# EXERCISE # 2.4

## (Question # 1)

Find $\frac{dy}{dx}$ by making suitable substitution in the following functions defined as:

(i) $y = \sqrt{\frac{1-x}{1+x}}$

Sol. $y = \sqrt{\frac{1-x}{1+x}}$

Let $u = \frac{1-x}{1+x}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \longrightarrow (1)$$

| | |
|---|---|
| $y = \sqrt{u}$ | $u = \frac{1-x}{1+x}$ |
| $\frac{d}{du}[y] = \frac{d}{du}\sqrt{u}$ | $\frac{d}{dx}[u] = \frac{d}{dx}\left[\frac{1-x}{1+x}\right]$ |
| $\frac{dy}{du} = \frac{1}{2\sqrt{u}}\frac{d}{du}(u)$ | $\frac{du}{dx} = \frac{(1+x)\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$ |
| $\frac{dy}{du} = \frac{1}{2\sqrt{u}}(1)$ | $\frac{du}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$ |

$$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{u}}}$$

$$\frac{du}{dx} = \frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}$$

$$\boxed{\frac{du}{dx} = \frac{-2}{(1+x)^2}}$$

① $\Longrightarrow$ $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{u}} \cdot \frac{-\cancel{2}}{(1+x)^2}$$

put the value of u

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{1-x}{1+x}}} \cdot \frac{-1}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{1-x}}{(1+x)^{1/2}}} \cdot \frac{-1}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x}\,(1+x)^{2-\frac{1}{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{\sqrt{1-x}\,(1+x)^{3/2}}}$$ Ans.

## Question # 1 (ii)

$$y = \sqrt{x+\sqrt{x}} \qquad \frac{dy}{dx} = ?$$

Sol.

$$y = \sqrt{x+\sqrt{x}}$$

Let $\quad u = x+\sqrt{x}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \longrightarrow (1)$$

$$y = \sqrt{u}$$

$$\frac{d}{du}(y) = \frac{d}{du}(\sqrt{u})$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \frac{d}{du}(u)$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}}(1)$$

$$\boxed{\frac{dy}{du} = \frac{1}{2\sqrt{u}}}$$

$$u = x+\sqrt{x}$$

$$\frac{d}{dx}[u] = \frac{d}{dx}(x+\sqrt{x})$$

$$\frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(\sqrt{x})$$

$$\frac{du}{dx} = 1 + \frac{1}{2\sqrt{x}} \frac{d}{dx}(x)$$

$$\frac{du}{dx} = 1 + \frac{1}{2\sqrt{x}}(1)$$

$$\boxed{\frac{du}{dx} = 1 + \frac{1}{2\sqrt{x}}}$$

(1) $\Rightarrow$ $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{u}}\right) \times \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \left(\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right)$$

put the value of $u$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x}}} \left(\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}}$$

Ans.

## Question # 1 (iii)

$$y = x\sqrt{\frac{a+x}{a-x}}$$

Sol.

$$y = x\sqrt{\frac{a+x}{a-x}}$$

$$y = \sqrt{x^2}\sqrt{\frac{a+x}{a-x}}$$

$$y = \sqrt{\frac{x^2(a+x)}{a-x}}$$

$$y = \sqrt{\frac{ax^2+x^3}{a-x}}$$

Let $u = \frac{ax^2+x^3}{a-x}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \longrightarrow (1)$$

$y = \sqrt{u}$

$\frac{d}{du}(y) = \frac{d}{du}\sqrt{u}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{u}}\frac{d}{du}(u)$

$u = \frac{ax^2+x^3}{a-x}$

$\frac{d}{dx}[u] = \frac{d}{dx}\left[\frac{ax^2+x^3}{a-x}\right]$

$\frac{du}{dx} = \frac{(a-x)\frac{d}{dx}(ax^2+x^3) - (ax^2+x^3)\frac{d}{dx}(a-x)}{(a-x)^2}$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}} \quad (1)$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$$

$$\frac{du}{dx} = \frac{(a-x)(2ax+3x^2)-(ax^2+x^3)(-1)}{(a-x)^2}$$

$$\frac{du}{dx} = \frac{2a^2x+3ax^2-2ax^2-3x^3+ax^2+x^3}{(a-x)^2}$$

$$\frac{du}{dx} = \frac{2a^2x+2ax^2-2x^3}{(a-x)^2}$$

$$\frac{du}{dx} = \frac{2x(a^2+ax-x^2)}{(a-x)^2}$$

(1) $\Longrightarrow$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \times \frac{2x(a^2+ax-x^2)}{(a-x)^2}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{\frac{ax^2+x^3}{a-x}}} \times \frac{\cancel{2}x(a^2+ax-x^2)}{(a-x)^2}$$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{x^2(a+x)}}{(a-x)^{1/2}}} \times \frac{x(a^2+ax-x^2)}{(a-x)^2}$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{a+x}} \times \frac{x(a^2+ax-x^2)}{(a-x)^{2-\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{x}\sqrt{a+x}} \times \frac{\cancel{x}(a^2+ax-x^2)}{(a-x)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = \frac{a^2+ax-x^2}{\sqrt{a+x}\,(a-x)^{3/2}}}$$

Ans.

## Question #1 (iv)

$$y=(3x^2-2x+7)^6 \qquad \frac{dy}{dx}=?$$

Sol.

$$y=(3x^2-2x+7)^6$$

let $u=3x^2-2x+7$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx} \longrightarrow (1)$$

$y=u^6$

$$\frac{d}{du}[y]=\frac{d}{du}[u]^6$$

$$\frac{dy}{du}=6u^{6-1}\frac{d}{du}(u)$$

$$\frac{dy}{du}=6u^5(1)$$

$$\frac{dy}{du}=6u^5$$

$u=3x^2-2x+7$

$$\frac{d[u]}{dx}=\frac{d}{dx}[3x^2-2x+7]$$

$$\frac{du}{dx}=\frac{d}{dx}(3x^2)-\frac{d}{dx}(2x)+\frac{d}{dx}(7)$$

$$\frac{du}{dx}=3(2x)-2(1)+0$$

$$\frac{du}{dx}=6x-2$$

$$(1) \Longrightarrow \frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}.$$

$$\frac{dy}{dx} = (6u^5)(6x-2)$$

$$\frac{dy}{dx} = (6u^5)\,2(3x-1)$$

$$\frac{dy}{dx} = 12u^5(3x-1)$$

put the value of $u$

$$\boxed{\frac{dy}{dx} = 12(3x^2-2x+7)^5(3x-1)}$$

Ans.

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## Question # 1 (v)

$$\sqrt{\frac{a^2+x^2}{a^2-x^2}} \qquad \frac{dy}{dx} = ?$$

Sol.

Let $y = \sqrt{\frac{a^2+x^2}{a^2-x^2}}$

Let $u = \frac{a^2+x^2}{a^2-x^2}$

$$y = \sqrt{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \longrightarrow (1)$$

$y = \sqrt{u}$

$\frac{d}{du}[y] = \frac{d}{du}[\sqrt{u}]$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}} \frac{d}{du}(u)$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}(1)$

$\boxed{\frac{dy}{du} = \frac{1}{2\sqrt{u}}}$

$u = \frac{a^2+x^2}{a^2-x^2}$

$\frac{d}{dx}[u] = \frac{d}{dx}\left[\frac{a^2+x^2}{a^2-x^2}\right]$

$\frac{du}{dx} = \frac{(a^2-x^2)\frac{d}{dx}(a^2+x^2) - (a^2+x^2)\frac{d}{dx}(a^2-x^2)}{(a^2-x^2)^2}$

$\frac{du}{dx} = \frac{(a^2-x^2)(0+2x) - (a^2+x^2)(0-2x)}{(a^2-x^2)^2}$

$\frac{du}{dx} = \frac{(a^2-x^2)2x + (a^2+x^2)2x}{(a^2-x^2)^2}$

$$\frac{du}{dx} = \frac{2a^2x - \cancel{2x^3} + 2a^2x + \cancel{2x^3}}{(a^2-x^2)^2}$$

$$\boxed{\frac{du}{dx} = \frac{4a^2x}{(a^2-x^2)^2}}$$

$$① \Longrightarrow \quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \times \frac{4a^2x}{(a^2-x^2)^2}$$

put the value of "u".

$$\frac{dy}{dx} = \frac{1}{\cancelto{1}{2}\sqrt{\frac{a^2+x^2}{a^2-x^2}}} \times \frac{\cancelto{2}{4}a^2x}{(a^2-x^2)^2}$$

$$\frac{dy}{dx} = \frac{2a^2x}{\frac{\sqrt{a^2+x^2}}{(a^2-x^2)^{1/2}} \cdot (a^2-x^2)^2}$$

$$\frac{dy}{dx} = \frac{2a^2x}{\sqrt{a^2+x^2}\,(a^2-x^2)^{2-\frac{1}{2}}}$$

$$\boxed{\frac{dy}{dx} = \frac{2a^2x}{\sqrt{a^2+x^2}\,(a^2-x^2)^{3/2}}} \quad \text{Ans.}$$

# EXERCISE # 2.4

## (Question # 2)

Find $\frac{dy}{dx}$ if:

(i) $3x + 4y + 7 = 0$

Sol.

$$3x + 4y + 7 = 0$$

$$\frac{d}{dx}[3x + 4y + 7] = \frac{d}{dx}[0]$$

$$\frac{d}{dx}(3x) + \frac{d}{dx}(4y) + \frac{d}{dx}(7) = 0$$

$$3\frac{d}{dx}(x) + 4\frac{d}{dx}(y) + 0 = 0$$

$$3(1) + 4\frac{dy}{dx} = 0$$

$$3 + 4\frac{dy}{dx} = 0$$

$$4\frac{dy}{dx} = -3$$

$$\boxed{\frac{dy}{dx} = -\frac{3}{4}}$$ Ans.

(ii) — {Question # 2(ii)} —

$$xy + y^2 = 2 \qquad \frac{dy}{dx} = ?$$

Sol.

$$xy + y^2 = 2$$

$$\frac{d}{dx}\left[xy + y^2\right] = \frac{d}{dx}(2)$$

$$\frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = 0$$

$$\left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] + \left[2y^{2-1}\frac{d}{dx}(y)\right] = 0$$

$$x\frac{dy}{dx} + y(1) + 2y^1\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$$

$$(x + 2y)\frac{dy}{dx} = -y$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x+2y}} \text{ Ans.}$$

## Question # 2 (iii)

$$x^2 - 4xy - 5y = 0 \qquad \frac{dy}{dx} = ?$$

Sol.

$$x^2 - 4xy - 5y = 0$$

$$\frac{d}{dx}(x^2 - 4xy - 5y) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(4xy) - \frac{d}{dx}(5y) = 0$$

$$2x^{2-1}\frac{d}{dx}(x) - 4\frac{d}{dx}(xy) - 5\frac{d}{dx}(y) = 0$$

$$2x^1(1) - 4\left[x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right] - 5\frac{dy}{dx} = 0$$

$$2x - 4\left[x\frac{dy}{dx} + y(1)\right] - 5\frac{dy}{dx} = 0$$

$$2x - 4x\frac{dy}{dx} - 4y - 5\frac{dy}{dx} = 0$$

$$2x - 4y = 4x\frac{dy}{dx} + 5\frac{dy}{dx}$$

$$2(x - 2y) = (4x + 5)\frac{dy}{dx}$$

$$\boxed{\frac{2(x-2y)}{4x+5} = \frac{dy}{dx}} \quad \text{Ans.}$$

## Question # 2(iv)

$$4x^2+2hxy+by^2+2gx+2fy+c=0$$

$$\frac{dy}{dx}=?$$

Sol:

$$4x^2+2hxy+by^2+2gx+2fy+c=0$$

$$\frac{d}{dx}\left(4x^2+2hxy+by^2+2gx+2fy+c\right)=\frac{d}{dx}(0)$$

$$\frac{d}{dx}(4x^2)+\frac{d}{dx}(2hxy)+\frac{d}{dx}(by^2)$$
$$+\frac{d}{dx}(2gx)+\frac{d}{dx}(2fy)+\frac{d}{dx}(c)=0$$

$$4\frac{d}{dx}(x^2)+2h\frac{d}{dx}(xy)+b\frac{d}{dx}(y^2)$$
$$+2g\frac{d}{dx}(x)+2f\frac{d}{dx}(y)+0=0$$

$$4(2x)+2h\left[x\frac{d}{dx}(y)+y\frac{d}{dx}(x)\right]+b\left[2y^{2-1}\frac{d}{dx}(y)\right]$$
$$+2g(1)+2f\frac{dy}{dx}=0$$

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$$8x + 2h\left(x\frac{dy}{dx} + y(1)\right) + b\left(2y\frac{dy}{dx}\right)$$

$$+ 2g + 2f\frac{dy}{dx} = 0$$

$$8x + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

$$2hx\frac{dy}{dx} + 2by\frac{dy}{dx} + 2f\frac{dy}{dx} = -8x - 2hy - 2g$$

$$(2hx + 2by + 2f)\frac{dy}{dx} = -2(4x + hy + g)$$

$$\frac{dy}{dx} = \frac{-2(4x + hy + g)}{(2hx + 2by + 2f)}$$

$$\frac{dy}{dx} = -\frac{\cancel{2}(4x + hy + g)}{\cancel{2}(hx + by + f)}$$

$$\boxed{\frac{dy}{dx} = \frac{-(4x + hy + g)}{(hx + by + f)}}$$

Ans.

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## Question#2(V)

$$x\sqrt{1+y}+y\sqrt{1+x}=0 \qquad \frac{dy}{dx}=?$$

Sol.

$$x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$\frac{d}{dx}\left(x\sqrt{1+y}+y\sqrt{1+x}\right]=\frac{d}{dx}[0]$$

$$\frac{d}{dx}[x\sqrt{1+y}]+\frac{d}{dx}[y\sqrt{1+x}]=0$$

$$\left[x\frac{d}{dx}\sqrt{1+y}+\sqrt{1+y}\frac{d}{dx}(x)\right]+\left[y\frac{d}{dx}\sqrt{1+x}+\sqrt{1+x}\frac{d}{dx}(y)\right]=0$$

$$\left[x\frac{1}{2\sqrt{1+y}}\frac{d}{dx}(1+y)+\sqrt{1+y}\,(1)\right]$$

$$+\left[y\frac{1}{2\sqrt{1+x}}\frac{d}{dx}(1+x)+\sqrt{1+x}\frac{dy}{dx}\right]=0$$

$$\left[\frac{x}{2\sqrt{1+y}}\left(0+\frac{dy}{dx}\right)+\sqrt{1+y}\right]+\left[\frac{y}{2\sqrt{1+x}}(0+1)+\sqrt{1+x}\frac{dy}{dx}\right]=0$$

$$\frac{x}{2\sqrt{1+y}}\frac{dy}{dx}+\sqrt{1+y}+\frac{y}{2\sqrt{1+x}}+\sqrt{1+x}\frac{dy}{dx}=0$$

$$\frac{x}{2\sqrt{1+y}}\frac{dy}{dx} + \sqrt{1+x}\,\frac{dy}{dx} = -\sqrt{1+y} - \frac{y}{2\sqrt{1+x}}$$

$$\left(\frac{x}{2\sqrt{1+y}} + \frac{\sqrt{1+x}}{1}\right)\frac{dy}{dx} = -\frac{\sqrt{1+y}}{1} - \frac{y}{2\sqrt{1+x}}$$

$$\left(\frac{x + 2\sqrt{1+y}\sqrt{1+x}}{2\sqrt{1+y}}\right)\frac{dy}{dx} = \frac{-\sqrt{1+y}\,(2\sqrt{1+x}) - y}{2\sqrt{1+x}}$$

$$\frac{dy}{dx} = \frac{-\left(2\sqrt{1+y}\sqrt{1+x} + y\right)}{\cancel{2}\sqrt{1+x}} \times \frac{(\cancel{2}\sqrt{1+y})}{(x + 2\sqrt{1+y}\sqrt{1+x})}$$

$$\frac{dy}{dx} = \frac{-2(\sqrt{1+y})^2\sqrt{1+x} - y\sqrt{1+y}}{x\sqrt{1+x} + 2\sqrt{1+y}(\sqrt{1+x})^2}$$

$$\frac{dy}{dx} = \frac{\left(-2\sqrt{(1+y)}\sqrt{1+x} - y\right)\sqrt{1+y}}{x\sqrt{1+x} + 2\sqrt{1+y}\sqrt{(1+x)}\sqrt{1+x}}$$

$$\boxed{\frac{dy}{dx} = \frac{-\left(2\sqrt{1+y}\sqrt{1+x} + y\right)\sqrt{1+y}}{\left(x + 2\sqrt{1+y}\sqrt{1+x}\right)\sqrt{1+x}}}$$ Ans.

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## Question #2(vi)

$$y(x^2-1) = x\sqrt{x^2+4} \qquad \frac{dy}{dx} = ?$$

Sol.

$$y(x^2-1) = x\sqrt{x^2+4}$$

$$\frac{d}{dx}\left[y(x^2-1)\right] = \frac{d}{dx}\left[x\sqrt{x^2+4}\right]$$

$$y\frac{d}{dx}[x^2-1] + (x^2-1)\frac{d}{dx}(y) = x\frac{d}{dx}\sqrt{x^2+4} + \sqrt{x^2+4}\frac{d}{dx}(x)$$

$$y(2x-0) + (x^2-1)\frac{dy}{dx} = x\frac{1}{2\sqrt{x^2+4}}\frac{d}{dx}(x^2+4) + \sqrt{x^2+4}\,(1)$$

$$2xy + (x^2-1)\frac{dy}{dx} = \frac{x}{2\sqrt{x^2+4}}[2x+0] + \sqrt{x^2+4}$$

$$2xy + (x^2-1)\frac{dy}{dx} = \frac{\cancel{2}x^2}{\cancel{2}\sqrt{x^2+4}} + \sqrt{x^2+4}$$

$$(x^2-1)\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+4}} + \sqrt{x^2+4} - 2xy$$

$$(x^2-1)\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+4}} + \frac{\sqrt{x^2+4}}{1} - \frac{2xy}{1}$$

$$(x^2-1)\frac{dy}{dx} = \frac{x^2 + \left(\sqrt{x^2+4}\right)^2 - 2xy\sqrt{x^2+4}}{\sqrt{x^2+4}}$$

$$(x^2-1)\frac{dy}{dx} = \frac{x^2+x^2+4-2xy\sqrt{x^2+4}}{\sqrt{x^2+4}}$$

$$\boxed{\frac{dy}{dx} = \frac{2x^2+4-2xy\sqrt{x^2+4}}{(x^2-1)\sqrt{x^2+4}}}$$

Ans.

put the value of y

given $\quad y(x^2-1) = x\sqrt{x^2+4}$

$$y = \frac{x\sqrt{x^2+4}}{x^2-1}$$

$$\frac{dy}{dx} = \frac{\frac{2x^2}{1}+\frac{4}{1}-2x\left(\frac{x\sqrt{x^2+4}}{x^2-1}\right)\sqrt{x^2+4}}{(x^2-1)\sqrt{x^2+4}}$$

$$\frac{dy}{dx} = \frac{\frac{2x^2(x^2-1)+4(x^2-1)-2x^2\left(\sqrt{x^2+4}\right)^2}{(x^2-1)}}{(x^2-1)\sqrt{x^2+4}}$$

$$\frac{dy}{dx} = \frac{\cancel{2x^4}-2x^2+4x^2-4-\cancel{2x^4}-8x^2}{(x^2-1)(x^2-1)\sqrt{x^2+4}}$$

$$\frac{dy}{dx} = \frac{-6x^2-4}{(x^2-1)^2\sqrt{x^2+4}} = \boxed{\frac{-2(3x^2+2)}{(x^2-1)^2\sqrt{x^2+4}}}$$

Ans. Book.

## Question # 3(i)

Find $\frac{dy}{dx}$ of the following parametric function.

$$x = \theta + \frac{1}{\theta} \quad \text{and} \quad y = \theta + 1$$

Sol.

$$\frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} \longrightarrow (1)$$

$$x = \theta + \frac{1}{\theta}$$

$$\frac{d}{d\theta}[x] = \frac{d}{d\theta}\left[\theta + \frac{1}{\theta}\right]$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\theta) + \frac{d}{d\theta}\left(\frac{1}{\theta}\right)$$

$$\frac{dx}{d\theta} = 1 + \frac{d}{d\theta}(\theta^{-1})$$

$$\frac{dx}{d\theta} = 1 + (-1)\theta^{-1-1}\frac{d}{d\theta}(\theta)$$

$$\frac{dx}{d\theta} = 1 - 1\theta^{-2}(1)$$

$$y = \theta + 1$$

$$\frac{d}{d\theta}[y] = \frac{d}{d\theta}[\theta + 1]$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}[\theta] + \frac{d}{d\theta}[1]$$

$$\frac{dy}{d\theta} = 1 + 0$$

$$\boxed{\frac{dy}{d\theta} = 1}$$

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$$\frac{dx}{d\theta} = 1 - \frac{1}{\theta^2}$$

$$\frac{dx}{d\theta} = \frac{\theta^2 - 1}{\theta^2}$$

Reciprocal

$$\boxed{\frac{d\theta}{dx} = \frac{\theta^2}{\theta^2 - 1}}$$

$$(1) \Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = (1) \times \left(\frac{\theta^2}{\theta^2 - 1}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{\theta^2}{\theta^2 - 1}}$$ Ans.

## Question # 3(ii)

$$\frac{dy}{dx} = ?$$

$$x = \frac{a(1-t^2)}{1+t^2}, \quad y = \frac{2bt}{1+t^2}$$

Sol.

$$\frac{dy}{dx} = ?$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \longrightarrow (1)$$

First we find $\frac{dy}{dt}$

$$y = \frac{2bt}{1+t^2}$$

$$\frac{d}{dt}[y] = \frac{d}{dt}\left[\frac{2bt}{1+t^2}\right]$$

$$\frac{dy}{dt} = 2b\frac{d}{dt}\left[\frac{t}{1+t^2}\right]$$

$$\frac{dy}{dt} = 2b\left[\frac{(1+t^2)\frac{d}{dt}(t) - t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$\frac{dy}{dt} = 2b\left[\frac{(1+t^2)(1) - t(0+2t)}{(1+t^2)^2}\right]$$

$$\frac{dy}{dt} = 2b\left[\frac{1+t^2-2t^2}{(1+t^2)^2}\right]$$

$$\boxed{\frac{dy}{dt} = 2b\left[\frac{1-t^2}{(1+t^2)^2}\right]}$$

Now find $\frac{dt}{dx}$

$$x = \frac{a(1-t^2)}{1+t^2}$$

$$\frac{d}{dt}[x] = \frac{d}{dt}\left[\frac{a(1-t^2)}{1+t^2}\right]$$

$$\frac{dx}{dt} = a\frac{d}{dt}\left[\frac{1-t^2}{1+t^2}\right]$$

$$\frac{dx}{dt} = a\left[\frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}\right]$$

$$\frac{dx}{dt} = a\left[\frac{(1+t^2)(0-2t) - (1-t^2)(0+2t)}{(1+t^2)^2}\right]$$

$$\frac{dx}{dt} = a\left[\frac{-2t - \cancel{2t^3} - 2t + \cancel{2t^3}}{(1+t^2)^2}\right]$$

$$\frac{dx}{dt} = \frac{-4at}{(1+t^2)^2}$$

Taking Reciprocal.

$$\boxed{\frac{dt}{dx} = \frac{(1+t^2)^2}{-4at}}$$

1 $\Rightarrow$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \cancel{2}b\left[\frac{1+t^2}{\cancel{(1+t^2)^2}}\right] \times \frac{\cancel{(1+t^2)^2}}{-\cancel{4}at}$$

$$\boxed{\frac{dy}{dx} = \frac{b}{-2at}(1+t^2)}$$ Ans.