6$

$f(6) = a(6) + b + 1$

$14 = 6a + b + 1$

$14 - 1 = 6a + b$

$6a + b = 13$… (ii)

Subtracting (i) from (ii) we have

$$\begin{array}{l} 6a + \cancel{b} = 13 \\ \pm 3a \pm \cancel{b} = \pm 7 \\ \hline 3a \quad\quad = 6 \end{array}$$

$a = \frac{6}{3} = 2 \Rightarrow a = 2$

putting value of a in eq(i)

$3(2) + b = 7$

$\Rightarrow 6 + b = 7 \quad \Rightarrow b = 7 - 6 = 1$

$\Rightarrow b = 1$

Hence $a = 2$, $b = 1$

Q5. Given that $g(x) = ax+b+5$, where a and b are constant numbers. If $g(-1)=0$ and $g(2)=10$, find the values of a and b.

Sol:

$g(x) = ax + b + 5$

$g(-1) = 0$, $g(2) = 10$

$g(x) = ax + b + 5$

putting $x = -1$

$g(-1) = a(-1) + b + 5$

$0 = -a + b + 5$

$\Rightarrow \quad a - b = 5$ …… (i)

and

$g(x) = ax + b + 5$

putting $x = 2$

$g(2) = a(2) + b + 5$

$10 = 2a + b + 5$

$10 - 5 = 2a + b$

$5 = 2a + b$

or $\quad 2a + b = 5$ …… (ii)

Adding (i) and (ii)

$$\begin{array}{l} 2a + b = 5 \\ a - b = 5 \\ \hline 3a \quad = 10 \end{array}$$

$\Rightarrow \boxed{a = \frac{10}{3}}$

Putting value of a in eq (i)

$\frac{10}{3} - b = 5$

$b = \frac{10}{3} - 5 = \frac{10 - 5}{3} = \frac{-5}{3}$

$\Rightarrow \boxed{b = \frac{-5}{3}}$

Q6. Consider the function defined by $f(x)=5x$. If $f(x)=32$, find the x value.

Sol: Given function:

$f(x) = 5x+1$ (i)

and $f(x) = 32$ (ii)

From (i) and (ii)

$5x + 1 = 32 \quad \Rightarrow 5x = 32 - 1$

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$\Rightarrow \quad 5x = 31 \quad \Rightarrow x = \dfrac{31}{5}$

Q7. Consider the function defined by $f(x)=cx^2+d$, where c and d are constant numbers. If $f(1)=6$ and $f(-2)=10$, then find the values of c and d.

Sol:

Give $f(x) = cx^2 + d$

$f(1) = 6, \; f(-2) = 10$

$f(x) = cx^2 + d$

Putting $x = 1$

$f(1) = c(1)^2 + d$

$\Rightarrow \quad 6 = c + d \qquad \text{(i)}$

and $f(x) = cx^2 + d$

putting $x = -2$

$f(-2) = c(-2)^2 + d$

$10 = 4c + d \qquad \text{(ii)}$

Subtracting (i) from (ii)

$$\begin{array}{l} 4c + d = 10 \\ \pm c \pm d = \pm 6 \\ \hline 3c \quad = 4 \end{array}$$

$\Rightarrow \boxed{c = \dfrac{4}{3}}$

Putting value of c in eq. (i)

$\dfrac{4}{3} + d = 6$

$d = 6 - \dfrac{4}{3} = \dfrac{18-4}{3} = \dfrac{14}{3}$

$\Rightarrow \boxed{d = \dfrac{14}{3}}$

REVIEW EXERCISE 3

1. Four options are given against each statement. Encircle the correct option.

i) The set builder form of the set $\left\{1, \dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{7}, \ldots\right\}$ is:

(a) $\left\{x \mid x = \dfrac{1}{n}, n \in W\right\}$ (b) $\left\{x \mid x = \dfrac{1}{2n+1}, n \in W\right\}$ ✓

(c) $\left\{x \mid x = \dfrac{1}{n+1}, n \in W\right\}$ (d) $\{x \mid x = 2n+1, n \in W\}$

ii) If A={ }, then P(A) is:

(a) { } (b) {1} (c) {{ }} ✓ (d) ϕ

iii) If U={1,2,3,4,5}, A={1,2,3} and B={3,4,5}, then U-(A$\cap$B) is:

(a) {1,2,4,5} ✓ (b) {2,3} (c) {1,3,4,5} (d) {1,2,3}

iv) If A and B are overlapping sets, then n (A-B) is equal to:

(a) n(A) (b) n(B) (c) (A$\cap$B) (d) n(A)-n(A$\cap$B) ✓

v) If $A \subseteq B$ and $B - A \neq \phi$, then n(B–A) is equal to:

(a) 0 (b) n(B) (c) n(A) (d) n(B) – n(A) ✓

vi). If n(A$\cup$B)=50, n(A)=30 n(B)=35, then n(A$\cap$B)=:

(a) 23 (b) 15 ✓ (c) 9 (d) 40

vii). If A= {1,2,3,4} and B= $\{x,y,z\}$, then Cartesian product of A and B contains exactly____ elements.

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(a) 13 (b) 12 ✓ (c) 10 (d) 6

viii). If $f(x) = x^2 - 3x + 2$, then the value of $f(a+1)$ is equal to:

(a) a+1 (b) a^2+1 (c) a^2+2a+1 (d) a^2-a ✓

ix). Given that $f(x) = 3x + 1$, if $f(x)$=28, then the value of x is:

(a) 9 ✓ (b) 27 (c) 3 (d) 18

x). Let A={1,2,3} and B={a,b} two non-empty sets and f :A→B be a function defined as f ={(1,a),(2,b),(3,b)}, then which of the following statement is true?

(a) f is injective (b) f is surjective ✓

(c) f is bijective (d) f is into only

Q2. Write each of the following sets in tabular forms:

Sr/#	Set	Table Form
i)	$\{x \mid x = 2n, n \in N\}$	{2,4,6,8,10,……}
(ii)	$\{x \mid x = 2m+1, m \in N\}$	{3,5,7,9,11…}
(iii)	$\{x \mid x = 11n, n \in W \wedge n < 11\}$	{0,11,22,33,44,55,66,77,88,99,110}
(iv)	$\{x \mid x \in E \wedge 4 < x < 6\}$	{ } or ϕ
(v)	$\{x \mid x \in O \wedge 5 \le x < 7\}$	{5}
(vi)	$\{x \mid x \in Q \wedge x^2 = 2\}$	{ } or ϕ
(vii)	$\{x \mid x \in Q \wedge x = -x\}$	{0}
(viii)	$\{x \mid x \in R \wedge x =\notin Q'\}$	Q $(\because Q \cup Q' = \mathbb{R})$

Q3. Let U={1,2,3,4,5,6,7,8,9,10}, A={2,4,6,8,10}, B={1,2,3,4,5} and C = {1,3,5,7,9}
List the members of each of the following sets:

(i) A′ (ii) B′ (iii) A∪B (iv) A – B
(v) A∩C (vi) A′∪C′ (vii) A′∪C (viii) U′

Sol:

(i) A′=U-A={1,2,3,4,5,6,7,8,9,10} – {2,4,6,8,10} = {1,3,5,7,9}

(ii) B′ = U-B = {1,2,3,4,5,6,7,8,9,10} – {1,2,3,4,5} = {6,7,8,9,10}

(iii) A ∪ B = {2,4,6,8,10} ∪ {1,2,3,4,5} = {1,2,3,4,5,6,8,10}

(iv) A – B = {2,4,6,8,10} – {1,2,3,4,5} = {6,8,10}

(v) A ∩ C = {2,4,6,8,10,} ∩ {1,3,5,7,9} = ϕ

(vi) A′= U – A = {1,2,3,4,5,6,7,8,9,10} – {2,4,6,8,10} = {1,3,5,7,9}

C′=U – C = {1,2,3,4,5,6,7,8,9,10} – {1,3,5,7,9} = {2,4,6,8,10}

$A' \cup C' \{1,3,5,7,9\} \cup \{2,4,6,8,10\} = \{1,2,3,4,5,6,7,8,9,10\}$

(vii) A′ = U – A = {1,2,3,4,5,6,7,8,9,10} – {2,4,6,8,10} = {1,3,5,7,9}

A′∪C = {1,3,5,7,9} ∪ {1,3,5,7,9} = {1,3,5,7,9}

(viii) U′ = U – U = {1,2,3,4,5,6,7,8,9,10} – {1,2,3,4,5,6,7,8,9,10} = ϕ

Q4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A′ (ii) A∩U (ii) A∩U (iv) A∪ϕ (v) $\phi \cap \phi$

Sol:

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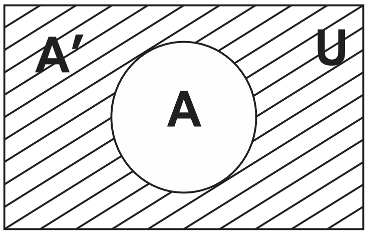
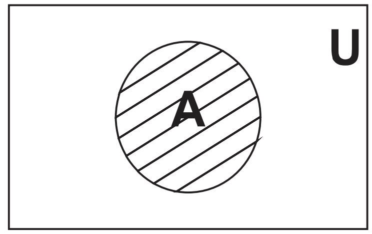
(i) A'



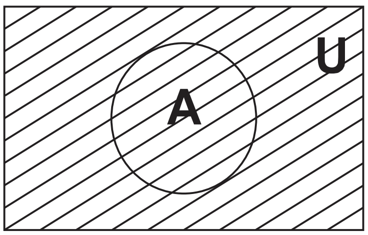
$A' = U - A$

(ii) $A \cap U$



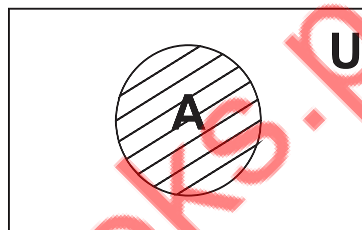
$A \cap U = A$

(ii) $A \cap U$



$A \cup U = A$

(iv) $A \cup \phi$



$A \cup \phi = A$

(v) $\phi \cap \phi$

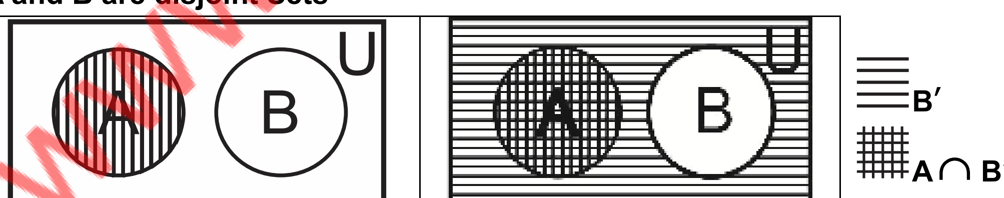
$\phi \cap \phi = \phi$

Q5: Use Venn diagrams, to verify the following:

i. $A - B = A \cup B'$

Sol: There are four cases

a) If A and B are disjoint Sets



$A - B$ **$A \cup B'$**

$A - B = A \cap B'$

b) If A and B are over lapping Sets

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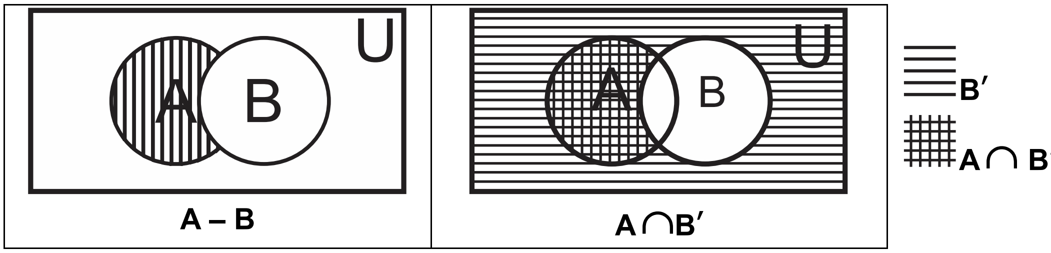
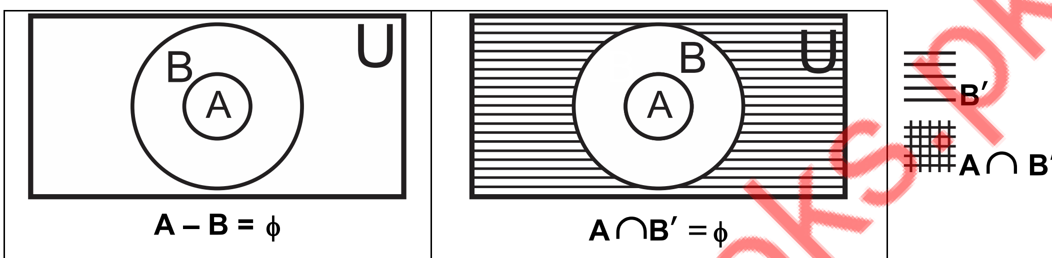


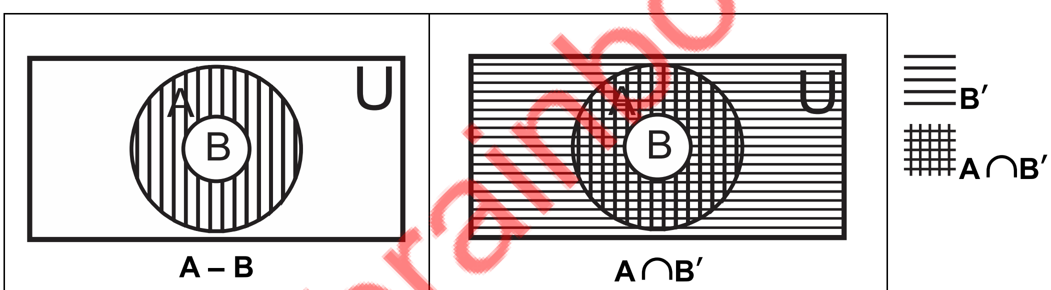
A – B = A ∩ B′

c) If A ⊆ B



A – B = A ∩ B′

d If B ⊆ A

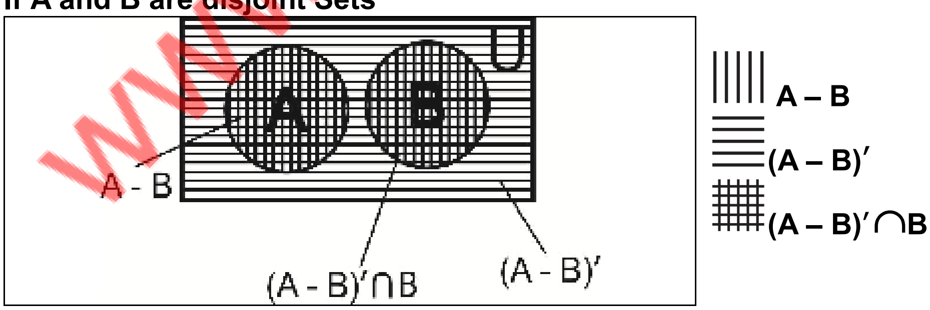


A – B = A ∩ B′

ii. (A – B)′ ∩ B = B

Sol: There are four cases

a) If A and B are disjoint Sets



A – B = A ∩ B′

b) If A and B are over lapping Sets

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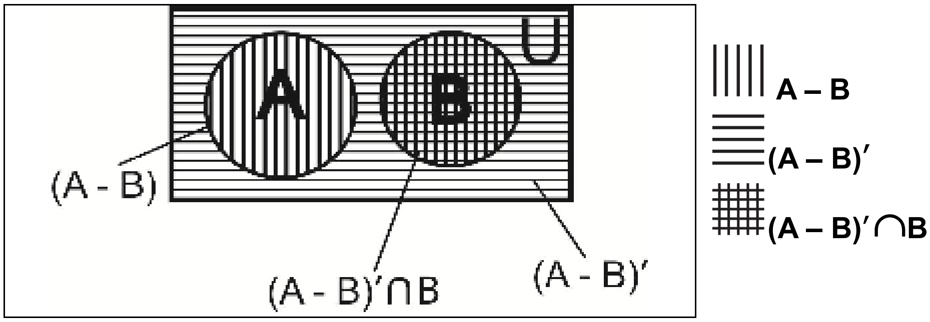
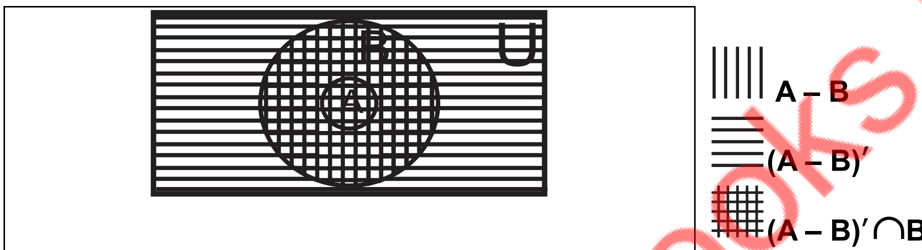


A – B = A ∩ B′

c) If A ⊆ B

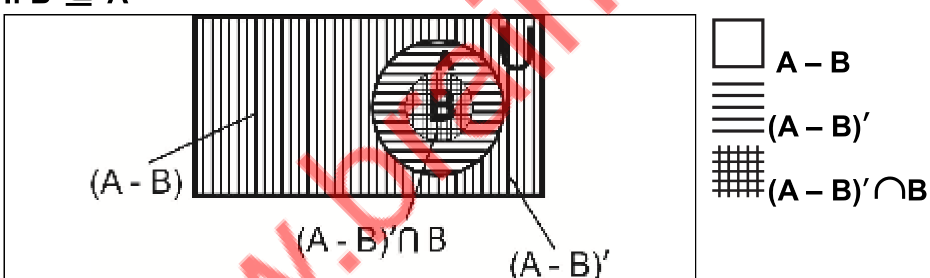


$A - B = \phi$

$(A - B)' = U$

$(A - B)' \cap B = U \cap B = B$

d If B ⊆ A



A – B = A ∩ B′

Thus $(A - B)' \cap B = B$

Q6: Verify the properties for sets A, B and C given below:

(i) Associativity of Union

(ii) Associativity of intersection.

(iii) Distributivity of Union over intersection.

(iv) Distributivity of intersection over union.

(a) A = {1,2,3,4}, B = {3,4,5,6,8}, C = {5,6,9,10} (b) A = ϕ, B = {0}, C = {0,1,2}

(c) A = N, B = Z, C = Q

Sol:

(a) A = {1,2,3,4}, B = {3,4,5,6,8}, C = {5,6,9,10}

(i) <u>Associativity Property of Union:</u>

$A \cup (B \cup C) = (A \cup B) \cup C$

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LHS $= (A \cup (B \cup C)$ $=\{1,2,3,4\} \cup (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\} = \{1,2,3,4,5,6,7,8,9,10\}$ (i)

RHS $= (A \cup B) \cup C = (\{1,2,3,4\} \cup \{3,4,5,6,7,8\}) \cup \{5,6,7,9,10\}$

$=\{1,2,3,4,5,6,7,8,9,10\}$ …… (ii)

From (i) and (ii)

LHS=RHS

$\Rightarrow A \cup (B \cup C) = (A \cup B) \cup C$

(ii) <u>Associativity Property of Intersection:</u>

$A \cap (B \cap C) = (A \cap B) \cap C$

LHS= $A \cap (B \cap C) = \{1,2,3,4\} \cap (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{5,6,7\} = \phi$ …..(i)

R.H.S. $= (A \cap B) \cap C$

$=(\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cap \{5,6,7,9,10\}$

$=\{3,4\} \cap \{5,6,7,9,10\} = \phi$ …..(ii)

From (i) and (ii)

LHS=RHS

$A \cap (B \cup C) = (A \cap B) \cap C$

(iii) <u>Distributive Property of Union over Intersection:</u>

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LHS $= A \cup (B \cap C) = \{1,2,3,4\} \cup (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$=\{1,2,3,4\} \cup \{5,6,7\} = \{1,2,3,4,5,6,7\}$ ……(i)

RHS $= (A \cup B) \cap (A \cup C)$

$=(\{1,2,3,4\} \cup \{3,4,5,6,8\}) \cap (\{1,2,3,4\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9.10\}$

$= \{1,2,3,4,5,6,7\}$ ……(ii)

Form (i) and (ii)

L.H.S. = R.H.S.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) <u>Distributive Property of Intersection over Union:</u>

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS $= A \cap (B \cup C)$

$=\{1,2,3,4\} \cap (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$ $= \{3,4\}$ …….(i)

RHS $= (A \cap B) \cup (A \cap C)$

$=(\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cup (\{1,2,3,4\} \cap \{5,6,7,9,10\}$

$= \{3,4\} \cup \{\ \} = \{3,4\}$ ……(ii)

From (i) and (ii)

LHR=RHS

$(A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) A=ϕ, B={0}, C={0,1,2}

(i) <u>Associative Property of Union:</u>

$(A \cap (B \cup C) = (A \cap B) \cup C$

LHS $= A \cup (B \cup C) = \phi \cup (\{0\} \cup \{1,1,2\}) = \phi \cup \{0,1,2\} = \{0,1,2\}$….(i)

RHS $= (A \cup B) \cup C = (\phi \cup \{0\}) \cup \{0,1,2\}$

$= \{0\} \cup \{0,1,2\} = \{0,1,2\}$.. (ii)

Form (i) and (ii) LHS = RHS

$A \cup (B \cup C) = (A \cup B) \cup C$

(ii) <u>Associative Property of Intersection:</u>

$A \cap (B \cap C) = (A \cap B) \cap C$

LHS $= A \cap (B \cap C) = \phi \cap (\{0\} \cap \{0,1,2\}) = \phi \cap \{0\} = \phi$ …… (i)

RHS $= (A \cap B) \cap C$

$= (\phi \cap \{0\}) \cap \{0,1,2\} = \phi \cap \{0,1,2\} = \phi$.. (ii)

Form (i) and (ii)

L.H.S. = R.H.S.

$A \cap (B \cap C) = (A \cap B) \cap C$

(iii) <u>Distributive Property of Union over Intersection:</u>

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LHS $= A \cup (B \cap C) = \phi \cup (\{0\} \cap \{0,1,2\}) = \phi \cup \{0\} = \{0\}$ ……. (i)

RHS $= (A \cup B) \cap (A \cup C) = (\phi \cup \{0\}) \cap (\{\phi \cup \{0,1,2\}) = \{0\} \cap \{0,1,2\} = \{0\}$ ……. (ii)

Form (i) and (ii)

LHS=RHS

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iv) <u>Distributive Property of Intersection over Union:</u>

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS $= A \cap (B \cup C) = \phi \cap (\{0\} \cup \{0,1,2\}) = \phi \cap \{0,1,2\} = \phi$ …….(i)

RHS $= (A \cap B) \cup (A \cap C) = (\phi \cap \{0\}) \cup (\phi \cap \{0,1,2\}) = \phi \cup \phi = \phi$ …….(ii)

Form (i) and (ii)

LHS=RHS

$A \cap B(B \cup C) = (A \cap B) \cup (A \cap C)$

(C) A=N,B=Z, C = Q

(i) <u>Associative property of Union:</u>

$A \cup (B \cup C) = (A \cup B) \cup C$

LHS $= A \cup (B \cup C)$

$= N \cup (Z \cup Q)$ $(\because Z \subset Q)$

$= N \cup Q$ $(\because N \subset Q)$

$= Q$ ……(i)

RHS $= (A \cup B) \cup C$

$= (N \cup Z) \cup Q$ $(\because N \subset Z)$

$= Z \cup Q$ $(\because Z \subset Q)$

$= Q$ ……(ii)

From (i) and (ii)

LHS = RHS

$A \cup (B \cup C) = (A \cup B) \cup C$

(ii) <u>Associative property of Intersection:</u>

$A \cap (B \cap C) = (A \cap B) \cap C$

LHS $= A \cap (B \cap C)$

RHS $= (A \cap B) \cap C$

$= (N \cap Z) \cap Q$ $(\because N \subset Z)$

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$=N\cap(Z\cap Q)$ $(\because Z\subset Q)$
$=N\cup Z$ $(\because N\subset Z)$
$=N$ ……(i)

$=N\cap Q$ $(\because N\subset Q)$
$=N$ ……(ii)

From (i) and (ii)
LHS = RHS
$A\cap(B\cap C)=(A\cap B)\cap C$

(iii) <u>Distributive Property of Union over Intersection:</u>

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

LHS= $A\cup(B\cap C)$ $=N\cup(Z\cap Q)$
$=N\cup Z$ $(\because Z\subset Q)$
$=N\cap Z$ $(\because N\subset Z)$
$=Z$ …….(i)

RHS = $(A\cup B)\cap(A\cup C)$

$\begin{pmatrix}\because N\subset Z\\ \because N\subset Q\\ \because Z\subset Q\end{pmatrix}$

$=(N\cup Z)\cap(N\cup Q)=Z\cap Q=Z$ (ii)
From (i) and (ii)
LHS = RHS
$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(iv) <u>Distributive Property of Interaction over Union:</u>

$A\cap(B\cup C)$ $=(A\cap B)\cup(A\cap C)$
LHS= $A\cap(B\cup C)$ $=N\cap(Z\cup Q)$
$=N\cap Z$ $(\because Z\subset Q)$
$=N$ …….(i) $(\because N\subset Q)$
RHS= $(A\cap B)\cup(A\cap C)=(N\cap Z)\cup(N\cap Q)$
$=N\cup N$ $(\because N\subset Z,\ N\subset Q)$
$=N$ …….(ii)
From (i) and (ii)
LHS = RHS
$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Q7. <u>Verify De Morgan's Laws for the following sets:</u>
U= {1,2,3,….20}, A={2,4,6,…20} and B= {1,3,5,…19}

(i) $(A\cup B)'=A'\cap B'$

(ii) $(A\cap B)'=A'\cup B'$

Sol: (i) $(A\cup B)'=A'\cap B'$
LHS $=(A\cup B)'$
$A\cup B=\{2,4,6,...20,\}\cup\{1,3,5,...,19\}=\{1,2,3,\ldots,20\}$
$(A\cup B)'=U-(A\cup B)$ $=\{1,2,3,...,20\}-\{1,2,3...,20\}=\phi$...(i)
RHS = $A'\cap B'$
$A'=U-A=\{1,2,3,\ldots,20\}-\{2,34,6,\ldots,20\}=\{1,3,5,7,\ldots,19\}$
and $B'=U-B=\{1,2,3,...,20\}-\{1,3,5,...,19\}$ $=\{2,4,6...,20\}=\{1,3,5,.........,19\}$
Now, $A'\cap B'=\{1,3,5,7,19\}\cap\{2,4,6,20\}=\phi$ …… (ii)

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Form (i) and (ii)
LHS=RHS
$(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$
LHS = $(A\cap B)'$
$(A\cap B)=\{2,4,6,\ldots 20\}\cap\{1,3,5,..,19\} = \phi$
$(A\cap B)' = U - (A\cap B) = \{1,2,3,\ldots 20\} - \phi = (1,2,3,\ldots,20\}$ .... (i)
RHS = $A'\cup B'$
$A' = U - A = \{1,2,3,\ldots 20\} - \{2,4,6,..,20\}$
and $B' = U - B = \{1,2,3,\ldots\ldots,20\} - \{1,3,5,\ldots\ldots.,19\} = \{2,4,6,\ldots\ldots,20\}$
Now,
$A'\cup B' = \{1,3,5\ldots 19\}\cup\{2,4,6,..,20\} = \{1,2,3,4,\ldots 20,\}$ (ii)
From (i) and (ii)
LHS = RHS
$(A\cap B)' = A'\cup B'$

Q8. Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$ Find $P\cap Q$

Sol. $P = \{x \mid x = 5m, m \in N\}$
$\Rightarrow$ $P = \{5,10,15,\ldots.\}$
and $Q = \{x \mid x = 2m, m \in N\}$
$\Rightarrow$ $Q = \{2,4,6,8\ldots.\}$
$P\cap Q = \{5,10,15,20,..\}\cap\{2,4,6,8,\ldots\ldots.\}$
$P\cap Q = \{10,20,30,40,50\ldots.\}$

Q9. From suitable properties of union and intersection, deduce the following results.

(i) $A\cap(A\cup B) = A\cup(A\cap B)$ (ii) $A\cup(A\cap B) = A\cap(A\cup B)$

(i) LHS = $A\cap(A\cup B)$
By distributive property of intersection over union.
$= (A\cap A)\cup(A\cup B) = A\cup(A\cap B)$ $(\because A\cap A = A)$
=RHS

(ii) LHS = $A\cup(A\cap B)$
By distributive property of intersection over union.
$= (A\cup A)\cap(A\cup B) = A\cap(A\cup B)$ $(\because A\cup A = A)$
=RHS

Q10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:

(i) **g (0)** **(ii)** **g (–1)** **(iii)** **S (x)** **(iv)** **S (–9)** **(v)** $s\left(\frac{7}{2}\right)$

Sol: $g(x) = 7x - 2$

(i) g (0) =7(0) – 2 = 0 – 2 = – 2
(ii) g (–1) =7(–1) – 2 = –7 – 2 = –9

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$$g\left(-\frac{5}{3}\right) = 7\left(-\frac{5}{3}\right) - 2 = \frac{-35}{3} - 2 = \frac{-35-6}{3} = \frac{-41}{3}$$

(iii) S (x) = $8x^2 - 3x$
S (1) = $8(1)^2 - 3 = 8 - 3 = 5$

(iv) S (–9) = 8 (–9) – 3 = 8(81) –3 = 648 – 3 = 645

(v) $s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2 - 3 = 8\left(\frac{49}{4}\right) - 3 = 98 - 3 = 95$

Q11. Give that $f(x) = ax + b$, Where a and b are constant numbers. If f (-2)=3 and f (4)=10, then find the values of a and b.

Sol:

$f(x) = ax + b$
$f(-2) = 3,\ f(4) = 10$
$f(x) = ax + b$
Putting $x = -2$
$f(-2) = a(-2) + b$
$3 = -2a + b$ ……(i)
and $f(x) = ax + b$
Putting $x = 4$
$f(4) = (4) + b$
$10 = 4a + b$ ……(ii)

Subtracting (i) From (ii)

$$\begin{array}{l} 4a+b = 10 \\ \underline{+2a \pm b = \pm 3} \\ 6a \quad = 7 \end{array}$$

$\Rightarrow a = \frac{7}{6}$

Putting value of a in eq(i)

$-2\left(\frac{7}{6}\right) + b = 3$

$b = 3 + \frac{7}{3} = \frac{9+7}{3} = \frac{16}{3}$

$\boxed{b = \frac{16}{3}}$

Q12. Consider the function defined by k(x) = 7x – 5. If k(x)=100, find the value of x.

Sol: K(x) = 7x – 5 (i)
If K(x) = 100 (ii)
Then from (i) and (ii)
$7x - 5 = 100$
$7x = 100 + 5 = 105$
$x = \frac{105}{7} = 15 \Rightarrow x = 15$

Q13: Consider the function g(x)=mx^2 + n, where m and n are constant numbers. If g(4) = 20 and g(0) = 5, find the values of m and n.

Sol:

g (x) = mx^2 + n
g (4) = 20,g(10) = 5
g (x) = mx^2 + n
Putting $x = 4$
g(4) = m$(4)^2$ +n
20 = 16m +n …..(i)
and g(x) = mx^2 +n

From (ii) n = 5
Put in (i)
20 = 16m + 5
20-5 = 16m
15 = 16m $\Rightarrow$ m = $\frac{15}{16}$

Putting $x=0$

$g(0) = m(0)^2 + n$

$\Rightarrow 5 = n$ .....(ii)

Q14. A shopping mall has 100 products from various categories labeled 1 to 100, A representing the universal set U. The products are categorized as follows:

- **Set A: Electronics, consisting of 30 products labeled from 1 to 30**
- **Set B: Clothing comprises 25 products labeled from 31 to 55.**
- **Set C: Beauty Products, comprising 25 products labeled from 76 to 100.**

Write each set in tabular form, and find the union of all three sets.

Sol: Total Products in the universal set

U =100

Set: A: Electronics = 30 Products labeled from 1 to 30.

Set: B: Clothing = 25 Products labeled from 31 to 55.

Set: C: Beauty products = 25 Products labeled from 76 to 100.

In tabular from:

U = {1,2,3,…,100}, A = {1,2,3,…,30}

B = {31,32,…55}, C = {76,77,…,100}

Union of all three sets:

$A \cup B \cup C = \{1,2,3,\ldots 30\} \cup \{31,32,..55\} \cup \{76,77,\ldots,100\}$

$= \{1,2,3,\ldots..,30,31,32,\ldots\ldots,55,76,77,\ldots.,100\}$

The union of all three sets contains 80 products.

Q15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

(b) How many did not pass either of the two tests.

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

Sol: U = {Total number of students appeared in the annual examination}

M = {Students who passed the math test}

S = {Students who passed the Science test}

From the statement of problems, We have

n(U) = 180, n(M) = 120

n(S) = 90, $n(M \cap S) = 60$

(a) We want to find the total number of students who passed either the math or science test. We are to find $n(M \cup S)$

Using the principle of inclusion and exclusion for 2 sets.

$n(M \cup S) = n(M) + n(S) - n(M \cap S) = 120+90-60 = 150$

(b) Students who did not pass any test

= Total students – students who passed either test.

$n(M \cup S)' = n(U) - n(M \cup S) = 180 - 150 - 30$

30 students did not pass either of the two tests.

(c) Students who passed only Science test

Using formula:

Science only = $n(S) - n(M \cap S) = 90 - 60 = 30$

30 students passed the science test but not the math test.

(d) To find the number of students who failed the science test.
Failed science = Total students – Students who passed the science test.
Failed science = n (U) – n(S) = 180 – 90 = 90
90 students failed the science test.

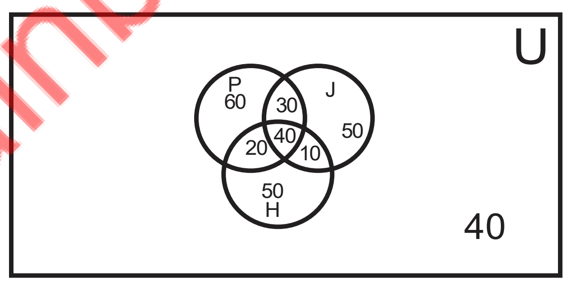
Q16. In a Software house of city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**

(a) How many developers use at least one of these languages?
(b) How many developers use only one of these languages?
(c) How many developers do not use any of these languages?
(d) How many developers use only PHP?

Sol. Let U = {Total number of software developers}
P = {Developers who like python}
J = {Developers who like Java}
H = {Developers who like PHP}

From the statement of problems we have
$n(U) = 300$
$n(P) = 150,\ n(J) = 130$
$n(H) = 120,\ n(P \cap J) = 70$
$n(P \cap H) = 60,\ n(J \cap H) = 50$
$n(P \cap J \cap H) = 40$



(a) $n(P \cap J \cap H) = n(P)+n(J) + n(H) - n(P \cap J) - n(J \cap H) - n(P \cap H)+n(P \cap J \cap H)$
$= 150 + 130 + 120 - 70 - 60 - 50 + 40 = 260$
260 developers use at least one of these languages.

(b) To find developers who use only one language:

(i) only python (P) $= n(P) - n(P \cap J) - n(P \cap H) + n(P \cap J \cap H)$
$= 150 - 70 - 60 + 40 = 60$

(ii) only Java (J)
$= n(J) - n(P \cap J) - n(J \cap H) + n(P \cap J \cap H)$

MULTIPLE CHOICE QUESTIONS (MCQs)

3.1 Mathematics as the Study of Patterns, Structures and Relationship

1. The sequence 0, 1, 1, 2, 3, 5, 8, 13, 21,…., is known as:
(A) Even (B) Odd (C) Fibonacci ✓ (D) Prime

2. The formula of Fibonacci sequence is:

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(A) $F_n = F_{n-1} + F_{n+2}$ (B) $F_n = F_{n-1} + F_{n-2}$ ✓
(C) $F_n = F_{n+1} + F_{n-2}$ (D) $F_n = F_{n-1} + F_{n+2}$

3.1.1 Basic Definitions

3. If U = $\{1,2,3,....,10\}$ and A = $\{3,4,5\}$ then A′ is:
(A) {1,2,3,4} (B) {3,4,5,6} (C) {4,5,6,7,8} (D) {1,2,6,7,8,9,10} ✓

4. A collection of well-known objects is called:
(A) subset (B) power set (C) set ✓ (D) none

5. The number of elements in a power set {a,b,c,d} is:
(A) 4 (B) 6 (C) 8 (D) 16 ✓

6. The number of elements in a power set {a,b} is:
(A) 1 (B) 2 (C) 3 (D) 4 ✓

7. A set $Q = \left\{\frac{a}{b} \mid a,b \in Z \wedge b \neq 0\right\}$ is called a set of _____:
(A) Whole Numbers (B) Natural Numbers
(C) Irrational Numbers (D) Rational Numbers ✓

8. Number of ways to describe a set:
(A) 1 (B) 2 (C) 3 ✓ (D) 4

9. A set containing no element is called:
(A) Subset (B) Empty set ✓ (C) Singleton set (D) Super set

10. A set having only one element is called:
(A) Empty set (B) Power set (C) Singleton set ✓ (D) Super set

11. The set $\{x \mid x \in W \wedge x \leq 100\}$ is called:
(A) Infinite set (B) Finite set ✓ (C) Null set (D) Subset

12. Power set of empty set is:
(A) ϕ (B) $\{a\}$ (C) $\{\{\phi\},\{a\}\}$ (D) $\{\phi\}$ ✓

13. Number of elements in power set of {1,2,3}:
(A) 4 (B) 6 (C) 8 ✓ (D) 9

14. Tabular form of $Q = \{x \mid x \in Q \wedge x^2 = 3\}$ is:
(A) $\{a\}$ (B) $\{\sqrt{3}\}$ (C) $\{\sqrt{3}, -\sqrt{3}\}$ (D) $\{\ \}$ ✓

15. $\{\phi\}$ is equal to:
(A) Empty set (B) Singleton set (C) Rational set (D) Complex set

16. If $A = \{0\}$, then $P(A) =$:
(A) 2 ✓ (B) 3 (C) 4 (D) 8

17. Which symbol is used to Membership of a set?
(A) $\wedge$ (B) $\vee$ (C) $\in$ ✓ (D) ~

18. The number of subsets of a set of four elements is equal to:
(A) 16 ✓ (B) 8 (C) 4 (D) 6

19. If $A = \{a, \{a,b\}\}$ then number of elements in $P(A)$.

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(A) 2 (B) 3 (C) 4 ✓ (D) 8

20. Number of ways in which a set can be described as:

(A) 1 (B) 2 (C) 3 ✓ (D) 4

21. The set $\{x|x\in R\vee x\neq x\}$ is:

(A) Empty set ✓ (B) Infinite set (C) Singleton set (D) Binary set

22. The tabular form of $\{x|x\in E\wedge 4<x<6\}$ is:

(A) $\{4\}$ (B) $\{5\}$ (C) $\{6\}$ (D) ϕ ✓

23. If every element of set A is also element of the set B, then:

(A) $A\subseteq B$ ✓ (B) $B\subseteq A$ (C) $A\cap B=\phi$ (D) $A\cap B=A^c$

24. $Q'\cup Q=$ _____ :

(A) N (B) P (C) E (D) R ✓

25. If A is a subset of B and A = B, then A is an _____ of B.

(A) Proper subset (B) Improper subset ✓
(C) Power set (D) Universal set

3.2 Operations of Two Sets

26. If A = {2,3,5} and B = {3,5,8} then $A\cap B=$ _____ :

(A) {3,5} ✓ (B) {5,8} (C) {3,8} (D) {2,3}

27. If B – A = B, then:

(A) $A\cap B=U$ (B) $B\cap A=B$ (C) $B\cap A=A$ ✓ (D) $A\cap B=\phi$

28. $A\cup(A\cap B)=$ _____ :

(A) B (B) $A\cup B$ (C) A (D) $A\cap B$

29. If the intersection of two sets is empty, the sets are said to be _____ set:

(A) Overlapping (B) Complement (C) Disjoint ✓ (D) Difference of two sets

30. Which of them is the set of all elements of U, which do not belong to A called:

(A) Disjoint sets (B) Difference of sets
(C) Complement of a set ✓ (D) Overlapping sets

31. Which of them is the set of all elements of U, which belong to A but do not belong to B is called:

(A) Disjoint sets (B) Difference of sets ✓
(C) Complement of a set (D) Overlapping sets

32. If the intersection of two sets is non-empty but neither is a subset of the other, the sets are called _____ sets:

(A) Disjoint (B) Difference (C) Complement (D) Overlapping ✓

3.2.1 Identification of Sets Using Venn Diagram

33. If $A\subseteq B$ then $A\cup B=$ _____ :

(A) A (B) B ✓ (C) ϕ (D) None of these

34. If $A\subseteq B$ then $A\cap B=$ _____ :

(A) A ✓ (B) B (C) ϕ (D) $A\cup B$

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35. $(A \cup B) \cup C =$ ____ :

(A) $A \cap (B \cup C)$ (B) $(A \cup B) \cap C$ (C) $A \cup (B \cup C)$ ✓ (D) $A \cap (B \cap C)$

36. $A \cup (B \cap C) =$ ____ :

(A) $(A \cup B) \cap (A \cup C)$ ✓ (B) $A \cap (B \cap C)$ (C) $(A \cap B) \cup (A \cap C)$ (D) $A \cup (B \cup C)$

37. If A and B are two disjoint sets then $A \cup B =$ ____ :

(A) A (B) B (C) ϕ (D) $A \cup B$ ✓

38. If A^c is complement of set A, then $A \cap B$:

(A) A (B) A^c (C) U (D) ϕ

39. If A and B are two disjoint sets then:

(A) $A \cap B = A$ (B) $A \cap B = B$ (C) $A \cap B = \phi$ (D) $A \cap B \neq \phi$ ✓

40. Venn diagram is useful only in case of:

(A) Concrete sets (B) Abstract sets ✓ (C) Subsets (D) Universal sets

41. If A and B are any subset U, then A – B =:

(A) $A \cup B^c$ (B) $(A \cup B)^c$ (C) $(A \cap B)^c$ (D) $A \cap B^c$

42. Which of them is the set of all elements that belongs to both A and B:

(A) Union of two sets (B) Intersection of two sets ✓

(C) Power set (D) Overlapping set

3.2.2 Operations on Three Sets

3.2.2.1 Properties of Union and Intersection

43. For any two set A and B, $(A \cap B)^c$ is equal to:

(A) A^c (B) B^c (C) $A^c \cup B^c$ ✓ (D) $A \cap B$

44. For any two subsets A and B of set U, then $(A \cup B)'$ is equal to:

(A) $A \cup B'$ (B) $A \cap B'$ (C) $A' \cup B'$ (D) $A' \cap B'$ ✓

45. $A \cup B = B \cup A$ **is known as:**

(A) Commutative property of union ✓ (B) Commutative property of intersection

(C) Associative property of union (D) Associative property of intersection

46. $A \cap B = B \cap A$ **is known as:**

(A) Commutative property of union (B) Commutative property of intersection ✓

(C) Associative property of union (D) Associative property of intersection

47. $A \cup (B \cup C) = (A \cup B) \cup C$ **is known as:**

(A) Commutative property of union (B) Commutative property of intersection

(C) Associative property of union ✓ (D) Associative property of union

48. $A \cap (B \cap C) = (A \cap B) \cap C$ **is known as:**

(A) Commutative property of union (B) Commutative property of intersection

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(C) Associative property of union (D) Associative property of intersection ✓

49. $A\cap(B\cup C)=(A\cup B)\cap(A\cup C)$ **is known as:**

(A) Distributive of union (B) De-Morgan's law
(C) Distributive property of union over intersection ✓
(D) Distributive property of intersection over union

50. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ **is known as:**

(A) Distributive of union (B) De-Morgan's law
(C) Distributive property of union over intersection
(D) Distributive property of intersection over union ✓

3.2.3 Real World Applications

51. If A = {1,3,5,7,9,11} then n(A) = ______:

(A) 2 (B) 4 (C) 6 ✓ (D) 11

52. In principle of inclusion and exclusion for two sets, $n(A\cup B)=$_____:

(A) $n(A)+n(B)-n(A\cap B)$ ✓ (B) $n(A)+n(B)+n(A\cap B)$

(C) $n(A)+n(B)-n(A\cup B)$ (D) $n(A)+n(B)+n(A\cup B)$

3.3 Binary Relations

53. The range of R = {(1,3),(2,2),(3,1),(4,4)} is:
(A) {2,3,4} (B) {1,2,3,4} ✓ (C) {1,3,4} (D) {1,2,3}

54. If X = {a,b,c} then number of elements is X×X are:
(A) 9 ✓ (B) 12 (C) 14 (D) 16

55. If set A has 3 elements and B has 4 then A×B has _____ elements.
(A) 3 (B) 4 (C) 12 ✓ (D) 7

56. If set A has 3 elements and B has 2 elements then number of binary relations of A×B are:
(A) 2^2 (B) 2^8 (C) 2^6 ✓ (D) 2^3

57. If R = {(0,2),(2,3),(3,3),(3,4)} then Dom (R) is:
(A) {0,3,4} (B) {0,2,3} ✓ (C) {0,2,4} (D) {2,3,4}

58. The range of R is if R = {(1,3),(2,2),(3,1),(4,4)}:
(A) {2,3,4} (B) {1,2,3,4} ✓ (C) {1,3,4} (D) {1,2,3}

59. Which of the following cannot be used as binary operation?
(A) Addition (B) Multiplication (C) Square root ✓ (D) Division

60. Which one of them is unary operation?
(A) Addition (B) Multiplication (C) Subtraction (D) Negation ✓

61. $A\times B=$______**:**

(A) $A\times B=\{(x,y)\mid x\in A, y\in B\}$ ✓ (B) $A\times B=\{(x,y)\mid x\in A, x\in B\}$

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(C) $A\times B=\{(x,y)\,|\,y\in A, y\in B\}$ (D) $A\times B=\{(x,y)\,|\,x\in A, y\notin B\}$

3.3.1 Relation as Table, Ordered Pair and Graph

62. Point (-1,4) lies in quadrant:
(A) I (B) II ✓ (C) III (D) IV

63. Each ordered pair consists of _____ coordinates:
(A) 2 ✓ (B) 3 (C) 4 (D) 5

64. In coordinates (x,y), x is known as:
(A) Abscissa ✓ (B) Ordinate (C) first element (D) second element

65. In coordinates (x,y), y is known as:
(A) Abscissa (B) Ordinate ✓ (C) first element (D) second element

66. Ordered pair is written as:
(A) x (B) y (C) (x,y) ✓ (D) (y,x)

3.3.2 Function and its Domain and Range

67. The relation R = {(1,2),(2,3),(3,3),(3,4)} is:
(A) Not a function ✓ (B) onto function
(C) one-one function (D) into function

68. In a function $f: A\to B$ is a such that Range $f=B$ then f is called:
(A) Injective (B) Surjective (C) Into (D) Periodic

69. A function $f: A\to B$ is surjective if:
(A) Range f = A (B) Range f = B ✓ (C) Range $f\neq B$ (D) Range $f\neq A$

70. A function which is onto is called:
(A) Injective (B) Subjective ✓ (C) Objective (D) Bijective

71. If a function $f: A\to B$ is such that Ran $f\subset B$ i.e., Ran $f\neq B$ then f is called:
(A) Into function ✓ (B) Onto function
(C) Injective function (D) Bijective function

72. A function $f: A\to B$ is called an onto function if:
(A) Range of $f=A$ (B) Range of $f\neq A$ (C) Range of $f=B$ ✓ (D) Range of $f\neq B$

73. How many types of function?
(A) 2 (B) 3 (C) 4 ✓ (D) 5

3.3.3 Notation of Function

74. If f = {(-1,1), (0,0), (1,1),(2,4),(3,9),(4,16)} then domain f is:
(A) {-1,0,1,2,3,4} ✓ (B) {0,1,4,9,16} (C) {1,-1,2,4,9,16} (D) {0,1,2,3,4}

75. If f = {(-1,1), (0,0), (1,1),(2,4),(3,9),(4,16)} then range f is:
(A) {-1,0,1,2,3,4} (B) {0,1,4,9,16} ✓ (C) {1,-1,2,4,9,16} (D) {0,1,2,3,4}

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3.3.4 Linear and Quadratic Functions

76. The graph of the function $f = \{(x, y) \mid y = mx + c\}$ is:

(A) Parabola (B) Straight line ✓ (C) Circle (D) Ellipse

77. If $f(x) = 2x - 1$ then $f(1)$ = _____:

(A) 0 (B) 1 ✓ (C) 2 (D) 3

78. If $f(x) = 2x - 1$ then $f(7)$ = _____:

(A) 10 (B) 11 (C) 12 (D) 13 ✓

79. If $g(x) = x^2 - 3$ then $g(-3)$ = _____:

(A) 2 (B) 4 (C) 6 ✓ (D) 8

80. If $g(x) = x^2 - 3$ then $g(4)$ = _____:

(A) 9 (B) 10 (C) 11 (D) 13 ✓

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